This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

MATHEMATICAL BOOKS.

Works by Barnard Smith, M.A.

Arithmetic and Algebra in their Principles and Applications; **with numerous systematically arranged Examples taken from the Cambridge Examination Papers. Ninth Edition. Crown 8vo., cloth, 10*s.* 6*d.***

Arithmetic for the use of Schools. New Edition. Crown 8vo., **cloth, 4*s.* 6*d.***

A Key to the Arithmetic for Schools. Fourth Edition. Crown **8vo., cloth, 8*s.* 6*d.***

Exercises in Arithmetic. Crown 8vo. 2*s.*; or, with Answers, **2*s.* 6*d.* Also, sold separately, in Two Parts, 1*s.* each. Answers 6*d.***

The School Class-Book of Arithmetic. Parts I. and II., 18mo., **limp cloth, price 10*d.* each. Part III. 1*s.*; or Three Parts in One Volume, price 3*s.* 18mo., cloth (forming one of Macmillan's Elementary School Class-Books).**

Key to Class-Book of Arithmetic. Complete, 18mo., cloth, price **6*s.* 6*d.*; or separately, Parts I. II. and III. 2*s.* 6*d.* each.**

A Shilling Book of Arithmetic, for National and Elementary **Schools. Part I. 2*d.* Part II. 3*d.* Part III. 7*d.* In One Vol. 1*s.* or, with Answers, cloth, 1*s.* 6*d.***

Works by Isaac Todhunter, M.A. F.R.S.

Euclid for Colleges and Schools. Second Edition. 18mo., bound **in cloth, 3*s.* 6*d.***

Algebra for Beginners. With numerous Examples. 18mo., bound **in cloth, 2*s.* 6*d.***

A Treatise on the Differential Calculus. With numerous Examples. **Fourth Edition.** Crown 8vo., cloth, 10*s.* 6*d.*

A Treatise on the Integral Calculus. Second Edition. With **numerous Examples. Crown 8vo., cloth 10*s.* 6*d.***

A Treatise on the Analytical Statics. With numerous Examples. **Second Edition. Crown 8vo., cloth, 10*s.* 6*d.***

A Treatise on the Conic Sections. With numerous Examples. **Third Edition. Crown 8vo., cloth, 7*s.* 6*d.***

Algebra for the use of Colleges and Schools. Third Edition. **Crown 8vo., cloth, 7*s.* 6*d.***

Plane Trigonometry for Colleges and Schools. Third Edition. **Crown 8vo., cloth, 5*s.***

A Treatise on Spherical Trigonometry for the use of Colleges and **Schools. Second Edition. Crown 8vo., cloth, 4*s.* 6*d.***

Critical History of the Progress of the Calculus of Variations **during the Nineteenth Century. 8vo. cloth, 12*s.***

Examples of Analytical Geometry of Three Dimensions. Second **Edition. Crown 8vo., cloth, 4*s.***

A Treatise on the Theory of Equations. Crown 8vo., cloth, price **7*s.* 6*d.***

MACMILLAN AND CO. LONDON.

MATHEMATICAL BOOKS.

An Elementary Treatise on Differential Equations. By the late GEORGE BOOLE, D.C.L., F.R.S., Professor of Mathematics in the Queen's University, Ireland. A new Edition, revised by I. TODHUNTER, F.R.S. Crown 8vo., cloth, 14*s.*
Supplementary Volume. Crown 8vo., cloth, 8*s.* 6*d.*

A Treatise on the Dynamics of a Particle. With numerous Examples. By PETER GUTHRIE TAIT, M.A., late Fellow of St Peter's College, Professor of Natural Philosophy in the University of Edinburgh; and the late WILLIAM JOHN STEELE, B.A., Fellow of St Peter's College. Second Edition. Price 10*s.* 6*d.*

The Cambridge Course of Elementary Natural Philosophy; being the Propositions in Mechanics and Hydrostatics in which those Persons who are not Candidates for Honours are examined for the Degree of B.A. Originally compiled by J. C. SNOWBALL, M.A., late Fellow of St John's College, Cambridge. New Edition, revised and enlarged, and adapted for the Middle-Class Examinations. By THOMAS LUND, B.D., late Fellow and Lecturer of St John's College, Editor of Wood's "Algebra." Fifth Edition. Crown 8vo., cloth, 5*s.*

The Elements of Plane and Spherical Trigonometry. By J. C. SNOWBALL, M.A., Fellow of St John's College, Cambridge. Tenth Edition. Crown 8vo., cloth, 7*s.* 6*d.*

An Elementary Treatise on Plane Trigonometry. With a numerous Collection of Examples. By R. D. BEASLEY, M.A., late Fellow of St John's College, Cambridge, Head Master of Grantham Grammar-school. Second Edition. Crown 8vo., cloth, 3*s.* 6*d.*

A Treatise on Elementary Mechanics. For the use of the Junior Classes at the University and the Higher Classes in Schools. With a Collection of Examples. By S. PARKINSON, D.D., President of St John's College, Cambridge. Third Edition, revised. Crown 8vo., cloth, 9*s.* 6*d.*

Elementary Hydrostatics. By J. B. PHEAR, M.A., Fellow of Clare College, Cambridge. Third Edition, accompanied by numerous Examples, with the Solutions. Crown 8vo., cloth, 5*s.* 6*d.*

A Geometrical Treatise on Conic Sections, with Copious Examples from the Cambridge Senate-House Papers. By W. H. DREW, M.A., of St John's College, Cambridge. Third Edition. Crown 8vo., cloth, 4*s.* 6*d.*

Solution to Problems contained in Mr Drew's "Treatise on Conic Sections." Crown 8vo., cloth, 4*s.* 6*d.*

Geometrical Conics, including Anharmonic Ratio and Projection. With numerous Examples. By C. TAYLOR, B.A., Scholar of St John's College, Cambridge. Crown 8vo., cloth, 7*s.* 6*d.*

An Elementary Treatise on Conic Sections and Algebraic Geometry. With a numerous collection of Easy Examples progressively arranged, especially designed for the use of Schools and Beginners. By G. HALE PUCKLE, M.A., Principal of Windermere College. Second Edition, enlarged and improved. Crown 8vo. cloth, 7*s.* 6*d.*

A Treatise on the Calculus of Finite Differences. By the late GEORGE BOOLE, D.C.L., Professor of Mathematics in the Queen's University, Ireland. Crown 8vo., cloth, 10*s.* 6*d.*

MACMILLAN AND CO. LONDON.

ELEMENTARY TEST QUESTIONS

IN

PURE AND MIXED MATHEMATICS,

WITH ANSWERS.

Cambridge:
PRINTED BY C. J. CLAY, M.A.
AT THE UNIVERSITY PRESS.

A COLLECTION

OF

ELEMENTARY TEST QUESTIONS

IN

PURE AND MIXED MATHEMATICS

WITH ANSWERS.

AND

APPENDICES
ON SYNTHETIC DIVISION
AND ON
THE SOLUTION OF NUMERICAL EQUATIONS
BY HORNER'S METHOD.

BY

JAMES R. CHRISTIE, F.R.S., F.R.A.S.
LATE FIRST MATHEMATICAL MASTER AT THE
ROYAL MILITARY ACADEMY, WOOLWICH.

London and Cambridge:
MACMILLAN AND CO.
1866

1207 c. 1.

BIBLIOTHECA · BODLEIANA ·

PREFACE.

THE Series of Mathematical Exercises here offered to the public is collected from those which the author has, from time to time, proposed for solution by his pupils during a long career at the Royal Military Academy: they are, in the main, original; and having well fulfilled the purpose for which they were first framed, it is hoped that they may be made still more widely useful.

The aim in proposing them was not so much to set before the pupil intricate and puzzling questions, as to determine, from the form of solution, whether his mind had fairly grasped the fundamental principles of the particular subject, and was capable of applying those principles: so that a student who finds that he is able to solve the larger portion of these exercises, may consider that he is thoroughly well grounded in the elementary principles of Pure and Mixed Mathematics.

It has not been considered desirable to place the questions strictly in order of presumed difficulty; first, because, on such a point, no two opinions would always agree; and secondly, because a student should be exercised to pass from one style of solution to another with as little effort as his mental capacity will allow.

It has been thought advisable to place the answers at the end of the volume, in a form which the author hopes will preclude the loss of time which such an arrangement usually entails. For this purpose the numbering of the questions is continuous throughout; and at the head of each page of answers are placed the index numbers of the solutions which commence and terminate the page.

Increasing attention is now being paid to the Method of Synthetic Division and to the Solution of Numerical Equations by Horner's Method, neither of which processes is given, in a form comprehensible by any but advanced students, in any of the treatises of the day on Elementary Algebra. It has therefore been found necessary to give a short and very practical outline of the principles upon which these methods are based; and the author is not without hope that his Second Appendix may lead to the introduction of the general numerical solution of equations in its natural position in every course of Elementary Algebra, immediately after the subject of Quadratic Equations, where it would tend to develope in the mind of the student a knowledge of the nature of algebraic functions, which would be of the utmost service throughout his subsequent course.

For the single purpose of the determination of the numerical roots of an equation, a vast amount of unnecessary and somewhat intricate investigation of the properties of equations is always entered into; and it is a real boon to the learner to clear away all redundancies and reduce the theory of solution to its simplest elements. This has been done in Appendix II. by basing the determination of a root, first upon the law of signs, for hypothetical position, and, ultimately, upon the test-fact of its satisfying the condition implied by the equation; this fact being shown in actual substitution by a process of synthetic division originated by

Horner for this very purpose. The simple logic of solution then becomes this: "If the roots be all real, we have found by the law of signs that one of them lies between *a* and *b*; treating this assumed root by Horner's process of developement, we find that the developed root, $a +$ &c., satisfies the equation more and more nearly according to the extent of its developement. There can be no doubt therefore that this value is a root, whether the remaining roots contain among them imaginary forms or not." By this means the mass of difficulties involved in the various theorems of Newton, Sturm, Fourier and others, regarding the limits of the roots, may be safely ignored until the learner is better prepared to undertake their study.

The method of dealing with approximately equal roots by a system of reciprocal equations, was first published by the author in 1842, and he still believes it to be the most efficient algorithm yet proposed for application to those delicate cases in which two or more roots are identical to more than one or two places of decimals, supposing this fact to be unknown to the worker.

The exercises in Practical Mechanics will, it is hoped, assist in calling attention to this important, though much neglected subject.

To prevent the necessity of reference to other books on mere matters of memory, a few pages of the more useful tables and elementary formulæ in all the subjects comprised in the present work have been given in a form which, while perfectly intelligible to students who have really studied the particular subjects, will probably be of little or no use to those who have merely acquired a smattering.

The collection of Examination Papers, also from original sources, will be useful to Instructors in ascertaining progress from time to time; and, with this view, the answers to them have been omitted.

In selecting and arranging questions from a mass of papers, spread over many years, it is very difficult to avoid the insertion of duplicates: although every effort has been made to do this, the author regrets that he has not been, in all cases, successful.

JAMES R. CHRISTIE.

9, Arundel Gardens,
Notting Hill,
9 *Feb.* 1866.

CONTENTS.

PAGE

ARITHMETIC 1
Algebra 10
Equations 19
Problems producing Equations 24
Progressions 31
Horner's Solution of Equations 37
Continued Fractions 48
Indeterminate Coefficients 49
Binomial Theorem 51
Permutations and Combinations 53
Piles of Shot 55
Logarithms 60
Geometrical Deductions 67
Application of Algebra to Geometry 70
Descriptive Geometry 72
Plane Trigonometry 73
Mensuration 87
Spherical Trigonometry 98
Practical Astronomy 103
Coordinate Geometry 111
Statics 121
Practical Mechanics 153
Dynamics 159
Hydrostatics 180

PAGE
Differential Calculus 187
Integral Calculus 193
Moments of Inertia 198
Centre of Oscillation 199
Motion in a Resisting Medium 200
Examination Papers 203
Answers 303
Tables and Formulæ 354
Appendix I. 375
Appendix II. 381

MATHEMATICAL EXAMINATION QUESTIONS.

I.

ARITHMETIC.

1. Supposing that a person can count two hundred in a minute, and that after counting incessantly for 30 years he dies, and his son goes on counting for 30 years and then dies, and so on; how many generations must elapse before one billion be counted?

2. A party of five persons agree to travel abroad, and they find that, after they have been absent from England a fortnight, they have expended 1000 francs; two of them are then obliged to return home; how long can the other three continue their tour with their remaining fund of 1500 francs?.

3. An estate is to be divided between *A*, *B* and *C*, in the ratios of 1 : 2 : 3, subject to the proviso that each of them pay to *D* one-eleventh of the whole; what would *D* receive, supposing *A*'s share to be £550?.

4. If a watch be two minutes and a half too fast by the clock of the R. M. Academy when the bell rings at 4 P.M. on Wednesday, what will be the time by that watch when the bell rings at noon on Monday? the daily rate of the watch being – 20 sec., and that of the clock + 18 sec.

5. If 6580 shot of 24 lbs. cost £725, how many 42 lb. shot can be purchased for £2100 when iron is five per cent. dearer?

6. If $30\frac{1}{2}$ miles of iron rails cost £15580 when iron was at £10. 5*s.* 6*d.* per ton, what would be the expense of $157\frac{5}{8}$ miles of the same rails when iron is at £12. 4*s.* 2*d.* per ton?

7. If a battery of 6 guns, firing three rounds in ten minutes, will breach a certain work in 60 hours, how many guns must be employed for the same purpose, firing two rounds in five minutes, in order that the breach may be made practicable in 15 hours?

8. *A* can run at the rate of eight miles an hour and *B* at the rate of seven and a half miles an hour; what is the greatest number of yards start that *A* may give *B* so as to beat him in a race of 440 yards?

9. If two 32-pounders can render a breach practicable in fifty hours, in how many hours can three 24-pounders, two 18-pounders, and one 12-pounder render practicable a breach requiring double the quantity of battering, supposing the effects of the different kinds of shot to be as 6, 5, 2 and 1?

10. A garrison of 1000 men was victualled for 30 days, but after 10 days it was reinforced, and the provisions were, in consequence, exhausted in 5 days: what was the number of men in the reinforcement?

11. A wall 1236 yards in length was to have been built by 60 men in 21 days, but at the end of 15 days it was found that only 824 yards had been completed; how many extra men must be employed in order that the work may be completed in the given time?

12. The proportions of sulphur, charcoal and nitre, in the composition of gunpowder being as 2, 3 and 15 respectively; what is the value of 5460 lbs. of powder, independent of the expense of manufacture, when sulphur is £1. 4*s.* 6*d.* per cwt., charcoal £1. 15*s.* 2*d.* per cwt., and nitre £1. 16*s.* 8*d.* per cwt.?

13. What is the value of five pounds Troy of an alloy of silver and gold, in which the weight of the gold is $\frac{3}{4}$ the weight of the silver, gold being £4. 5*s.* and silver 6*s.* per oz.?

14. Gunpowder being composed of 75 per cent. of nitre, 12·5 of charcoal, and 12·5 of sulphur, how much of each of these substances is there in 5 tons of powder?

15. If two ounces of gun-cotton produce the same effect as five ounces of gunpowder, and the spaces occupied by the same weight of each be as 10 to 3 (the cotton being the more bulky); what must be the length of a musketry cartridge, filled with cotton, which is to be used instead of a gunpowder cartridge two inches long?

16. If 25 labourers can dig a trench 220 yards long, 3 feet 4 inches wide, and 2 feet 6 inches deep, in 32 days of 9 hours each; how many would it require to dig a trench half-a-mile long, 2 feet 4 inches deep, and 3 feet 6 inches wide, in 36 days of 8 hours each?

17. Supposing the rates of marching of two columns of infantry to be as 4 to 3, and the one to be three miles in advance of the other and marching at the rate of 2½ miles per hour; in what time will the column in the rear overtake the other?

18. If five bricklayers can lay 45 bricks in five minutes, and the bricks made use of be 10 inches long, 4 inches broad, and 3½ inches thick; how long (with such bricks) will it take 8 bricklayers to build a wall which would contain 95000 bricks 8 inches long, 5 inches broad, and 3 inches thick; the day being 12 hours long?

19. Six thousand of a certain kind of shot weigh 64 tons 5 cwt. 2 qrs. 24 lbs., how many 9 lb. shot will weigh as much as 3810 such shot?

20. What is the time when the hour and minute hands of a watch are exactly together between 9 and 10 o'clock?

21. Find the time, between three and four o'clock, when the hour and minute hands of a watch are (1) coincident, (2) in exactly opposite directions, and (3) at right angles to each other.

22. By what part of its present value must the farthing be increased or diminished if a decimal coinage were introduced in which 1 sovereign = 10 royals, 1 royal = 10 groats, and 1 groat = 10 farthings; the value of the sovereign remaining unaltered?

23. The French Mètre being the ten-millionth part of the distance, at the sea-level, from the pole to the equator of the earth, find the length of an entire meridian in miles, the mètre being 39·370089 English inches.

24. The French *Gramme* is the weight of a cubic *Centimètre* of water, and the superior denominations, ascending decimally, are the *Décagramme*, *Hectogramme*, *Kilogramme* and *Myriagramme*; find the difference of weight of a French gun weighing 263 Myriagrammes and an English gun weighing 50 cwt. 45 lbs., supposing a cubic foot of water to weigh 1000 oz., and the centimètre to be ·3937 of an English inch.

25. A reservoir of water, which will hold 2641500 gallons, is to be filled by a pipe which discharges 1000 French litres per minute; how long will it take to fill it?

26. If 50 sappers can dig a trench 500 yards long in 25 days; how long will it take 10 sappers to dig a trench 75 hectomètres long, of the same depth, but twice the width of the former?

27. It being required to re-cast a quantity of captured shot and guns, weighing 105730 kilogrammes, into the form of 13-inch shells weighing 196 lbs. each; what number of shells will be obtained, allowing 6 per cent. for waste; the kilogramme being 2·2047 of the pound avoirdupois?

28. If £1 sterling be worth 25 francs, 60 centimes; and also worth 6 thalers, 20 silber groschen; how many francs and centimes is a thaler worth, when 1 thaler is equal to 30 silber groschen, and 1 franc equal to a 100 centimes?

29. The ditch of a fortress can be filled by one sluice alone in 12 hours, and by another in 15 hours: in what time will it be filled by both open together?

30. A cistern which holds 820 gallons is filled in 20 minutes by three pipes, one of which conveys 10 gallons more, and the other 5 gallons less per minute than the third; how much water flows through each pipe per minute?

31. The imperial gallon contains 277·274 cubic inches, and the hectolitre contains 6102·379 cubic inches; determine the value of the litre in terms of the quart.

32. Reduce the fraction $\dfrac{\frac{5}{6}+\frac{1}{2}\left(\frac{2}{3}-\frac{1}{2}\right)-\frac{6}{7}\left(\frac{4}{5}+\frac{1}{9}\right)}{\frac{3}{4}\left(\frac{7}{8}-\frac{1}{2}\right)-\frac{1}{3}\left(\frac{1}{7}-\frac{1}{10}\right)}$ to its simplest form.

33. Reduce $\dfrac{\frac{1}{2}\cdot\frac{5}{6}+\frac{2}{3}\cdot\frac{7}{8}-\frac{2}{5}\left(\frac{1}{4}-\frac{5}{7}\right)}{\frac{3}{7}\left(\frac{4}{3}-\frac{1}{2}\right)-\frac{1}{9}\left(\frac{1}{4}-\frac{3}{28}\right)}$ to its lowest terms.

34. Find the value of $\dfrac{\frac{2247}{1017}}{\frac{301}{339}}\times\dfrac{\frac{774}{615}}{\frac{1926}{565}}$ of a £ as the decimal of a £50 note.

35. Express $\dfrac{3\frac{2}{5}}{2\frac{1}{3}-\dfrac{4}{3\frac{1}{3}}}$ cwt. as the decimal of a ton.

36. Reduce $\dfrac{\dfrac{3}{5\frac{1}{7}}}{2\frac{3}{4}+\dfrac{7}{6\frac{1}{8}-\dfrac{2}{2\frac{2}{7}}}}$ of a £ to the fraction of a shilling.

37. Add together $\frac{3}{4}$ of a guinea, $\frac{3}{32}$ of a £, $\frac{3}{5}$ of 7*s.* 6*d.*, and subtract $\frac{3}{4}$ of 2*d.* from the result..

38. Reduce $\frac{4}{15}$ of 7*s.* 0*d.* 1½*f.* to the fraction of a half-crown.

39. What part of five shillings is five-sixths of sevenpence-halfpenny?

40. What part of 4*s.* 6*d.* is $\left(\frac{1}{15} - \frac{1}{120} + \frac{1}{40} + \frac{1}{24}\right)$ of £1. 4*s.*?

41. What part of 5*s.* 4*d.* is two-sevenths of 17*s.* 6*d.*?

42. Reduce 4*s.* 3½*d.* to the decimal of a crown; and to the fraction of a Napoleon of 20 francs; the franc being $\frac{1}{25}$th of a £.

43. What is the value of 100 shells, each weighing 196 lbs., when cast iron is 5*s.* 6½*d.* per cwt., and the expense of manufacture 1*s.* 3*d.* per shell?

44. A person by selling a certain horse lost $\frac{3}{7}$ of $\frac{42}{6}$ of $\frac{1}{18}$ of £60, and by selling another he gained $\frac{1}{4}$ of $\frac{28}{3}$ of $\frac{18}{7}$ of £2, when he found that the price received for both together was $\frac{15}{6}$ of $\frac{24}{20}$ of $\frac{42}{49}$ of $\frac{14}{6}$ of £17; what did he give for the two?

45. From an ammunition waggon containing 27 cwt. of ammunition $\frac{3\frac{7}{11}}{4\frac{2}{7}}$ of $\frac{10\frac{5}{7}}{7\frac{1}{2}}$ of $\frac{77}{540}$ of the whole was delivered to Battery *A*, and $\frac{3}{5}$ of the remainder to Battery *B*; what quantity was delivered to each?

46. If $\left(\frac{1}{2} + \frac{2}{3} - \frac{1}{6} + \frac{3}{8} - \frac{1}{12}\right)$ of an estate be worth £62000, what will be the value of $1\frac{1}{12}$ of $\frac{3}{8}$ of $\frac{25}{26}$ of $\frac{64}{75}$ of it?

47. What part of $\frac{5}{6}$ of 17 shillings is $\frac{2}{3}$ of $\frac{7}{5}$ of $\frac{\frac{1}{4}}{\frac{3}{8}}$ of 16*s.* 8*d.*?

48. If $\frac{3}{5}$ of $\frac{2}{3}$ of $\frac{5}{4}$ of a ton of coals cost $\frac{1}{2}$ of $\dfrac{\frac{14}{15}}{\frac{7}{10}}$ of a £, what is the price per cwt.?

49. If $\frac{4}{17}$ of $\frac{3}{11}$ of $\frac{2}{9}$ of a mass of metal weighed $\frac{400}{33}$ lb., what was the weight of $\dfrac{1\frac{7}{8}}{4\frac{1}{4}}$ of $\frac{2}{5}$ of the same mass?

50. Owning $\frac{4}{17}$ of a ship, I sold $\frac{3}{11}$ of $\frac{2}{9}$ of my share for £$\frac{400}{33}$; what was the value of $\dfrac{1\frac{7}{8}}{4\frac{1}{4}}$ of $\frac{2}{5}$ of the vessel at the same rate?

51. Multiply 162·5473 by 8726·47231, contracting the operation to four places of decimals.

52. Multiply 1854·362 by ·000087931, contracting to six places of decimals.

53. Multiply ·0073654281 by 37584·26, contracting the work to six places of decimals.

54. Multiply 13·50629 by ·0036472, contracting to six places of decimals; and divide the second by the first, contracting to eight places of decimals.

55. Divide 15·63214725 by ·0057123, contracting to three places of decimals; and multiply 730·6581 by ·08652, contracting also to three places of decimals.

56. Divide 634·7538292 by ·0657391, contracting to three places of decimals; and multiply the quotient by the divisor, contracting to four places of decimals.

57. Divide 6·38572164 by ·00752681, contracting the work so that the quotient may contain four places of decimals.

58. What place in the decimal scale will the first figure of the quotient of ·0003279634 by 286·3471 hold? State how the result is obtained.

59. Divide ·0073628439 by ·000265847, contracting the work to five places of decimals; and then multiply the quotient by the dividend, contracting the work to six places of decimals.

60. Divide ·00034984 by 37627·15, giving the quotient to twelve places of decimals.

61. Find the value of the recurring decimal $2{\cdot}54\dot{3}1\dot{2}$, &c.

62. Find the vulgar fraction equivalent to the recurring decimal ${\cdot}9\dot{2}4\dot{3}$.

63. Find the values of ·13888 &c. of a shilling; ${\cdot}2\dot{3}$ of a £; and ${\cdot}\dot{0}\dot{4}$ of a yard.

64. Divide $2{\cdot}05\dot{0}\dot{5}$ by $31{\cdot}\dot{2}$; and show the correctness of the result by reducing it and each of the above recurring decimals to its equivalent fraction.

65. Divide the recurring decimal ${\cdot}0\dot{3}5\dot{7}$ by the recurring decimal $25{\cdot}\dot{8}\dot{4}$, giving the quotient to eight places of decimals.

66. Reduce each of the above decimals to its equivalent fraction; and prove the correctness of the division by means of these fractions.

67. Extract the square root and also the cube root of 534·267351, giving the root in both cases to five places of decimals.

68. Extract the square root of ·00000475850596.

69. Extract the square root and also the cube root of 2 to six places of decimals.

70. Extract the cube root of 5·76 to five places of decimals.

71. Extract the cube root of 17 to nine places of decimals.

72. Extract the cube root of 28 to ten places of decimals.

73. Find the cube root of 7 to ten places of decimals.

74. The population of Great Britain in 1851 was 21,121,967, and the increase during the previous half century had been 93·47 per cent. What was the population in 1801?

75. The Income Tax upon a certain income at 1*s.* 4*d.* in the pound amounts to £20; find the sum invested in the 3 per cents. from which the income is derived.

76. The simple interest on a certain sum for 9 months, at 5 per cent. per annum, is £150 less than the simple interest on the same sum for 15 months, at 4 per cent. per annum. Find the principal.

77. An estate purchased at £84. 7*s.* 6*d.* per acre was sold at £90. 14*s.* 0$\frac{3}{4}$*d.* per acre; find the gain per cent.

78. The investment of a certain sum at 3 per cent. produces an income of £501. 15*s.* A portion of this, sufficient, when re-invested at 5 per cent., to produce the same income as the whole formerly did, is called in. Find the amount of income derived from the whole when this re-investment has been effected.

79. Multiply together 11 ft. 4 in. and 4 ft. 7 in., by duodecimals, stating clearly the value of the superficial unit in each denomination.

80. Multiply 3 feet 2 inches 8 lines by 1 foot 8 inches 10 lines, using the duodecimal method; a line being a twelfth part of an inch; and state what each term of the result expresses.

81. Multiply 6 feet 5 inches by 4 feet 9 inches, according to the duodecimal notation, and exhibit the result in the decimal scale, the unit of measure being a square foot.

82. Multiply 17 feet 9 inches 5 lines by 3 feet 2 inches 7 lines by duodecimals, and state the value of the superficial unit in each denomination. Find the result also by decimals, and shew that the two agree.

83. Multiply 5237684 by 4539, in the duodecimal scale, using α to express ten, and β to express eleven.

84. Multiply 1432312 by 31422 in the quinary scale; and transform the former to the denary scale.

85. Express the undenary number $57a4685$ in the octenary scale: where a expresses 10.

86. Convert the senary number 5321425 to the quinary scale.

87. Transform the quinary number 3241342 to the octenary scale.

II.

ALGEBRA.

INTRODUCTORY OPERATIONS.

88. Find the numerical value of

$$3a^3 - 2b\{a^3 - 3c(b^3 - 2a) + c^2\} - 4c(a - b)^2,$$

when $a = 7$, $b = 5$, and $c = 2$..

89. Find the numerical value of

$$\sqrt[2]{\frac{a\sqrt[3]{(b-a)} - \{cd(d-e)(b-d) - f^2\}}{a + f\{c + d - d(a+b)(c-b)\}}}$$

to five places of decimals; when $a = 10$, $b = 2$, $c = 7$, $d = 5$, $e = -3$ and $f = 6$.

90. Find the numerical value of

$$\frac{ab^2 - c\{a-(c+b)\}}{\sqrt{2ab-(a^2-c)+4b}},$$

when $a=6$, $b=2$, and $c=8$.

91. Give the numerical values of the following expressions, when $a=5$, $b=\cdot25$, $c=1$, $d=7$, $e=0$ and $f=\frac{1}{3}$:

$$2c-b\,(1-2d+3f)-5a\,.\,\frac{6b-4c}{3a-2d}\;\ldots\ldots\ldots(1),$$

$$\frac{12\,(a^2-c^2)-5e\sqrt{3b-10d}}{4\,\{(a+c)^2-4ac\}}\;\ldots\ldots\ldots\ldots\ldots\ldots(2),$$

$$\frac{2\,(a+b)\,.\,(5c-6f)\,8\,(bd-a^2df)^2}{5\sqrt{15af}-(3b^2d+9)^2}\;\ldots\ldots\ldots\ldots(3),$$

$$\frac{a^2b^4d-4ef^2c^3+9b^3d^5}{\sqrt[3]{\{5b^2d\,(a^4-6ac)\}}}\,.\,(a^2-bd)\,.\,e\;\ldots\ldots\ldots(4),$$

$$\frac{b^2-6ef+10\,(a+d^3)}{abe\sqrt{\{5af^2\,(c+bd)\}}}\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5).$$

92. Find the numerical value of the expression

$$\sqrt{\frac{a^2+b^2}{a-b}+\frac{a^2-b^2}{a+b}}-\sqrt{\frac{(a+b)^2-c^2}{a+b+c}},$$

when $a=16$, $b=2$, and $c=9$; having previously reduced it to its simplest form.

93. Reduce

$$\frac{(4a+1)^3-64a-48a(2a-1)-1}{12}+3\,.\,a\,.\,(2a-1)$$

to the form

$$\frac{2a}{3}(8a-5)\,(a+1)\,;$$

and give this result in terms of b when $b=a-1$.

94. Find the value of

$$\frac{\sqrt{3+\sqrt{5}}-\sqrt{5-\sqrt{5}}}{\sqrt{3+\sqrt{5}}+\sqrt{5-\sqrt{5}}}$$

to six places of decimals, having previously reduced it to its simplest form.

95. Find the value of

$$\frac{1}{4}\sqrt{24}-\sqrt{\frac{2}{3}}+2\,.\sqrt{3-\sqrt{5}}\,.\sqrt{3+\sqrt{5}}$$

to five places of decimals.

96. Reduce $\sqrt{a}$, $\sqrt[3]{b^3}$, $\sqrt[5]{c^2}$, $\sqrt[4]{3a^5}$ and $d^{-\frac{7}{2}}$ to a common index.

97. Subtract $\frac{x}{y}\sqrt{\frac{2y^3}{x}}$ from $\frac{3x^3}{7y^2}\sqrt{\frac{49y^3}{18x^3}}$, and add the result to $\frac{3y^2}{5x^3}\sqrt{\frac{50x^5}{81y}}$.

98. Prove that $\sqrt[4]{(-1)}+\sqrt{\{-\sqrt{(-1)}\}}=\pm\sqrt{2}$.

99. Prove that

$$\sqrt{[2\{a+\sqrt{(a^2+b^2)}\}]}=\sqrt{\{a+b\sqrt{(-1)}\}}+\sqrt{\{a-b\sqrt{(-1)}\}}.$$

Simplify the following expressions :

100. $\frac{\sqrt{18}}{2\sqrt{2}-\sqrt{3}}$.

101. $\dfrac{\dfrac{x}{xy}+\dfrac{y}{x+y}}{\dfrac{x}{x-y}-\dfrac{y}{x+y}}$.

102. $\frac{1}{3+2\sqrt{2}}$.

103. $\frac{2}{5}\sqrt{\frac{3}{5}}+\frac{7}{8}\sqrt{\frac{20}{147}}-\frac{1}{2}\sqrt{\frac{5}{3}}$.

104. $\frac{\sqrt{(x+y)}+\sqrt{(x-y)}}{\sqrt{(x+y)}-\sqrt{(x-y)}}-\frac{\sqrt{(x+y)}-\sqrt{(x-y)}}{\sqrt{(x+y)}+\sqrt{(x-y)}}$.

105. $\frac{\{x-\sqrt[3]{(xy^2)^4}\}\{x+\sqrt[3]{(xy^2)^2}\}}{x^2+y\sqrt[3]{(x^2y)}-2\sqrt[3]{(x^4y^2)}}$.

106. $\frac{a^4+b^4}{a^4-b^4}+\frac{a^2-b^2}{a^2+b^2}-\frac{a^2+b^2}{a^2-b^2}$.

107. $$\sqrt{(x+y)^2(x^2-y^2)-2(x^2y+xy^2)\frac{2(x^3-y^3)}{x^2+xy+y^2}}.$$

108. $$\sqrt[3]{\frac{x^2\{2(x+3y)-(x+3y)\}+4xy^2}{x^3-y^3-3xy(x-y)}-\frac{y^2}{x^2-2xy+y^2}}.$$

109. $$\frac{x^a+1+x^{3a}+x^{2a}}{x^a-1+x^{3a}-x^{2a}}.$$

110. $$\frac{\sqrt{x}+(a-b)\sqrt{(-1)}}{\sqrt{x}-(a+b)\sqrt{(-1)}}+\frac{\sqrt{x}-(a-b)\sqrt{(-1)}}{\sqrt{x}+(a+b)\sqrt{(-1)}}.$$

111. $$\frac{1}{\sqrt{x^2-1}}\cdot\left\{\frac{x+\sqrt{x^2-1}}{x-\sqrt{x^2-1}}-\frac{x-\sqrt{x^2-1}}{x+\sqrt{x^2-1}}\right\}.$$

112. $$\frac{7x^{\frac{5}{4}}}{9y^{\frac{6}{5}}}\times\frac{3}{2}\sqrt{\frac{9y^{\frac{8}{5}}}{98x^{\frac{3}{2}}}}.$$

113. $$\frac{\sqrt{(x^2+1)}+\sqrt{(x^2-1)}}{\sqrt{(x^2+1)}-\sqrt{(x^2-1)}}+\frac{\sqrt{(x^2+1)}-\sqrt{(x^2-1)}}{\sqrt{(x^2+1)}+\sqrt{(x^2-1)}}.$$

114. $$\frac{\sqrt{\left\{x^2+\frac{1}{x^2}+2\left(x-\frac{1}{x}\right)-1\right\}}}{x^2+x-1}.$$

115. $$\left\{\frac{a^2x}{(a+x)^{\frac{5}{4}}}\right\}^{-\frac{1}{5}}$$

116. $$\frac{a^{\frac{n+1}{n}}+b\left(ab^{\frac{1-n}{n}}+a^{\frac{1}{n}}\right)+b^{\frac{n+1}{n}}}{a^2-b^2}.$$

117. $$\frac{3x^2-2xy-y^2}{4x^3-2x^2y-3xy^2+y^3}.$$

118. $$\frac{x^4-3ax^3+4a^3x}{x^5-17a^2x^3+36a^3x^2-20a^4x}.$$

119. $$\frac{\{\sqrt[3]{(a^2b)}-\sqrt{(ab^3)^3}\}}{1-3a^{-\frac{1}{6}}b^{\frac{7}{6}}+3a^{-\frac{1}{3}}b^{\frac{7}{3}}-a^{-\frac{1}{2}}b^{\frac{7}{2}}}.$$

120. $$\sqrt{\frac{(a-b)^6+8ab(a-b)^4+16a^2b^2(a-b)^2}{a^6b^2-2a^4b^4+a^2b^6}}.$$

121. $$\frac{\frac{1+\sqrt{5}}{2}x-2}{x^2-\frac{1+\sqrt{5}}{2}x}+\frac{\frac{1-\sqrt{5}}{2}x-2}{x^2-\frac{1-\sqrt{5}}{2}x}.$$

122. $$\frac{\sqrt{(a^2+b^2).(a+b)^2-4a^3b-4ab^3}}{\sqrt{(a+b\sqrt{-1})}.\sqrt{\{(a-b)^2+4ab\}}}.$$

123. $$\frac{1-\frac{a^2-x^2}{a-x}}{\frac{\sqrt{(a^2-x^2)}}{\sqrt{(a-x)}}+1}.$$

124. $$\frac{x-y}{xy}+\frac{z-x}{xz}+\frac{y-z}{yz}.$$

125. $$\frac{x+\sqrt{x^2-y^2}}{x-\sqrt{x^2-y^2}}-2.\frac{x^2}{y^2}+1.$$

126. $$\frac{\sqrt{x^2+a}+\sqrt{x^2-a}}{\sqrt{x^2+a}-\sqrt{x^2-a}}+\frac{\sqrt{x^2+a}-\sqrt{x^2-a}}{\sqrt{x^2+a}+\sqrt{x^2-a}}.$$

127. $$\frac{\sqrt{\left\{(a-b)^2+4a(a+b)-4a^2\right\}.\left\{a^2\left(1-\frac{2b}{a}\right)+b^2\right\}}}{\sqrt[3]{a^2\left\{3b^2(b^2-a^2)+a^4\right\}-b^6}}.$$

128. $$\frac{x^2+y^2}{x^2-y^2}+\frac{2x}{x+y}\left\{\frac{xy-x^2}{(x-y)^2}+\frac{x+y}{x-y}\right\}.$$

129. $$\frac{x^{\frac{2}{m}}-y^{\frac{2}{n}}}{x^3-y^3}\times\frac{y^6-x^6}{y^{\frac{1}{n}}+x^{\frac{1}{m}}}.$$

130. $$\frac{1}{4x^3(x+y)}+\frac{1}{2x^2(x^2+y^2)}+\frac{1}{4x^3(x-y)}.$$

131. $$\frac{x+y\sqrt{-1}}{x-y\sqrt{-1}}+\frac{x-y\sqrt{-1}}{x+y\sqrt{-1}}.$$

132. $$\sqrt{\frac{\sqrt{4(\sqrt{a}+\sqrt{b})^2-16\sqrt{ab}}+\sqrt[4]{16\{a^2+b^2+4\sqrt{ab}(a+b)+6ab\}}}{\sqrt{a}}}.$$

133. $$\sqrt[3]{\frac{x^4y - 3x^3y^2 + 3x^2y^3 - xy^4}{x^3 + 3x^2y + 3xy^2 + y^3}} - \sqrt[3]{\frac{x^2 + 2xy + y^2}{x^3 - 3x^2y + 3xy^2 - y^3}} \cdot \sqrt[3]{x^2y + xy^2}.$$

134. Simplify the expression:

$$\frac{(1+x)^{\frac{1}{2}} + (1-x)^{\frac{1}{2}}}{(1+x)^{\frac{1}{2}} - (1-x)^{\frac{1}{2}}},$$

first by rationalizing the numerator, and then by rationalizing the denominator; and show that the sum of the results is, as it ought to be, the double of the original expression.

135. Multiply $7x^3 - 8x^2 - 5x + 3$ by $3x^2 - 11x - 9$, using detached coefficients; and divide the result by $3x^2 - 9x - 6$ by the same method.

136. Write down the coefficient of x^5 in the product of $ax^6 + bx^5 - cx^4 + ex^2 + fx - h$ by $kx^4 + lx^3 + mn - n$ without actual multiplication; and state how the result is obtained.

137. Multiply together

$$\sqrt{x^{2n} + x^ny^n + y^{2n}},\ \ \sqrt{x^n - y^n},\ \ \sqrt{x^{2n} - x^ny^n + y^{2n}} \text{ and } \sqrt{x^n + y^n}.$$

138. Multiply together

$$(a^2 + ab + b^2)^{\frac{1}{q}},\ \ (a - b)^{\frac{1}{q}},\ \ (a - b)^{\frac{1}{p}} \text{ and } (a^2 + ab + b^2)^{\frac{1}{p}}.$$

139. Multiply $4a^3b - 2a^2b^2 - 6ab^3 + 3b^4$ by $ab^3 - 2a^{-1}b^5$, using detached coefficients.

140. Square the expression $\sqrt[4]{-1} + \sqrt{-\sqrt{-1}}$.

141. Cube the expression $a^2\sqrt{x} - \sqrt{ba}\sqrt[3]{y}$.

142. Extract the square root of

$$\frac{4}{9}y^4z^2 - 8y^3z^3 + 34y^2z^4 + 18yz^5 + \frac{9}{4}z^6.$$

143. Extract the square root of

$$x - \frac{3}{2}x^{\frac{5}{6}} + \frac{9}{16}\sqrt[3]{x^2} + 2\sqrt{x} - \frac{3}{2}\sqrt[3]{x} + 1.$$

144. Extract the square root of

$$1 + \frac{41}{16}a - \frac{3+3a}{2}\sqrt{a} + a^2.$$

145. In any equation $x + \sqrt{y} = a + \sqrt{b}$, which involves rational quantities and quadratic surds, show that the rational parts on each side are equal, and also the irrational parts: and extract in the form of a binomial surd the square root of

$$ab + c^2 + \sqrt{\{(a^2 - c^2)(b^2 - c^2)\}}.$$

146. Extract the square root of $3\sqrt{3} - 2\sqrt{6}$, without using a formula: show that $2 + \sqrt{3}$ is the reciprocal of $2 - \sqrt{3}$, and state generally what must be the connexion between the two terms w and $\sqrt{x}$, so that their sum may be the reciprocal of their difference.

Find the square roots of the following binomial surds, without using a formula:

147. $\sqrt{27} - 2\sqrt{6}$.

148. $\sqrt{20} - \sqrt{15}$.

149. $56 - 2\sqrt{55}$.

150. $43 - 30\sqrt{2}$.

151. $7 + 2\sqrt{10}$.

152. $21 + 12\sqrt{3}$.

153. Divide $x^6 + (a^2 - 2b^2)x^4 - (a^4 - b^4)x^2 - a^6 - 2a^4b^2 - a^2b^4$ by $x^2 - a^2 - b^2$, by *Horner's* or the "*synthetic*" method.

154. Arrange $6(x^3+y^3)+(18xy-4)(x+y)-8(x^2+y^2)-16xy-120$ and $x^2 + y^2 + 2x(1+y) + 2y + 6$ according to the powers of $(x+y)$, and divide the former by the latter, by the synthetic method.

155. Divide $-10a^3x^4y - 60ax^6y^3 + 6a^4x^3 - 9a^2x^5y^2$
by $3x^7y^4 - 5ax^6y^3 + a^3x^4y$,
by the synthetic method, giving six terms of the quotient, arranged according to the *ascending* powers of y.

156. Divide $3x^3 - 9x + \frac{4}{x^3}$ by $4x^2 + 8$, using the *synthetic method*, and giving the quotient as far as the term involving x^{-6}.

157. Divide $x^9 - 3x^8 - 31x^7 + 25x^6 + 3x^5 - 15x^4 - 8x^3 + 19x^2 + 3x + 10$ by $3x^4 - 21x^3 + 9x - 6$, by the synthetic method; showing the "*final remainder*," and then carrying on the quotient to twelve terms.

158. Expand $\frac{1-x}{1+x-x^2}$ and $\frac{3x^4 - 8x}{2x^3 - 6x^2 + 10x - 4}$ in series by synthetic division.

159. Expand $\frac{x^2 - x + 1}{x - 1}$ in a series to five terms, by Horner's mode of division; and show the arithmetical equality of the fraction and resulting series, when $x = 3$.

160. Divide

$$x^9 - 4x^8 - x^7 + 14x^6 - 10x^5 - 21x^4 + 22x^3 - 8x^2 - 10x + 3$$

by $x^4 - 4x^3 + 2x^2 - 2$, showing the *final remainder*, by the synthetic method.

161. Find the remainder left after dividing

$$5y^7 - 37y^6 - 5y^5 - 27y^4 + 7y^3 - 10y - 6$$

by $4y^4 - 28y^3 - 16y^2 - 24y + 8$,

synthetically; the quotient involving only positive powers of y.

162. Divide

$$2a^{-1}b^6 + 10b^5 - 4ab^4 - 32a^2b^3 + 16a^3b^2 - 37a^4b + 2a^5 + 66a^6b^{-1} - 8a^7b^{-2}$$

by $2a^{-5} + 4a^{-4}b^{-1} - 6a^{-3}b^{-2} - 8a^{-2}b^{-3}$, by Horner's method, showing the remainder.

163. Divide
$4x^7y-23x^6y^2z-36x^5y^3z^2+26x^4y^4z^3-52x^3y^5z^4-30x^2y^6z^5+16xy^7z^6+35y^8z^7$ by $4x^2yz^{-2}-3xy^2z^{-1}-7y^3$ to such extent that the quotient may involve no negative powers of x, and give the remainder.

164. Divide $2a^5b^{-2}+9a^4b^{-1}-9a^3-$&c., the remaining coefficients being $-21+69-74+68-15-36+29$,

$$\text{by } 2a^{-1}b^4+3a^{-2}b^5-8a^{-3}b^6+4a^{-4}b^7,$$

giving the remainder whose first term involves $a^{-2}b^5$.

165. Divide

$$2x^4y^{-2}-14x^2+22xy-22y^2+16x^{-1}y^3+9x^{-2}y^4-12x^{-3}y^5$$

by $2x^{-1}y^7-4x^{-2}y^8-2x^{-4}y^{10}$, by Horner's method, and give the remainder.

166. Divide $3x^2yz^{-1}+4x^3+6x^4y^{-1}z-21x^5y^{-2}z^2+16x^6y^{-3}z^3-11x^7y^{-4}z^4-8x^8y^{-5}z^5-5x^9y^{-6}z^6-38x^{10}y^{-7}z^7+33x^{11}y^{-8}z^8$ by $3x^4yz^{-3}-2x^5z^{-2}+x^6y^{-1}z^{-1}-5x^7y^{-2}$, and give the final remainder.

167. Divide $18x^{-7}y^4+6x^{-6}y^3+20x^{-5}y^2+31x^{-4}y+2x^{-3}+6x^{-2}y^{-1}+39x^{-1}y^{-2}-64y^{-3}+20xy^{-4}$ by $3x^{-4}y^{-2}-x^{-3}y^{-3}+5x^{-2}y^{-4}-2x^{-1}y^{-5}$, by Horner's method, and give the remainder.

168. Divide $8a^{-5}.b^8+17a^{-4}b^7+a^{-3}b^6-9a^{-2}b^5+8a^{-1}b^4-7b^3+6ab^2-5a^2b+4a^3-4a^4b^{-1}+a^5b^{-2}$ by $a^3+3a^4b^{-1}+2a^5b^{-2}-a^6.b^{-3}$ by Horner's method.

169. Divide $3ax^{-1}y^7+7y^6-5a^{-1}xy^5+35a^{-2}x^2y^4-22a^{-3}x^3y^3$
$+18a^{-4}x^4y^2+30a^{-5}x^5y-60a^{-6}x^6+18a^{-7}x^7y^{-1}+5a^{-8}x^8y^{-2}$

by $3x^{-5}y^2-2a^{-1}x^{-4}y+7a^{-2}x^{-3}-5a^{-3}x^{-2}y^{-1}-a^{-4}x^{-1}y^{-2}$, by Horner's method; giving the remainder whose first term involves $a^{-6}x^6$.

170. Expand $\dfrac{x^2+2}{x^2+2x-3-x^{-1}}$ in a series, and show the arithmetical equality of the fraction and its expansion, when $x=1$; taking four terms of the series.

171. Find the greatest common measure of

$$x^5 - x^4 - x + 1 \text{ and } 5x^4 - 4x^3 - 1.$$

172. Find the greatest common measure of

$$6a^4x^3 - 9a^3x^2y - 10a^2xy^2 + 15ay^3$$

$$\text{and } 10a^5x^6y^2 - 15a^4x^5y^3 + 8a^3x^4y^4 - 12a^2x^3y^5.$$

173. Find the greatest common measure of

$$x^6 + 4ax^5 - 3a^2x^4 - 16a^3x^3 + 11a^4x^2 + 12a^5x - 9a^6$$

$$\text{and } 6a^2x^5 + 20a^3x^4 - 12a^4x^3 - 48a^5x^2 + 22a^6x + 12a^7.$$

EQUATIONS.

Solve the following equations:

174. $$\frac{1-x}{4x} - \frac{2-3x}{3} + \frac{3-x}{4} = \frac{3(1-x)(1+2x^2)}{4x(3-2x)}.$$

175. $$3x - 2 \cdot \frac{x-4}{5} + 5 \cdot \frac{8-7x}{3} = 6x - 7 - \frac{x-2}{4}.$$

176. $$\frac{3x+7}{14} + 7 - \frac{2x-14}{21} + \frac{5}{12} = \frac{x+16}{4}.$$

177. $$\frac{7x+5}{23} + \frac{9x-1}{10} - \frac{x-9}{5} + \frac{2x-3}{15} = \frac{70}{3}.$$

178. $$\sqrt{(1+x)} + \sqrt{\{1+x+\sqrt{(1-x)}\}} = \sqrt{(1-x)}.$$

179. $$\left\{\begin{aligned} \frac{x-2}{5} - \frac{10-x}{3} &= \frac{y-10}{4} \\ \frac{2y-4}{3} - \frac{2x+y}{8} &= \frac{x+13}{4} \end{aligned}\right\}.$$

2—2

180. $$\frac{\frac{3}{x}-1}{2}-\frac{9\left(\frac{1}{2x}-1\right)-\frac{2}{5}\left(\frac{9}{2x}-4\right)}{\frac{3}{x}-4}=\frac{\frac{9}{x}+19}{6}.$$

181. $$\frac{1-5x}{\sqrt{(5x)}+1}=\frac{1-\sqrt{(5x)}}{2}-1.$$

182. $$\frac{x-6}{\sqrt{x}-\sqrt{6}}+\frac{\sqrt{(x-6)}}{\sqrt{x}}=\sqrt{x}-\frac{\sqrt{x}}{\sqrt{(x-6)}}.$$

183. $x-1=2+\dfrac{2}{\sqrt{x}}.$ 184. $\sqrt[3]{(24+x)}+\sqrt[3]{(24-x)}=5.$

185. $2\sqrt{(x^2-5x+2)}-x^2+8x=3x-78.$

186. $$\frac{a-\sqrt{(a^2-x)}}{a+\sqrt{(a^2-x)}}=\frac{1}{a}.$$

187. $$\frac{4x-22}{35}-\frac{4x^2-1}{5x}=-\frac{5x^2+3x+1}{7x}.$$

188. $$\frac{(y-2)^3}{y-1}+\frac{5}{(y-1)(y-2)}=5.\frac{y-3}{y-2}.$$

189. $$\frac{x}{\sqrt{x}+\sqrt{(a-x)}}+\frac{x}{\sqrt{x}-\sqrt{(a-x)}}=\frac{b}{\sqrt{x}}.$$

190. $\dfrac{x}{x+1}+\dfrac{x+1}{x}=\dfrac{13}{6}.$ 191. $\dfrac{x-1}{x-2}+\dfrac{x-2}{x-1}=1.$

192. $x^2-\sqrt{(x^2-9)}=21.$ 193. $\dfrac{\sqrt{(1+x)}}{1+\sqrt{(1+x)}}=\dfrac{\sqrt{(1-x)}}{1-\sqrt{(1-x)}}.$

194. $\sqrt{x}-4(\sqrt{x}+13)^{\frac{1}{2}}+7=x-2\sqrt{x}-9.$

195. $$\frac{a+x}{\sqrt{a}+\sqrt{(a+x)}}=\frac{a-x}{\sqrt{a}-\sqrt{(a-x)}}.$$

196. $$\sqrt{\frac{x}{a}}+\sqrt{\left\{\left(1-\frac{c}{b}\right)\left(1-\frac{bx}{ac}\right)\right\}}=1.$$

197. $4x^2+12x.\sqrt{(1+x)}=27(1+x).$

198. $\dfrac{x+\sqrt{(x^2-9)}}{x-\sqrt{(x^2-9)}}=(x+2)^2.$

199. $11-3x^3+5x^6=9x^3+299-x^6.$

200. $\dfrac{1}{x-\sqrt{(2-x^2)}}=\dfrac{1}{x+\sqrt{(2-x^2)}}+1.$

201. $x^2+\dfrac{1}{x^2}-a^2-\dfrac{1}{a^2}=0.$

202. $\sqrt[3]{x}+\dfrac{5}{2\sqrt[3]{x}}=\dfrac{13}{4}.$

203. $x=7-\dfrac{y+12}{7},\quad 4y-\dfrac{x+10}{5}=5.$

204. $\left\{\begin{array}{l} x^3-y^3=3xy(x-y)+8, \\ x^2y+xy=4(x+1). \end{array}\right\}$

205. $\left\{\begin{array}{l} \dfrac{4}{x}+\dfrac{12}{y}=13, \\ 7\left(\dfrac{10y}{x}+1\right)=\dfrac{98}{x}. \end{array}\right\}$

206. $\left\{\begin{array}{l} \dfrac{x-2}{5}-\dfrac{10-x}{3}=\dfrac{y-10}{4}, \\ \dfrac{2y+4}{3}-\dfrac{2x+y}{8}=\dfrac{x+13}{4}. \end{array}\right\}$

207. $\left\{\begin{array}{l} (x^2+y^2)(x+y)=2336, \\ (x^2-y^2)(x-y)=576. \end{array}\right\}$

208. $9x^2-6xy+3x-7y^2-2y=12$ and $3x-2y=7$.

209. $(x+y)^2-x+y=4xy+20.$
$x+y=12.$

210. $y^2+y=14-x^2+x,$
$xy=4.$

211. $\left\{\begin{array}{l} \dfrac{x+1}{x-1}-\dfrac{x-1}{x+1}=xy, \\ y(x+2)=1. \end{array}\right\}$

212. $\left\{\begin{array}{l} x^4+y^4=97, \\ x+y=5. \end{array}\right\}$

213. $\left\{\begin{array}{l} x^{\frac{3}{4}}-y^{\frac{3}{4}}=a, \\ x^{\frac{1}{4}}-y^{\frac{1}{4}}=b. \end{array}\right\}$

214. $\left\{\begin{array}{l} \sqrt{y}-\sqrt{(a-x)}=\sqrt{(y-x)}, \\ \dfrac{\sqrt{(y-x)}+\sqrt{(a-x)}}{\sqrt{(a-x)}}=\dfrac{5}{2}. \end{array}\right\}$

180. $$\frac{\frac{3}{x}-1}{2}-\frac{9\left(\frac{1}{2x}-1\right)-\frac{2}{5}\left(\frac{9}{2x}-4\right)}{\frac{3}{x}-4}=\frac{\frac{9}{x}+19}{6}.$$

181. $$\frac{1-5x}{\sqrt{(5x)}+1}=\frac{1-\sqrt{(5x)}}{2}-1.$$

182. $$\frac{x-6}{\sqrt{x}-\sqrt{6}}+\frac{\sqrt{(x-6)}}{\sqrt{x}}=\sqrt{x}-\frac{\sqrt{x}}{\sqrt{(x-6)}}.$$

183. $x-1=2+\dfrac{2}{\sqrt{x}}.$ 184. $\sqrt[3]{(24+x)}+\sqrt[3]{(24-x)}=5.$

185. $2\sqrt{(x^2-5x+2)}-x^2+8x=3x-78.$

186. $$\frac{a-\sqrt{(a^2-x)}}{a+\sqrt{(a^2-x)}}=\frac{1}{a}.$$

187. $$\frac{4x-22}{35}-\frac{4x^2-1}{5x}=-\frac{5x^2+3x+1}{7x}.$$

188. $$\frac{(y-2)^3}{y-1}+\frac{5}{(y-1)(y-2)}=5\,.\frac{y-3}{y-2}.$$

189. $$\frac{x}{\sqrt{x}+\sqrt{(a-x)}}+\frac{x}{\sqrt{x}-\sqrt{(a-x)}}=\frac{b}{\sqrt{x}}.$$

190. $\dfrac{x}{x+1}+\dfrac{x+1}{x}=\dfrac{13}{6}.$ 191. $\dfrac{x-1}{x-2}+\dfrac{x-2}{x-1}=1.$

192. $x^2-\sqrt{(x^2-9)}=21.$ 193. $\dfrac{\sqrt{(1+x)}}{1+\sqrt{(1+x)}}=\dfrac{\sqrt{(1-x)}}{1-\sqrt{(1-x)}}.$

194. $\sqrt{x}-4\,(\sqrt{x}+13)^{\frac{1}{2}}+7=x-2\sqrt{x}-9.$

195. $$\frac{a+x}{\sqrt{a}+\sqrt{(a+x)}}=\frac{a-x}{\sqrt{a}-\sqrt{(a-x)}}.$$

196. $$\sqrt{\frac{x}{a}}+\sqrt{\left\{\left(1-\frac{c}{b}\right)\left(1-\frac{bx}{ac}\right)\right\}}=1.$$

197. $4x^2+12x\,.\sqrt{(1+x)}=27\,(1+x).$

198. $\dfrac{x+\sqrt{(x^2-9)}}{x-\sqrt{(x^2-9)}}=(x+2)^2.$

199. $11-3x^3+5x^6=9x^3+299-x^6.$

200. $\dfrac{1}{x-\sqrt{(2-x^2)}}=\dfrac{1}{x+\sqrt{(2-x^2)}}+1.$

201. $x^2+\dfrac{1}{x^2}-a^2-\dfrac{1}{a^2}=0.$

202. $\sqrt[3]{x}+\dfrac{5}{2\sqrt[3]{x}}=\dfrac{13}{4}.$

203. $x=7-\dfrac{y+12}{7},\quad 4y-\dfrac{x+10}{5}=5.$

204. $\begin{Bmatrix} x^3-y^3=3xy\,(x-y)+8, \\ x^2y+xy=4\,(x+1). \end{Bmatrix}$

205. $\begin{Bmatrix} \dfrac{4}{x}+\dfrac{12}{y}=13, \\ 7\left(\dfrac{10y}{x}+1\right)=\dfrac{98}{x}. \end{Bmatrix}$

206. $\begin{Bmatrix} \dfrac{x-2}{5}-\dfrac{10-x}{3}=\dfrac{y-10}{4}, \\ \dfrac{2y+4}{3}-\dfrac{2x+y}{8}=\dfrac{x+13}{4}. \end{Bmatrix}$

207. $\begin{Bmatrix} (x^2+y^2)\,(x+y)=2336, \\ (x^2-y^2)\,(x-y)=576. \end{Bmatrix}$

208. $9x^2-6xy+3x-7y^2-2y=12$ and $3x-2y=7.$

209. $(x+y)^2-x+y=4xy+20.$
$x+y=12.$

210. $y^2+y=14-x^2+x,$
$xy=4.$

211. $\begin{Bmatrix} \dfrac{x+1}{x-1}-\dfrac{x-1}{x+1}=xy, \\ y\,(x+2)=1. \end{Bmatrix}$

212. $\begin{Bmatrix} x^4+y^4=97, \\ x+y=5. \end{Bmatrix}$

213. $\begin{Bmatrix} x^{\frac{3}{4}}-y^{\frac{3}{4}}=a, \\ x^{\frac{1}{4}}-y^{\frac{1}{4}}=b. \end{Bmatrix}$

214. $\begin{Bmatrix} \sqrt{y}-\sqrt{(a-x)}=\sqrt{(y-x)}, \\ \dfrac{\sqrt{(y-x)}+\sqrt{(a-x)}}{\sqrt{(a-x)}}=\dfrac{5}{2}. \end{Bmatrix}$

215. $$\left\{\begin{aligned} x+y&=a-\sqrt{(x+y)},\\ x-y&=b.\end{aligned}\right\}$$

216. $$\left\{\begin{aligned} \frac{x}{a}+\frac{y}{b}&=1-\frac{x}{c},\\ \frac{y}{a}+\frac{x}{b}&=1+\frac{y}{c}.\end{aligned}\right\}$$

217. $$\left\{\begin{aligned} x+y&=xy,\\ x^2+y^2&=a-x-y.\end{aligned}\right\}$$

218. $$\left\{\begin{aligned} x^2+y^2&=89,\\ x-y&=3.\end{aligned}\right\}$$

219. $$\left\{\begin{aligned} x^3-y^3&=a^3,\\ x-y&=b.\end{aligned}\right\}$$

220. $$\left\{\begin{aligned} x^3-y^3&=36+x+y,\\ x^2y^2&=36\,(x+y).\end{aligned}\right\}$$

221. $$\left\{\begin{aligned} x^4+y^4&=641,\\ x+y&=7.\end{aligned}\right\}$$

222. $$\left\{\begin{aligned} x^2+x\,.\,\sqrt[3]{(xy^2)}&=a,\\ y^2+y\,.\,\sqrt[3]{(x^2y)}&=b.\end{aligned}\right\}$$

223. $x^2+y^2=8$, and $\dfrac{1}{x^2}+\dfrac{1}{y^2}=\dfrac{1}{2}$.

224. $$\left\{\begin{aligned} x^2+y^2&=34,\\ 2x^2-3xy&=23-2y^2.\end{aligned}\right\}$$

225. $$\left\{\begin{aligned} (x+y)^m-3a^2\,(x+y)^{-m}&=2a,\\ (x-y)^n+5b^2\,(x-y)^{-n}&=6b.\end{aligned}\right\}$$

226. $$\left\{\begin{aligned} \frac{10x+y}{xy}&=3,\\ 9\,(y-x)&=18.\end{aligned}\right\}$$

227. $$\left\{\begin{aligned} x^2+y^2-x-y&=78,\\ xy+x+y&=39.\end{aligned}\right\}$$

228. $$\left\{\begin{aligned} \sqrt{x}+\sqrt{y}&=3,\\ x^{\frac{3}{2}}+y^{\frac{3}{2}}&=9.\end{aligned}\right\}$$

229. $$\left\{\begin{aligned} x+y&=10,\\ x^4+y^4&=1552.\end{aligned}\right\}$$

230. $$\left\{\begin{aligned} (x^2+y^2)\,xy&=78,\\ x^2\,(x^2+6y^2)+y^4&=313.\end{aligned}\right\}$$

231. $$\left\{\begin{aligned} x+y+x^2+y^2&=18,\\ xy&=6.\end{aligned}\right\}$$

232. $$\left\{\begin{aligned} x^2-xy&=2,\\ 2x^2+y^2&=9.\end{aligned}\right\}$$

233. $$\left\{\begin{aligned} \sqrt{(y-x)}+\sqrt{(a-x)}&=\frac{5}{2}\sqrt{(a-x)},\\ \sqrt{y}-\sqrt{(a-x)}&=\sqrt{(y-x)}.\end{aligned}\right\}$$

234. $$\left\{\begin{aligned} xy&=1225,\\ \sqrt{x}+\sqrt{y}&=12.\end{aligned}\right\}$$

235. $$\left\{\begin{aligned} \frac{1+x}{1-y}+\frac{1+y}{1-x}&=a,\\ \frac{1+x}{1+y}+\frac{1-y}{1-x}&=b.\end{aligned}\right\}$$

236. $$\left\{\begin{array}{r} \dfrac{x+y}{x-y}+\dfrac{x-y}{x+y}=\dfrac{5}{2}, \\ x^2+y^2=20. \end{array}\right\}$$

237. $$\left\{\begin{array}{c} x^3y+xy^3=3\sqrt{2}, \\ x^4+y^4=5. \end{array}\right\}$$

238. $$\left\{\begin{array}{l} x^2+xy=2, \\ 2x^2+y^2=9. \end{array}\right\}$$

239. $$\left\{\begin{array}{l} \dfrac{x^3}{y}-\dfrac{y^3}{x}=\dfrac{15}{2}, \\ \dfrac{x}{y}-\dfrac{y}{x}=\dfrac{3}{2}. \end{array}\right\}$$

240. $$\left\{\begin{array}{r} x+y=72, \\ \sqrt[3]{x}+\sqrt[3]{y}=6. \end{array}\right\}$$

241. $$\left\{\begin{array}{r} \sqrt{\dfrac{5x}{x+y}}+\sqrt{\dfrac{x+y}{5x}}=\dfrac{3}{2}\sqrt{2}, \\ xy-(x+y)=1. \end{array}\right\}$$

242. $$\left\{\begin{array}{c} \sqrt{x}-\sqrt{y}=2, \\ (x+y)\sqrt{(xy)}=510. \end{array}\right\}$$

243. $$\left\{\begin{array}{l} x^2-xy=45, \\ xy-y^2=20. \end{array}\right\}$$

244. $$\left\{\begin{array}{l} x^2+xy+y^2=13, \\ x^4+x^2y^2+y^4=91. \end{array}\right\}$$

245. $$\left\{\begin{array}{l} x+2y+3z=16, \\ 2x+3y+4z=23, \\ 3x+4y+2z=18. \end{array}\right\}$$

246. $$\left\{\begin{array}{l} x+2y+3z=17, \\ 2x+3y+z=12, \\ 3x+y+2z=13. \end{array}\right\}$$

247. $$\left\{\begin{array}{l} y-z=2, \\ y(x+z)=65, \\ z(x+y)=45. \end{array}\right\}$$

248. $$\left\{\begin{array}{l} ax+by+cz=d, \\ a_{,}x+b_{,}y+c_{,}z=d_{,}, \\ a_{,,}x+b_{,,}y+c_{,,}z=d_{,,}. \end{array}\right\}$$

249. $$\left\{\begin{array}{l} a_1x+b_1y+c_1z=e_1, \\ a_2x+b_2y+c_2z=e_2, \\ a_3x+b_3y+c_3z=e_3. \end{array}\right\}$$

250. $$\left\{\begin{array}{l} \dfrac{1}{x}+\dfrac{1}{y}=5, \\ \dfrac{1}{y}+\dfrac{1}{z}=6, \\ \dfrac{1}{z}+\dfrac{1}{x}=7. \end{array}\right\}$$

251. $$\left\{\begin{array}{l} 2x+3y+4z=19, \\ 3x+4y-2z=9, \\ 5x-2y+3z=-33. \end{array}\right\}$$

252. $$\left\{\begin{array}{l} ax+by-cz=d, \\ bx-cy+az=-e, \\ cx+ay-bz=f. \end{array}\right\}$$

253. $$\left\{\begin{array}{l} \frac{x}{1}+\frac{y}{2}+\frac{z}{3}=32, \\ \frac{x}{2}+\frac{y}{3}+\frac{z}{4}=22, \\ \frac{x}{3}+\frac{y}{4}+\frac{z}{5}=17. \end{array}\right\}$$

254. $$\left\{\begin{array}{l} ax+by+cz=k, \\ bx+cy+az=l, \\ cx+ay+bz=m. \end{array}\right\}$$

255. $$\left\{\begin{array}{l} x+2y+3z=14, \\ 2x+5y+7z=33, \\ 3x-4y+2z=5. \end{array}\right\}$$

256. $$\left\{\begin{array}{l} x+2y+3z=17, \\ 2x+3y+z=12, \\ 3x+y+2z=13. \end{array}\right\}$$

257. $$\left\{\begin{array}{l} x-3y+2z=5, \\ y-3z+2x=-11, \\ 3x+3y+3z=24. \end{array}\right\}$$

258. $$\left\{\begin{array}{l} x(y+z)=20, \\ y(x+z)=35, \\ z(x+y)=27. \end{array}\right\}$$

259. $$\left\{\begin{array}{ll} x^2+y^2+z^2 & =3037, \\ y^2+x & =871, \\ y^2+z & =877. \end{array}\right\}$$

260. $$\left\{\begin{array}{l} v+2x+3y=6a, \\ x+2y+3z=6b, \\ y+2z+3v=6c, \\ z+2v+3x=6d. \end{array}\right\}$$

PROBLEMS PRODUCING EQUATIONS.

261. In a battery of 2 guns *A* and *B*, it is found that *A* fires 2 rounds more than *B* in half an hour, and if *A* then cease firing for 10 minutes, the number of rounds fired by *A* is to the number by *B*, in the ratio of 4 to 5; what is the number of rounds fired by each?

262. A light six-pounder gun weighs two-thirds as much as the carriage, and both together weigh half as much again as the limber when filled with ammunition; but the weight of the gun is three-fourths that of the empty limber; what are the respective weights of the gun, the carriage, and the empty limber, supposing the weight of the ammunition to be 4 cwt.?

263. Two companies of sappers were ordered to throw up a field-work, each company to do one-half of the work; No. 1 commenced operations half an hour before No. 2, and both stopped to rest for one hour at 12 o'clock, when it was observed that the work was just half finished; No. 2 completed its portion at 7 P.M., but No. 1 had not finished until a quarter to 10. At what time did each company commence work in the morning?

264. Having bought as many muskets as cost £8500, I reserved 800, and sold the remainder for £8400, gaining 10*s.* on each; how many muskets did I purchase, and what did each cost?

265. A railway-train runs a certain distance at a certain rate; had the rate been increased by 5 miles an hour, the distance would have been performed in $\frac{4}{5}$ of the time; but had the rate been diminished by 5 miles an hour, the time would have been increased by $2\frac{1}{2}$ hours. Find the time and distance.

266. A battalion of 820 men is raised in twenty weeks from three recruiting districts, one of which supplies 10 men more, and the other 5 men less, per week, than the third: how many men are supplied per week by each district?

267. An officer was sent by government to expend £10,000 in the purchase of cavalry horses. Being despatched to the various depots, 25 of the horses died on the road; and this had the effect of increasing the average price of each surviving horse by £1. 13*s.* 4*d.* What was the number of horses purchased?

268. Seven Enfield rifle bullets weigh 210 grains less than eight of the old spherical bullets, and each of the former weighs 40 grains more than each of the latter; find the weight of each.

269. Two companies of sappers could have completed a certain work in 16 days, but, after 4 days, one of them is ordered off to other duty, and the remaining company completed the work in 40 days from the commencement: in what time could each company have completed the work separately?

270. A train leaves B at 9 A.M. and runs to C at the rate of 15 miles an hour, and another train leaves A at noon and, running, through B, to C at 25 miles an hour, arrives half an hour later than the train from B. Find the distance from A to C, the distance from A to B being 15 miles.

271. A detachment from an army received a reinforcement containing one-third as many men as the front of the detachment consisted of when drawn up "four deep"; had the depth however been increased by 2, and the whole been drawn up together, the number of men in front would have been 125 fewer than it was before: find the number of men in the original detachment.

272. After a cavalry action, the commanding officer found that the number of his men killed was seven-ninths of the number of horses he had lost, and that the number of men still mounted was five-sixths of his original force; with how many men did he commence the action, supposing that he now required 48 horses to restore to the ranks his dismounted troopers?

273. A contractor supplied 3560 muskets and 1650 cutlasses for £10,000, and found that if 50 were added to the number of cutlasses sold for £100, it would be the same as the number of muskets for £500: what was the price of each musket and cutlass?

274. A trench was dug by A and B, whose rates of working were as 9 to 8; after having worked together for 8 days, the remainder was completed by A alone in one-sixth of the time that B would have taken to do the whole: how long would each have taken to do the work separately?

275. At a review, the infantry were drawn up in close column 40 deep, and it was found that there were just one-fifteenth as many men in the fronts of all the columns together as there were cavalry and artillery with the army: and that had the depth of each column been increased by five, and the troopers and artillerymen been drawn up with the infantry, the number of

men in the whole front would have been 100 more than before. What was the whole number of men in the army?

276. A retreating enemy is ten miles in advance of the pursuing army, which marches at the rate of three miles an hour; how far will the enemy be able to retreat before being overtaken, supposing that their rate per hour is $\frac{13}{405}$ of the number of hours they are on their march? and what is their rate of march?

277. What number consisting of three digits, is greater by 99 than when its digits are inverted; greater by 126 than the sum of its digits; and greater by 27 than when its second and third digits are transposed?

278. A troop of horse-artillery on the march from Woolwich to Chatham met a company of marines marching to Woolwich at the rate of three miles an hour; four miles further on, and four hours after leaving Woolwich, two stragglers from this company were met hastening on at the rate of four miles an hour, which pace would just enable them to join the company as it marched into Woolwich: at what rate was the troop marching, and at what distance from Woolwich did it meet the company of marines? Give an interpretation of the negative result.

279. Three gunners, A, B and C, were discharged from the regiment; A's pension was treble that of B, and, if increased by 7*d.* per diem, would have been double those of B and C together; but if C's had been increased by the same sum, it would have been only three-fourths of those of A and B together. What was the pension of each per week?

280. A detachment of field-artillery had to receive 216 rounds of ammunition, but before it arrived three of the guns were disabled, and the rest, in consequence, received each six rounds more than they otherwise would have done; how many guns were there at first?

281. If a cwt. of sulphur and b cwt. of nitre were bought for £c, and it were found that d cwt. more of sulphur were bought

for £e than of nitre for £f; what would be the price per cwt. of each?

282. Find five numbers, having equal differences, and such that their sum shall be 25, and the sum of their cubes 1225.

283. Two steamers A and B started from Woolwich at 11 o'clock to proceed, with the tide, to Greenwich; and it was found that their rates of speed were then as 3 to 2. A arrived at Greenwich at five and twenty minutes to 12; and, after remaining there five minutes, commenced her return, and met B just half a mile from Greenwich. At what rate was the tide running, supposing the distance between Woolwich and Greenwich to be 5 miles?

284. A detachment of 5000 men, ten miles to the rear of the British army, to which it belonged, was ordered to the front, that it might take part in an action just commenced with the French, whose numbers exceeded the British originally engaged by 2700 men. At what rate was the detachment marched if it arrived just in time to cause a similar preponderance on the side of the British, supposing the men to have fallen at the rate of 1130 British and 1250 French per hour?

285. Two railway-trains start, the one from London for Bristol at 6 A.M., travelling at the rate of 32 miles per hour; the other from Bristol for London at 5 A.M. How far from London will the trains meet, supposing the number of miles per hour the Bristol "up train" travels to be less by 2 than ten times the number of hours the London train takes to meet it, and the distance of Bristol from London to be 118 miles? (State under what circumstances the negative result is explicable.)

286. A and B are 300 miles distant from each other and start at the same time on foot to meet; A goes 6 miles the first day, 10 the next, 14 the next, and so on; B, on the contrary, goes 27 miles the first day, 22 the next, 17 the next, and so on; find how far they were from one another at the end of the

eleventh day; and shew how it happens that *B* is so short a distance from his starting-point.

287. Supposing that at the termination of a journey it were known that each ordinary wheel of the locomotive engine had made ten thousand more revolutions than the driving-wheel, and that the circumferences of these wheels were 25 feet and 8 feet respectively; what would be the distance passed over?

288. A cistern has three taps, two of which are of equal dimensions: when all three are open they will empty the cistern in 9 hrs. 36 min. And if one of the equal taps be stopped, the others will empty seven-ninths of the cistern in 10 hrs. 40 min. In how many hours would each tap empty the cistern by itself?

289. A column of infantry is known to contain 264 men more than a column of cavalry which has the same number of men in front, but six men less in depth: had the infantry, however, been increased by having ten men more in depth, with the same front as before, the number of men would have been double that of the cavalry: what was the number of men in each column?

290. Two parties of sappers, *A* and *B*, were ordered to work at two distinct saps; *A* commenced at 5 o'clock and *B* at 11, and, when ordered to cease, *A* had completed 360 yards and *B* 250 yards: now if *B* had commenced work at 5 and *A* at 11 o'clock, one party would have done just the same quantity of work as the other. At what time did they leave off working, and how many yards of sap did each complete in an hour?

291. A requisition for 35,000 lbs. of bread for the consumption of an army was made, in equal portions, upon a certain number of villages, but two of them having afterwards fallen into the hands of the enemy, it was found that each of the others had to supply 2000 lbs. more than the original share in order to make up the deficiency: what was the original number of villages?

292. There are 15 guns of a certain kind in the fort *A*, and 12 guns of another sort in the fort *B*: all the guns in *A* with one

from B are worth the same sum as all the guns in B with one from A, viz. £1969. Find the value of a gun in each fort.

293. A triangular piece of garden ground, measuring 180 feet, 240 feet, and 300 feet in the sides, is to have a path of uniform width, and to contain one-tenth of the area of the ground, cut round it. Find the width of the path.

294. Two contractors engage to supply, equally, a certain quantity of stores, in equal weekly deliveries, in three months; but one of them fails at the end of the first month, and the other, in consequence, supplies half as much again as before: in what time will the whole of the stores have been delivered?

295. Of 20 sheep purchased by a farmer, he lost one and sold the remainder at two shillings a head more than they cost, thereby gaining ten shillings on the bargain. Find the cost price.

296. Two squads of sappers can throw up a breast-work in h hours when working together; but, when working separately, No. 1 can do the same work in k hours less than thrice the time No. 2 can do it in: how many hours will each take to do the work?

297. A troop of horse-artillery and a company of sappers are ordered to march to Chatham, and the sappers start at half-past seven and the horse-artillery at eight; they both rested one hour at noon, and it appeared that the two together had then marched as much as the whole distance; at what time would they severally arrive at their destination, supposing the artillery to arrive one hour before the sappers?

298. A wine-merchant bought a cask of wine, holding 38 gallons, for £25; after drawing off 8 gallons for his own use, he sold the remainder at such a price as to gain 8 per cent. upon the whole; at what price per gallon did he sell the wine?

PROGRESSIONS.

299. If a be the first term, b the common difference, and n the number of terms of an arithmetical progression, find the value of s the sum of the series; and apply this formula to the summation of the series

$$5-2-9-16-\text{\&c. to eight terms,}$$

and $$\sqrt{\frac{1}{2}}+\sqrt{2}+3\sqrt{\frac{1}{2}}+\text{\&c. to twenty terms.}$$

300. If a be the first term, l the last term, b the common difference, n the number of terms, and s the sum of a series of numbers in arithmetic progression, show that

$$s=\frac{2a+(n-1)b}{2}.n \text{ and } n=\frac{l-a+b}{b}.$$

301. The first term of an arithmetical progression is 5, and the fiftieth is 95; find the series and the sum of the fifty terms.

302. Find an expression for the sum of n terms of the series

$$5+11+18+26+35+45+56+\text{\&c.}$$

303. Find the tenth term of the Arithmetic Progression whose first and sixteenth terms are 3 and 48: and the sum of those eight terms the last of which is 60.

304. The sum of the pth terms of the two series

$$1+4+7+10+\text{\&c. and } 3+7+11+15+\text{\&c.}$$

is 172; determine those terms.

305. Insert five arithmetic means between 10 and 8.

306. Insert four arithmetic means between -2 and -16.

307. Find the sum of 20 terms of the series $7+\frac{13}{2}+6+\text{\&c.}$

308. The sum of the first three terms of an arithmetic progression is 15, and the sum of their squares is 83; find an expression for the sum of n terms.

309. Find the sum of the series $27 + 22\frac{1}{2} + 18 + 13\frac{1}{2} + \&c.$ to sixteen terms; and also *in infinitum*.

310. What is the expression for the sum of n terms of an Arithmetical Progression whose first term is $\frac{3}{2}$, and the difference of whose third and seventh terms is 3?

311. The first and ninth terms of an arithmetic progression are 5 and 22; find the sum of 21 terms.

312. The difference between the first and tenth terms of an increasing arithmetic series is 3, and the sum of ten terms is 45; determine the series.

313. Find the sum of thirty terms of the series

$$7 + \frac{13}{2} + 6 + \&c.$$

314. The first term of an arithmetic progression is 41 and the fifth is $26\frac{1}{3}$; find the sum of ten terms.

315. The sum of 20 terms taken in a particular part of the series 4, 8, 12, &c., is 1240; what is the first term?

316. How many terms must be taken from the commencement of the series $1 + 5 + 9 + 13 + 17 + \&c.$, so that the sum of the 13 succeeding terms may be 741?

317. There are two arithmetic series which have the same common difference; the first terms are 3 and 5 respectively, and the sum of seven terms of the one is to the sum of seven terms of the other as 2 to 3. Determine the series.

318. There are five numbers in arithmetical progression whose sum is to the sum of their squares as 9 to 89, and the sum of the first four is 32; find the numbers.

319. An aid-de-camp is sent off to a division at the distance of 30 miles from head quarters, with important orders, and with directions to ride at the rate of 6 miles an hour, in case it should be necessary to countermand them; what interval does this allow, supposing it likely that a change of circumstances will render it absolutely necessary to overtake the aid-de-camp before he reaches the division, and that the fleetest horse at command can gallop 4 miles the first ten minutes, 3·8 miles the next 10 minutes, 3·6 the third, and so on?

320. The number of troops in a besieged town being 3600 and the number disabled the first day being 30, the second day 35, the third day 40, and so on; how long will the siege last before the active garrison is reduced to 3025 men, supposing that on the third day 3 men come out of hospital, on the fourth day 7, on the fifth day 11, and so on?

321. Find three numbers in arithmetical progression, such that their product is 120 and their sum 15.

322. If there be two series of numbers in arithmetical progression, each consisting of n terms and having the same first term a, the one increasing and the other diminishing by the same common difference b; shew that

$$\frac{S-S_1}{S+S_1}=\frac{b}{a}\cdot\frac{n-1}{2},$$

where S and S_1 are the sums of the respective series.

323. An army on the march is advancing at the rate of 12 miles a day, when a detachment, 50 miles in the rear, is ordered to join it; how long will it take to do so, supposing that, from increasing impediments on the roads, it can advance 25 miles the first day, 24 the second, 23 the third, and so on?

324. A and B are 170 miles distant and start at the same time, on foot, to meet. A walks 5 miles the first day, 7 miles the second day, 9 miles the third day, and so on. B walks 30 miles the first day, 24 miles the next, 18 miles the next, and so on.

find the time at which they meet and their distance from the starting-point of B.

325. If S_m, S_n, S_{m+n} express, respectively, the sum of m terms, the sum of n terms, and the sum of $m+n$ terms of the same arithmetical progression; shew that

$$S_m - S_n : S_{m+n} :: m-n : m+n.$$

326. Investigate the expression for the sum of a geometric series, and show what it becomes when n is infinite and r a proper fraction.

327. The third and seventh terms of a geometric series are 12 and 192; determine the tenth term.

328. The difference between two numbers is 48 and the arithmetic mean exceeds the geometric by 18: what are the numbers?

329. Sum the series $\sqrt{2} - 2 + 2\sqrt{2} -$ &c. to ten terms, and find the twelfth term.

330. Show that the ratio of the sums of n terms of two geometric series having the same common ratio, is the ratio of their pth terms.

331. If f and t be the fourth and seventh terms of a geometric progression, find an expression for the sum of six terms.

332. Prove that, in a geometric progression, when the common ratio is a proper fraction, and S_∞ expresses the sum *in infinitum*,

$$S_\infty = \frac{a}{1-r}.$$

333. Find the sum of ten terms of the progression

$$3 - 2 + \frac{4}{3} - \frac{8}{9} + \text{\&c.};$$

and also the sum *in infinitum*.

334. Sum the series $\sqrt[6]{3} + \sqrt[6]{6} + \sqrt[6]{12} + \&c.$ to 12 terms; and find the nineteenth term.

335. Sum the series $2 - \sqrt{2} + 1 - \frac{1}{2}\sqrt{2} + \frac{1}{2} - \&c.$ to eight terms, and also *in infinitum*.

336. A series of numbers whose third term is $\frac{1}{3}$ is found to decrease in geometrical progression, and the first term is to the sixth term as 243 to 1; find the sum of the six terms.

337. Sum the series $\sqrt[4]{\frac{5}{8}} - \frac{1}{2}\sqrt[4]{5} + \frac{1}{2}\sqrt[4]{2\frac{1}{2}} - \&c.$ to 12 terms; and find the ninth term.

338. Sum the series $2\sqrt{2} - 4 + 4\sqrt{2} - 8 + \&c.$ to ten terms.

339. Find an expression for the sum of n terms of the series

$$1 + 4p + 7p^2 + 10p^3 + \&c.$$

340. Find the sum of eight terms of the series

$$1 + \sqrt{3} + 3 + \&c.$$

341. The first and seventh terms of a geometric progression are 2 and $\frac{1}{2}$; find the intermediate terms.

342. Sum the series $\frac{1}{2} + \frac{1}{4\sqrt{(-1)}} - \frac{1}{8} - \frac{1}{16\sqrt{(-1)}} + \&c.$ *in infinitum*.

343. Sum the series $\frac{1}{\sqrt{3}} + \frac{1}{3} + \frac{1}{\sqrt{27}} + \&c.$ to six terms; and also *in infinitum*.

344. The garrisons of three towns are in geometric progression, and it is known that the whole number of men in the two smaller ones is one-third of the number in the two larger; what is the number of men in each garrison, supposing the total number to be 13000?

345. The fourth term of a geometric progression is 192, and the seventh term is 12288; find the sum of the first three terms.

346. If a geometric progression consist of $4n$ terms, shew that the ratio of the sum of the first n terms and last n terms is to the sum of the remaining $2n$ terms as $r^{2n} - r^n + 1$ to r^n.

347. Prove that if quantities be in geometrical progression their differences are also in geometrical progression, having the same common ratio as before.

348. The first and sixth terms of a geometric series are 1 and 243; find the sum of six terms, commencing at the third.

349. The sum of a geometric series is 117186, the first term 6, and the common ratio 5; determine the number of terms, by logarithms.

350. Show that the sum of n terms of the geometric series whose common ratio is r and first term a, when divided by the sum of its first two terms, is equal to $\frac{1}{r+1}$ of the sum of n terms of the series whose first term is 1 and common ratio r; whatever be the value of a.

351. If S_n and S'_p represent the sum of n terms, and the sum of p terms of two geometric progressions having the same first term, and whose common ratios are respectively r and t, show that the ratio of S_n to S'_p may be expressed by the fraction

$$\frac{r^{n-1} + r^{n-2} + r^{n-3} \ldots\ldots + r^2 + r + 1}{t^{p-1} + t^{p-2} + t^{p-3} \ldots\ldots + t^2 + t + 1}.$$

352. If four quantities of the same kind be proportionals, the greatest and least of them together are greater than the other two together.

353. Prove that if a, b, and c be three quantities in harmonic progression, $\frac{1}{a}$, $\frac{1}{b}$ and $\frac{1}{c}$ are in arithmetic progression.

354. If an ordinary train leave the terminus at 4 P.M. at the rate of ten miles an hour, and increase its speed at the rate of two miles an hour every ten minutes; and an express train leave the same terminus at 4 hrs. 10 min. P.M. starting at the same speed as the other, but increasing its speed every ten minutes, in the ratio of 5 to 7; what would be the relative position of these trains at the end of fifty minutes after the starting of the express?

355. Find the number by which the numbers 20, 50, and 100 being severally increased, the results may be

(1) In geometrical progression:

(2) In harmonical progression.

356. Shew that three different quantities cannot be in harmonic progression and also in arithmetic progression; but that, if they *could*, they must also be in geometric progression.

357. If a, b, c be in harmonic progression, show that

$$\frac{1}{a}+\frac{1}{c}+\frac{1}{a-b}+\frac{1}{c-b}=0.$$

HORNER'S SOLUTION OF NUMERICAL EQUATIONS.

358. Show that if a polynomial in z be divided by $z-v$, the final remainder will be the same as the original polynomial, only having v in the place of z.

359. Apply the property stated in the last question to the determination of the value of the expression

$$v^5-5v^4-3v^3-5v+2,$$

when $v=-\cdot 5$.

360. Find, by synthetic division, the values of the expression

$$w^4 - 3w^3 - 12w^2 + w - 53,$$

when $w = 3$, $w = 6$, $w = \cdot 58$, and $w = -2\cdot 73$.

361. State the principle upon which the method of determining the value of an algebraical expression by synthetic division depends, and apply it to the determination of the value of

$$y^4 - 4y^3 + 9y + 3, \text{ when } y = -\cdot 3.$$

362. Find, by synthetic division, the values of the following fractions, when $x = 3$:

$$\frac{x^4 - 3x^3 - 7x^2 + 25x - 12}{x^3 + 3x^2 - 40x + 66}; \quad \frac{x^5 - 10x^4 + 60x^2 + 10x + 2}{x^4 + 2x^3 - 15x^2 - 2x + 6}.$$

363. Find the value of $x^5 - 7x^3 + 5x + 8$; when $x = -1\cdot 2$.

364. What is the numerical value of

$$y^5 + 7y^3 - 5y^2 - 1,$$

when $y = -\cdot 16$?

365. Find the value of

$$a^5 - 6a^4 + 12a^2 + 3a + 8$$

when $a = -1\cdot 1$; by synthetic division.

366. Determine the value of

$$x^5 - 2x^2 + 5x + 10$$

when $x = \cdot 12$, by synthetic division.

367. Find the value of $x^5 - 17x^3 + 2x^2 + 20$, when $x = 4$; by division.

368. Show that if any equation be divisible without remainder by $x - a$, a is a root.

369. Show that every equation has the same number of roots as there are units in the highest exponent of the unknown quantity in it.

370. Form the equation whose roots are -3, $2+3\sqrt{(-1)}$, $2-3\sqrt{(-1)}$ and 5, without multiplication, and prove its correctness by multiplication.

371. Find the *middle term* of the equation whose roots are 1, -2, 3 and -4; without determining any other term.

372. Determine by inspection the roots of the equation

$$ax^3-(b+ac-a^2d)x^2+(bc-abd-a^2cd)x+abcd=0.$$

373. Determine the last term but one of the equation whose roots are 1, -2, 3, -4, 5 and -6, *without determining any other term.*

374. Of how many products is the coefficient of the middle term of an equation having 8 roots, the sum?

375. Form the equation whose roots are 2, -5, and $4\pm\sqrt{(-7)}$.

376. Find the last term but one of the equation whose roots are 1, 2, -3, -4 and 5.

377. The second term of an equation is $9x^4$, its last term is 2520, and three of its roots are 5, -6 and -7; find the remaining roots.

378. Two roots of the equation

$$x^4-35x^2+90x-56=0$$

are 1 and 2; find the remaining roots.

379. The roots of the equation

$$x^3-6x^2-4x+24=0$$

are in arithmetical progression: find them.

380. Show that if the roots of the equation

$$ax^4+bx^3+cx^2+dx+e=0$$

be of the form α, β, $\frac{1}{\alpha}$ and $\frac{1}{\beta}$, then $a=e$ and $b=d$.

381. Two roots of an equation whose second and last terms are respectively $-11x^3$ and $+30$, are 1 and 5; what are its other roots?

382. Determine the last term but three of the equation whose roots are 1, 2, 3, -1, -2, and -3; without finding any other term.

383. Assuming that if an equation in x be divisible by $x-r$, r is a root of it; state how it appears that if p, q, s, t, v, &c. be also roots of such an equation, the successive quotients arising from the division of the original equation by any number of the binomials $x-p$, $x-q$, $x-r$, $x-s$, &c. will, when equated to 0, be equations whose roots are, in each case, the second terms of the binomials by which it has *not* been divided.

384. Find all the roots of the equation

$$x^5-2x^4-10x^3+20x^2+9x-18=0.$$

385. Determine whether any of the digits less than 6 are roots of the equation

$$x^6-6x^5+9x^4+2x^3-13x^2+26x-15=0.$$

386. Find all the roots of the equation

$$x^4-8x^3+24x^2-32x-9=0.$$

387. For what value of n will the roots of the equation $2x^2+8x+n=0$ be equal?

388. Show that if an equation have p roots each equal to r, and q roots each equal to s, the limiting equation will have $p-1$ roots each equal to r, and $q-1$ each equal to s: and state the law of the formation of the coefficients of the limiting equation from those of the original.

389. Supposing

$$(y-p)^a \cdot (y-q)^b \cdot (y-r)^c \cdot (y-s)^3 \cdot (y-t)^2 \cdot (y-v)$$

to be the form into which the greatest common measure between an equation and its limiting equation can be transformed; state what you would infer therefrom with regard to the roots of that equation.

390. The equation

$$4x^5 - 20x^4 + 25x^3 + 10x^2 - 20x - 8 = 0$$

has equal roots; find them.

391. Determine the equal roots of the equation

$$x^5 - 5x^3 + 5x + 2 = 0;$$

and, by means of them, complete its solution.

392. Determine the equal roots of the equation

$$x^4 - 16x^3 + 90x^2 - 208x + 169 = 0.$$

393. By means of its equal roots solve the equation

$$x^3 - x^2 - 16x - 20 = 0.$$

394. Solve the equation

$$x^4 - 6x^3 + 15x^2 - 18x + 9 = 0,$$

which has equal roots.

395. Determine the equal roots of the equation

$$x^5 + 5x^4 - 5x^3 - 45x^2 + 108 = 0.$$

396. By means of its equal roots, solve the equation

$$y^4 - 6y^3 + 15y^2 - 20y + 12 = 0.$$

397. The two equations

$$x^3 - 6x^2 + 11x - 6 = 0,$$

$$\text{and } x^3 - 14x^2 + 63x - 90 = 0,$$

have one root common, find it; and thence determine the remaining roots of both.

398. Prove that if the alternate signs of an equation be changed, the roots of the new equation will be the same as those of the original with contrary signs.

399. Prove that if we wish to form, from any equation, another, whose roots shall be greater, or less, by a given quantity

s, than those of the original, the coefficients of the new equation will be the successive final remainders left after dividing the original continually by $x+s$, or by $x-s$; and state when the positive, and when the negative sign is to be used.

400. From the equation $x^3-12x-15=0$ form two others, one whose roots are *greater* by $3{\cdot}97$, and another whose roots are *less* by $1{\cdot}5$ than the roots of the original.

401. Determine the equation whose roots are the roots of the equation

$$x^5+10x^4+40x^3+80x^2+80x+32=0,$$

increased by 2; and from the form of the result state what are the roots of the given equation.

402. Eliminate the second term from the equation

$$x^4-3x^3-7x^2+6x-2=0.$$

403. Supposing the roots of the equation

$$ax^n+bx^{n-1}+cx^{n-2}\,\ldots\ldots+kx^2+lx+m=0$$

to be p, q, r, s, &c., show that the equation whose roots are $\frac{1}{p}$, $\frac{1}{q}$, $\frac{1}{r}$, $\frac{1}{s}$, &c. will be

$$mx^n+lx^{n-1}+kx^{n-2}\,\ldots\ldots+cx^2+bx+a=0.$$

404. If the roots of the equation $ax^4+bx^3+cx^2+dx+e=0$ be r, s, t, and v, what is the equation whose roots are $\frac{1}{r}$, $\frac{1}{s}$, $\frac{1}{t}$, and $\frac{1}{v}$?

405. Give the principle upon which the reciprocal equation may be made available for the determination of the imaginary roots of an equation.

406. Define a *recurring equation;* state what is the characteristic property of the roots of such an equation; and also why this property enables us always to assign the value of one root of a recurring equation of an *odd* degree.

407. Solve completely the recurring equations

(1) $x^4 + 5x^3 + 2x^2 + 5x + 1 = 0.$

(2) $x^5 - 8x^4 + 9x^3 - 9x^2 + 8x - 1 = 0.$

408. Solve the following, as recurring equations:

(1) $x^4 + 2x^3 + 3x^2 + 2x + 1 = 0,$

(2) $x^5 - 5x^4 + 7x^3 - 7x^2 + 5x - 1 = 0.$

409. Solve completely the recurring equations

(1) $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0.$

(2) $x^5 - 6x^4 + 7x^3 - 7x^2 + 6x - 1 = 0.$

410. Solve completely the recurring equation

$$x^5 + 9x^4 + 16x^3 - 16x^2 - 9x - 1 = 0.$$

411. Solve completely the recurring equation

$$x^5 - 9x^4 + 16x^3 + 16x^2 - 9x + 1 = 0,$$

and show that the roots are of the recurring form.

412. Solve the recurring equation

$$x^5 - 11x^4 + 36x^3 - 36x^2 + 11x - 1 = 0,$$

and show that to each root there is a corresponding one which is its reciprocal.

413. Solve completely the recurring equation

$$x^5 - 13x^4 + 36x^3 + 36x^2 - 13x + 1 = 0;$$

and show that the roots are of the recurring form.

414. Solve the recurring equation

$$x^5 + 2x^4 - 5x^3 + 5x^2 - 2x - 1 = 0,$$

giving all the roots, and showing that half of them are the reciprocals of the other half.

415. Solve completely the recurring equation

$$x^5 - 2x^4 + 5x^3 - 5x^2 + 2x - 1 = 0.$$

416. Solve completely the recurring equation

$$x^5 - 2x^4 + 3x^3 - \&c. = 0.$$

417. If $+ - + - - - + - + +$ be the series of signs in any equation, show that if a positive root be introduced there will be *at least* one more variation than before in the result, and if a negative root be introduced there will be *at least* one more permanence than before; and state the rule of signs to which this gives rise.

418. State and show the truth of De Gua's criterion of the presence of imaginary roots in an equation.

419. What is the smallest and what the greatest number of imaginary roots which can exist in an equation of the form

$$A_1x^7 + A_3x^5 - A_6x^2 + A_8 = 0?$$

420. Apply De Gua's criterion to the determination of the number of imaginary roots of the equation

$$x^{12} + 4x^{10} - 7x^8 + 3x^6 - 2x^5 - 6x^4 - 8x + 5 = 0.$$

421. Determine the number of imaginary roots indicated by the absent terms of the equation $x^7 + 3x^5 - 2x^2 + 1 = 0$.

422. Can a cubic equation have all its roots imaginary? Give a reason for your answer, and state generally what class of equations must have *at least* one real root.

423. What is the least number of imaginary roots which an equation of the form $x^7 - ex^3 + h = 0$ can have?

424. Two roots of the equation

$$x^6 + 3x^5 - 14x^4 - 12x^3 + 47x^2 + 21x - 70 = 0$$

are 2 and -5; find all the other roots.

425. One root of the equation $x^4 - 4x^3 + 5x^2 + 8x - 14 = 0$ is $2 + \sqrt{(-3)}$; find all the other roots.

426. Determine the positions of the roots of the equation

$$8x^4 - 104x^3 + 465x^2 - 806x + 388 = 0,$$

by showing that each root lies between numbers different from those which limit any other root.

427. Show that the two positive roots of the equation $x^4 - 4x^3 + x^2 + 6x + 2 = 0$ are real, and find the first decimal of each of them.

428. Solve the equation $x^3 + 2x^2 - 3x + 4 = 0$, by Horner's method; giving such roots as are real to six places of decimals.

429. Find the character and positions of all the roots of the equation $x^4 - 4x^3 + 8x^2 - 5x - 6 = 0$, and find the negative root to five places of decimals.

430. Solve the equation $x^3 - 3x^2 + 5x + 10 = 0$, by Horner's method, giving the roots to six places of decimals.

431. Solve the equation $x^3 - 11x + 12 = 0$, by Horner's method, giving the smallest positive root to five places of decimals, and the integer figures of the other roots.

432. Find the real roots of the equation

$$x^4 - 11x^3 + 33x^2 - 40x + 19 = 0$$

to seven places of decimals.

433. One root of the equation $x^4 - 4x^3 + 5x^2 + 2x + 52 = 0$ is $3 - 2\sqrt{(-1)}$; determine the remaining roots.

434. Solve the equation, giving all the roots,

$$x^3 + 17x^2 - 46x + 29 = 0.$$

435. Solve the equation $x^3 - 8x + 12 = 0$, by Horner's method, giving the real root to eight places of decimals; and show that the other two roots are imaginary.

436. Determine the position and character of the roots of the equation $x^4 - 6x^3 - 14x^2 - 16x + 8 = 0$, and find the greatest root to nine places of decimals.

437. Solve the equation $x^3 - 4x^2 + 3x + 5 = 0$, by Horner's method, giving the real root to five places of decimals.

438. Determine the positions of the roots of the equation

$$x^4 - 14x^3 + 55x^2 - 12x - 184 = 0,$$

and find the least positive root to eight places of decimals.

439. Solve the equation $y^4 - 2y^3 + 3y^2 - 4y - 5 = 0$, giving such of the positive roots as are real, to seven places of decimals; and determining merely the *position* of the negative root.

440. Solve the equation $x^4 - 3x^2 - 4x + 7 = 0$, by Horner's method, giving all the real roots to six places of decimals.

441. Find such of the positive roots of the equation

$$x^4 - 2x^3 + 9x - 14 = 0$$

as are real, commencing the contraction at the third decimal; and show how it appears that the others are imaginary.

442. Determine all the roots of the equation

$$x^4 - 12x^3 + 44x^2 - 48x + 16 = 0.$$

443. Determine the characters and positions of the roots of the equation $x^4 - 19x^3 + 132x^2 - 302x + 200 = 0$.

444. Solve the following equations by Horner's method, giving the roots to eight places of decimals:

$$(1)\quad x^4 - 6x^3 + 8x^2 + 12x - 20 = 0.$$

$$(2)\quad x^4 - 12x^3 + 48x^2 - 72x + 36 = 0.$$

445. Solve the equation $4x^4 - 20x^3 + 133x^2 - 120x + 29 = 0$, giving all the roots.

446. Solve completely the equation $x^4 - 2x^2 + 7x - 6 = 0$.

447. Find the positions of the roots of the equation

$$x^4 + 2x^3 - 12x^2 + 15x + 5 = 0;$$

and determine the least negative root, by Horner's method, to eight places of decimals.

448. Solve the equation $x^4 - 3x^3 + 9x^2 + 2x - 1 = 0$, giving all the real roots.

449. Find the roots between 4 and 5 of the equation

$$x^4 \pm 0x^3 - 75x^2 + 318x - 322 = 0$$

by the method of reciprocals.

450. In the equation $z^3 + 3cz - 2d = 0$, show that the value of z is

$$\sqrt[3]{\{d + \sqrt{(d^2 + c^3)}\}} - \frac{c}{\sqrt[3]{\{d + \sqrt{(d^2 + c^3)}\}}}.$$

451. Investigate Cardan's formula for the solution of a cubic equation; and state when and why the formula is inapplicable.

452. Solve the equation $x^3 - 12x^2 + 5x + 2 = 0$, by Cardan's method, if possible.

453. Solve the equation $x^3 - 6x^2 + 15x - 18 = 0$, by the method of Cardan.

454. Find one root of the equation $x^3 - 9x^2 + 28x - 34 = 0$, by Cardan's method.

455. Solve the equation $x^3 - 9x^2 + 24x - 10 = 0$ by Cardan's method.

456. Solve the equation $x^3 - 6x^2 + 18x = 22$, by Cardan's method, giving the root to four places of decimals.

457. Find one root of the equation $x^3 - 15x - 22 = 0$ by trial and error, to four places of decimals.

458. Find by trial one root of the equation $x^3 - 27x - 36 = 0$, giving the root accurate to three places of decimals.

459. Find the integer values of x and y which satisfy the equation $5x + 11y = 70$.

460. An army of between 20,000 and 25,000 men was composed of infantry regiments, each containing 800 men, and cavalry,

each containing 600 men, and there were 10,000 more infantry than cavalry; how many infantry and cavalry regiments were there respectively?

CONTINUED FRACTIONS.

461. Find a series of converging fractions approximating to the ratio of the lb. troy, containing 5760 grains to the lb. avoirdupois, containing 7000 grains.

462. Determine a series of converging fractions approximating to the ratio of the side of a square to its diagonal, that ratio being 1 : 1·414214 nearly.

463. Find a series of converging fractions expressing the ratio of the French *Are* to the *Square Chain* from the equality

$$1 \textit{ Fr. Are} = {\cdot}2471 \textit{ Sq. Ch.}$$

464. The rate of exchange between France and England being such that the English sovereign = 25·42 francs; show, by means of a series of converging fractions, that the ratio 37 : 47 would be a close approximation to that of the franc to the shilling.

465. Determine a series of converging fractions approximating to the ratio of the French *litre* to the English *quart,* the litre being 1·761 *pint;* and determine *by inspection* whether the fraction $\frac{81}{92}$ must be increased or diminished to obtain the true ratio.

466. The mètre being 1·09361 yard, derive a series of converging fractions approximating to the ratio of the English mile to the myriamètre.

467. Express the ratio of the 48 lb. shot to the weight of the French shot of 24 kilogrammes, in a series of converging fractions; the kilogramme being 2·205 lbs.

467*. The mean diameter of the earth being 7912 miles, and that of Mars 4189 miles, find a series of converging fractions approximating to the ratio of the mean diameters of these two planets.

INDETERMINATE COEFFICIENTS.

468. Find an expression for the sum of n terms of the series

$$2 + 7 + 15 + 26 + 40 + \&c.$$

by the method of indeterminate coefficients.

469. Find an expression for the sum of n terms of the series

$$1 . 5 + 2 . 6 + 3 . 7 + 4 . 8 + \&c.$$

by the method of indeterminate coefficients.

470. Find an expression for the sum of n terms of the series

$$2 . 5 + 3 . 6 + 4 . 7 + 5 . 8 + \&c.$$

by the method of indeterminate coefficients.

471. Find an expression for the sum of n terms of the series

$$1 . 3 + 4 . 6 + 7 . 12 + 10 . 24 + \&c.$$

472. Sum the series $1 . 3 + 3 . 9 + 5 . 27$ + &c. to n terms.

473. Find an expression for the sum of n terms of the series

$$1 . 3 + 2 . 9 + 3 . 27 + \&c.$$

474. Sum the series $1^3 + 2^2 + 3^2 + 4^3$ + &c. to n terms.

475. Investigate the expression for the number of shot in a triangular pile of n courses, by the method of indeterminate coefficients.

476. Find an expression for the sum of the fourth powers of the natural numbers, by the method of indeterminate coefficients.

477. Find an expression for the sum of n terms of the series

$$1^2 + 3^2 + 5^2 + 7^2 + 9^2 + \&c.$$

478. Find the three component fractions of $\dfrac{x^2}{x^3 - 7x^2 + 36}$.

479. Decompose $\dfrac{x^2 - 3x + 2}{x^3 - 6x^2 - x + 30}$ into three partial fractions having denominators of the first degree.

480. Separate the fraction $\dfrac{6x^2 + 34x - 76}{(x-1)(x-3)(x+5)}$ into three component fractions having denominators of the first degree.

481. Decompose $\dfrac{3x^3 + 4x - 2}{3x^3 - 19x^2 + 30x - 8}$ into three partial fractions, by the method of *substitution.*

482. Decompose $\dfrac{11x^2 - 41x - 64}{x^3 - 8x^2 + 29x - 52}$ into more simple fractions.

483. Decompose $\dfrac{-x^2 + 14x - 27}{x^3 - 7x^2 + 17x - 14}$ into more simple fractions.

484. Decompose $\dfrac{37x^2 - 313x + 696}{x^3 - 6x^2 - x + 30}$ into three partial fractions having denominators of the first degree.

485. Decompose $\dfrac{x^2 - 7x + 36}{x^3 - 2x^2 - 16x + 32}$ into three partial fractions, by the method of indeterminate coefficients, and *substitution.*

486. Decompose $\dfrac{x^2 - 17x - 12}{x^3 - 6x^2 - 7x + 60}$ into three partial fractions.

487. Decompose $\dfrac{x^3 + 5x^2 - 3x + 7}{x^4 - 7x^3 - 13x^2 + 103x - 84}$ into four fractions having denominators of the first degree.

488. Decompose $\dfrac{18x^3 - 23x^2 + 57x - 100}{4x^4 + 16x^3 - 85x^2 - 4x + 21}$ into four fractions having denominators of the first degree.

489. Decompose $\dfrac{4x^2 - 18x + 62}{x^3 - 9x^2 + 33x - 65}$ into more simple fractions.

490. Decompose $\dfrac{5x^2 - 15x + 13}{x^3 - 6x^2 + 13x - 10}$ into more simple fractions.

491. Decompose $\dfrac{3x^2 + 48x + 17}{x^3 + 2x^2 - 15x - 36}$ into fractions having denominators of a lower degree.

492. Decompose $\dfrac{x^3 - 16x - 9}{x^4 - 6x^3 + 3x^2 + 26x - 24}$ into four partial fractions.

493. Decompose $\dfrac{10x^3 + 113x^2 + 264x + 138}{2x^4 + 13x^3 + 19x^2 - 10x - 24}$ into four partial fractions.

BINOMIAL THEOREM.

494. Prove that the first two terms of the expansion of $(a+x)^n$ are $a^n + na^{n-1}x$, whether n be integral or fractional, positive or negative.

495. In the equation

$$(a+x)^n = a^n + A_{/}a^{n-1}x + A_{//}a^{n-2}x^2 + A_{///}a^{n-3}x^3 + \&c.,$$

assume $A_{/} = n$, and thence determine $A_{//}$ and $A_{///}$.

496. Expand $\dfrac{1}{\sqrt{a^2 + 2x^2}}$ in a series arranged according to the descending powers of x; and write down the p^{th} term.

Expand by the binomial theorem, and give the p^{th} term of the following expressions :

497. $\sqrt{(v^5 - a^3x^2)^5}$.

498. $\sqrt{(a^2x + b)^3}$.

499. $(a^2 - x^2)^{\frac{3}{2}}$.

500. $\dfrac{1}{(a^2 - x^2)^{\frac{3}{4}}}$.

501. $\dfrac{1}{(a^2 - bx)^{\frac{2}{5}}}$.

502. $\dfrac{a}{\sqrt{(c^2 - 2x)}}$.

503. $\dfrac{a}{\sqrt[3]{(a^3 - y^3)^2}}$.

504. $\dfrac{a}{(a^4 - x)^{\frac{1}{3}}}$.

505. $\dfrac{ax}{(a^3 - x^2)^{\frac{1}{4}}}$.

506. $\sqrt[3]{\dfrac{a^2}{(a-y)^2}}$.

507. $\sqrt{\dfrac{a^3}{a^2 - x^3}}$.

508. $\sqrt[5]{\dfrac{x^3}{x-y}}$.

509. $\dfrac{x^2}{\sqrt[3]{x^4 - y}}$.

510. $\sqrt{\dfrac{a^5}{(a-b)^3}}$.

511. $\dfrac{a^2}{\sqrt[3]{a^3 - ab^2}}$.

512. $\dfrac{1}{\sqrt[3]{(a^2 - bc^3)^5}}$.

513. $\dfrac{a}{\sqrt[3]{a^2x + b^2}}$.

514. $\sqrt{\dfrac{xy}{x^2 - y^2}}$.

515. $\dfrac{x^2}{\sqrt[3]{x^4 - y}}$.

516. $\sqrt[4]{\dfrac{y}{y^2 - z^3}}$.

517. $\left(\dfrac{a-x}{a+x}\right)^{\frac{1}{2}}$.

518. $\dfrac{a^3}{\sqrt[5]{(a - 2y^3)}}$.

519. $\dfrac{ax}{\sqrt[3]{a^3x^2 - z}}$.

520. $\dfrac{a^2b^{\frac{1}{2}}}{\sqrt[3]{a^2 - bc}}$.

521. $\sqrt[3]{a - \sqrt{b}}$.

522. Expand $\sqrt{\dfrac{a+x}{a-x}}$ by the binomial theorem; and employ the result for the determination of the value of $\sqrt{2}$, to five places of decimals.

523. Find the sixth term of the expansion of $\dfrac{x^3y^2}{\sqrt[3]{(x^2+y)}}$ by the binomial theorem, without determining any other term.

PERMUTATIONS AND COMBINATIONS.

524. How many signals may be made with six flags which can be hoisted any number at a time and of which no two are of the same colour ?

525. What number of signals can be made with eight flags of different colours, which can be hoisted any number at a time above one another ?

526. How many rounds can be fired by a field battery of four guns, supposing the guns to be fired in a different order each round ?

527. How many different signals can be made with ten flags, of which three are white, two red and the rest blue, always hoisted, all together, above one another ?

528. How many signals can be made with 7 flags, viz. 2 red, 1 white, 3 blue, and 1 yellow, when all displayed together, one above another, for each signal ?

529. Find the number of signals which can be made with four lights of different colours which can be displayed in any number at a time, arranged either above one another, side by side, or diagonally.

530. How many night signals can be made with four different coloured lights capable of being hoisted two at a time, both above one another and side by side ?

531. In how many different ways may the eight men serving a field gun be arranged, so that the same man may always lay the gun ?

532. How many signals could be made by five bugle-notes, two of which are the same, blown either singly or any number in succession as one signal, and in any order ?

533. How many signals can be made with five lights of different colours, which can be displayed either singly or any number at a time side by side, or one above another?

534. A regiment consisting of 10 companies has to detach two of them each day for guard-mounting; how many days can this service continue to be performed before there is a necessity for any, the same, two companies being on guard again together?

535. How many distinct sound signals can be made by firing, in different orders of succession, four 6-pounders and two 18-pounders; supposing all of the guns to be used for each signal?

536. In how many different ways can the letters of the word *Examination* be written, using all the letters?

537. With ten soldiers and eight sailors, how many different parties can be made, each party consisting of three soldiers and three sailors?

538. How many different arrangements may be made of eleven cricketers, supposing the same two always to bowl?

539. How many signals can be made with three blue and two white flags, which can be displayed singly or any number at a time one above the other?

540. From a company of 90 men, 20 are detached for mounting guard each day; how long will it be before the same 20 men are on guard together, supposing the men to be changed as much as possible, and how often will each man have been on guard?

541. Five flags of different colours are capable of being hoisted either singly or any number at a time one above another; how many distinct signals can be made with them?

542. A chess player being allowed by his opponent to change the position of his remaining pieces, consisting of two knights, five pawns, a bishop, a castle and the king, in any way he pleases,

provided he use always the same squares; how long would he be trying all these changes, supposing each change to occupy one minute, and that he sit at the board nine hours each day?

543. A piquet of 24 men is required to mount double sentries at four important posts of observation; and it is desirable that no two men should be posted together more than once: how many times can the posts be relieved, supposing each of the three reliefs to consist always of the same eight men?

544. Supposing that a man can place himself in three distinct attitudes, clearly visible at a distance, how many signals can be thus made by four men all placed side by side?

PILES OF SHOT.

545. Investigate the expression for the number of shot in a square pile of n courses.

546. From the formula for the number of shot in a square pile of n courses, deduce the formula for a rectangular pile, having n and $m+n$ shot respectively in the two sides of the lowest course.

547. Investigate the formula for the number of shot in a triangular pile of n courses, and determine what must be the number of shot in the side of a triangular pile which is intended to contain 286 shot.

548. Three rows of the base of a square pile of twenty courses, with the shot supported by them, were removed; how many were thus obtained?

549. The number of shot in the upper course of a square pile is 169 and in the lowest course 1089; how many shot are there in the pile?

550. Find the number of shot in a rectangular pile having 17 shot in one side and 42 in the other side of the base; find also the number when there are only 12 courses complete.

551. Find the number of shot in an unfinished rectangular pile of shot having 27 and 9 shot respectively in the shorter sides of the base and upper course, and 50 in the longer side of the base.

552. Give the formula for the number of shot in a triangular pile; and find by how many shot one side of the base of a complete square pile of 15 courses must be increased, so that the rectangular pile so formed may contain 1960 shot.

553. The value of a triangular pile of 18 lb. shot is £76. 10*s.*; how many shot are there in its base; iron being 14*s.* per cwt.?

554. How many shot must be removed from a rectangular pile containing 884 shot so as to leave 8 courses, the number in the longest side of the bottom row being 15?

555. By how many shot must the shortest side of a rectangular pile, having 24 and 40 shot in the bottom rows, be diminished, in order that the remaining pile may contain 7070 shot?

556. The number of shot in the longest side of a rectangular pile is 21, and the number of shot in the pile is 2176; what is the number of shot in the other side?

557. How many shot must there be in the lowest course of a triangular pile so that 10 courses of it, commencing at the base, may contain 36780 shot?

558. Find the number of shot in a complete rectangular pile of 15 courses having 20 shot in the longest side of its base.

559. Find the number of shot in an unfinished rectangular pile of 20 courses, the number in the sides of its upper course being 11 and 18.

560. If from a complete square pile of shot a triangular pile of the same number of courses be formed; shew that the remaining shot will be just sufficient to form another triangular pile, and find the number of shot in its side.

561. What is the number of shot required to complete a rectangular pile having 15 and 6 shot respectively in the sides of its upper course?

562. Investigate the expression for the number of shot in a square pile of n courses; and, by means of it, determine the number of shot in a square pile of 10 courses, having 16 shot in the bottom row.

563. Find the number of shot in the longer side of the lowest course but three of a rectangular pile when the short side of that course contains 17 shot and the whole pile 3920.

564. In a rectangular pile of six courses, the number of shot in the top course is 198, and the number in the longest side of the base is 27; how many shot are required to complete the pile?

565. Determine the number of shot included between the fourth and twelfth courses (commencing at the base) of each kind of pile; the number of courses in each complete pile being 17, and the value of m in the rectangular pile being 7.

566. Find the number of shot in the bottom row of a square pile which contains 2600 more shot than a triangular pile of the same number of courses.

567. Find the number of shot in a complete square pile, in which the difference between the number of shot in the base and the number in the fifth course above it is 225.

568. Find an expression for the sum of n terms of the series $1 . 2 + 2 . 3 + 3 . 4 + \&c.$; and shew, by the series itself, as well as by the form of the result, that the sum is equal to twice the number of spherical shot in a triangular pile of n courses; or to

the number of spherical shot in a square pile of n courses together with the number in one of its faces.

569. Find the number of shot in two unfinished piles of 14 courses each; the one being triangular, the other a square pile, and both having 25 shot in the bottom row.

570. The number of shot in a triangular pile is to the number in a square pile of the same number of courses as 22 : 41; find the number of shot in each pile.

571. It is found that ten times the number of shot in a particular triangular pile is equal to six times the number in a square pile of the same number of courses; how many are there in that square pile?

572. The number of shot in a triangular pile is to the number in a square pile of the same number of courses, as 9 : 17; find the number of shot in each pile.

573. Find the number of shot in two unfinished piles of 14 courses each; the one being triangular, the other a square pile, and both having 25 shot in the bottom row.

574. Having at command a space convenient in shape for the erection of a square pile of shot, with a triangular pile of the same number of courses at one of its sides, how many shot must I place at the bottom row of each, so that the whole may contain 64975 shot; the bottom row of one side being common to both piles?

575. Show that the number of shot in a complete triangular pile is four times the number in a complete square pile of half the number of courses: and find the number of shot in a complete rectangular pile having 43 and 17 shot in the sides of its base.

576. Find expressions in terms of n for the number of shot in the fifth course (commencing at the base) of each kind of pile of n courses; using $m+1$ to represent the number of shot in the top ridge of the rectangular pile.

577. Show that if there be a rectangular pile, a square pile, and a triangular pile of shot, all of the same number of courses, and if the number of shot in the top ridge of the rectangular pile be one-third of the number obtained by adding 2 to the number of courses, then the number of shot in these piles must be in arithmetical progression whatever be the number of courses.

578. Show that when the three kinds of pile have the same number of courses, the difference between the number of shot in the complete rectangular pile, and the number in the complete square pile, is m times the number in the lowest course of the triangular pile; m being the difference of the number of shot in the two sides of the base of the rectangular pile.

579. Prove that the difference between the number of shot in a square pile and in a triangular pile of any, the same, number (n) of courses, is $\frac{n}{6}(n^2-1)$; and show that this difference is equal to one-third of the number of shot in a triangular face of either pile, multiplied by one unit less than the number of courses.

580. Find the number of shot in a rectangular pile in which the number in the lowest course is 600 and in the top ridge 11.

581. There are three piles of shot, a triangular, a square, and a rectangular pile, in which the number of courses is the same and the numbers of shot are in arithmetical progression; show that this can only be the case when $n-1$ is a multiple of 3; and find the number of shot in the two former piles when the number in the rectangular pile is 5566.

582. How many eight-inch shells would there be in a square pile erected upon the ground recently occupied by a square pile of 4900 thirteen-inch shells?

583. The number of shot in an incomplete square pile is equal to six times the number of shot required to complete it; and the number of completed courses is equal to the number required to complete it: find the number of shot in the incomplete pile.

584. Show that the sum of the number of shot in a square pile, and in a triangular pile of the same number of courses (n), is $\frac{n(n+1)^2}{2}$, and the difference $\frac{n(n^2-1)}{6}$.

585. Show that the arithmetical mean between the number of shot in a complete triangular and in a complete square pile is equal to $n+1$ times half the number of shot in one of the faces of either pile; n being the number of courses in each pile.

586. A square pile consists of n courses; investigate an expression for determining the number of shot in the pile: and find the number of shot in an incomplete square pile of 12 courses, the top course containing 169 shot.

587. If there be a triangular pile of shot and a square pile of the same number of courses; show that three times the number of shot in the first is to the number in both together, as $(n+2)$ to $(n+1)$.

588. Sum the series $a+1+2(a+2)+3(a+3)+4(a+4)+$ &c. to n terms; and show, by a figure, the relation of the series to a rectangular pile of spherical shot.

LOGARITHMS.

589. Give a definition of the logarithm of a number, and write down the equation which must subsist between a number (n), its logarithm (z), and the base of the system (a).

590. Define the term "*Logarithm*," and show that, in the expansion

$$a^x = 1 + Ax + \frac{A^2x^2}{1.2} + \frac{A^3x^3}{1.2.3} + \text{\&c.},$$

$$A = \log_e a\text{; and that } \log_e a \,.\, \log_a e = 1,$$

where e is the value of a, which makes $A = 1$.

591. Prove that

$$\log_e a = a - 1 - \frac{1}{2}(a-1)^2 + \frac{1}{3}(a-1)^3 - \&c.$$

592. Assuming the complete expansion of a^x, show that the value of a which makes the above value of $A_{,}$ equal to unity, is

$$1 + 1 + \frac{1}{1\,.\,2} + \frac{1}{1\,.\,2\,.\,3} + \frac{1}{1\,.\,2\,.\,3\,.\,4} + \&c.$$

and give the approximate numerical value of this series.

593. Show that $\log_e a \,.\, \log_a e = 1$, e being the base of the Napierian system, and a any other number; show also that it is true when $a = e$.

594. Prove that $\log(xy) = \log x + \log y$,

$$\log \frac{x}{y} = \log x - \log y,$$

and $$\log\left(x^{\frac{1}{n}}\right) = \frac{1}{n}\log x,$$

all the logarithms being in the same system.

595. Show that $\log_a x = \dfrac{\log_e x}{\log_e a}$, a and x being any real positive quantities, and e being the base of the Napierian system of logarithms. And point out the utility of this equation, showing also what it becomes when $a = e$.

596. Define the *logarithm* of a number; and from your definition find the logarithm of 144 to the base $2\sqrt{3}$. Prove also that $\log_a n : \log_b n :: \log b : \log a$, and thence show that

$$\log_e a \times \log_a e = 1.$$

597. What is the log 81 in the system whose base is 3?

598. What is the logarithm of $\frac{1}{3}$ in the system whose base is 27?

599. Given $\log_e 3 = 1{\cdot}0986123$ and $\log_e 2 = {\cdot}6931472$, find $\log_2 6$.

600. Given $\log 2 = 0{\cdot}30103$ and $\log 70 = 1{\cdot}8450980$, find $\log 1{\cdot}96$ and $\log {\cdot}00448$.

601. State the equation by means of which, from a table of Napierian logarithms, we may determine the logarithms of the same numbers in another system whose base is one of the numbers in that table.

602. Given $\log 2 = {\cdot}30103$, find $\log 3{\cdot}2$, $\log 2{\cdot}5$, and $\log 12{\cdot}8$.

603. Given $\log_e 2 = {\cdot}6931472$ and $\log_e 5 = 1{\cdot}6094379$; find the common logarithm of $6{\cdot}4$.

604. Given $\log_{10} 2 = {\cdot}30103$ and $\log_{10} 3 = {\cdot}4771213$; find $\log_{10} 15$ and $\log_{10} \sqrt{({\cdot}0075)}$.

605. Given

$$\begin{aligned} \log_e 10 &= 2{\cdot}3025851, \\ \log_e 6 &= 1{\cdot}7917595, \\ \log_e 2 &= 0{\cdot}6931472, \end{aligned}$$

find com. $\log 9$, and the moduli of the systems whose bases are respectively 2, 3, and 5.

606. Given

$$\begin{aligned} \log_e 10 &= 2{\cdot}3025851, \\ \log_e 7 &= 1{\cdot}9459101, \\ \log_e 2 &= 0{\cdot}6931472, \end{aligned}$$

find the common $\log 2{\cdot}8$, and also $\log_2 7$.

607. Given $\log_e 5 = 1{\cdot}6094379$ and $\log_e 2 = 0{\cdot}6931472$, find common $\log {\cdot}8$.

608. Given $\log_e 3 = 1{\cdot}0986123$ and $\log_e 6 = 1{\cdot}7917595$; find $\log_3 12$.

609. Given $\log 16 = 1{\cdot}20412$, find $\log 2{\cdot}5$.

610. Given $\log 2 = {\cdot}30103$, and $\log 3 = {\cdot}4771213$; find $\log {\cdot}005$, and $\log 6{\cdot}48$.

611. Given $\log 2 = 0\cdot30103$, and $\log 3 = 0\cdot4771213$; find $\log 5$, $\log 18$, $\log 135$, and $\log 750$.

612. If $x = \log_a n$, and $y = \log_b n$; show that

$$\log_b a = \frac{y}{x}, \text{ and } \log_a b = \frac{x}{y}.$$

613. Assuming that you know the Napierian logarithm of 2, what values would you substitute for n and z in the formula

$$\log_e (n+z) = \log_e n + 2\left\{\frac{z}{2n+z} + \frac{z^3}{3(2n+z)^3} + \frac{z^5}{5(2n+z)^5} + \&c.\right\}$$

in order to obtain the Napierian logarithm of 12?

614. Assuming the "Exponential Theorem," show that

$$\log_e \frac{1+x}{1-x} = 2 \,.\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \&c.\right).$$

And prove from first principles that

$$\log_a x = \frac{\log_e x}{\log_e a}.$$

615. Write down the expression for $\log(n+z)$ and state the steps you would take to calculate the numerical value of $\frac{1}{\log_e 10}$, the modulus of the common system of logarithms.

616. Given $\log_e 5 = 1\cdot60944$, calculate $\log_e 7$ to five places.

617. Find the value of x in the equation $5^x = 300$.

618. Find the values of x and y in the equations

$$2^x = 7y; \quad 3^x = 10y.$$

619. Find the values of x and y from the equations

$$\frac{3^x}{2^y} = 4 \text{ and } 7^x = 3^y.$$

620. Find the values of x and y in the equations

$$3^y = \frac{27}{3^x}, \quad 2^y = 2^x \times 4.$$

621. Given $2^x . 7^y = 80000$ and $3^y = 500$, find x and y.

622. Find the value of x in the equations:

(1) $3^x = 729$. (2) $a^{4x} . b^{2x} = c$.

623. Find the values of x and y in the simultaneous equations $a^{5x} . b^{3x-7} = c^{2y}$ and $d^y = b^{4x}$.

624. If a series of numbers be in geometric progression, prove that their logarithms are in arithmetic progression.

625. What term of the series

$$\frac{2}{5} + \frac{8}{15} + \frac{32}{45} + \frac{128}{135} + \&c. \text{ is } \frac{2097152}{295245}?$$

626. Find, by logarithms, the value of $\sqrt[5]{(bc)} + \sqrt{\left(\frac{ac}{b}\right)}$,

when $a = 6340{\cdot}518$, $b = 7{\cdot}360591$, $c = {\cdot}003758426$.

627. Point out the principle and use of the "*Arithmetic Compliment*" in logarithmic calculations and calculate, by logarithms, the value of

$$\sqrt[5]{\frac{{\cdot}00754326^2 \times 78{\cdot}34295 \times 8172{\cdot}371^{\frac{1}{3}} \times {\cdot}00052}{64285{\cdot}71^{\frac{1}{3}} \times 154{\cdot}27^4 \times {\cdot}001 \times 586{\cdot}7983^{\frac{1}{2}}}},$$

taking care to arrange the work in a proper form.

628. Find the value of

$$\sqrt[5]{\frac{15{\cdot}832^3 \times 5793{\cdot}64^{\frac{1}{3}} \times {\cdot}7842613}{{\cdot}000327^{\frac{1}{3}} \times 768{\cdot}94^2 \times 3015{\cdot}28 \times {\cdot}007^{\frac{1}{2}}}},$$

by the aid of logarithms.

629. Find the value of

$$\frac{3\cdot5724^2}{408\cdot62} \times \sqrt[5]{\frac{\cdot00753286 \times 3426\cdot002^2 \times 7\cdot854}{30\cdot47269^3 \times \sqrt{\cdot03278971} \times \cdot5163084^2}},$$

by logarithms.

630. Find the value of

$$\sqrt[7]{\frac{\cdot00357 \times 628\cdot4931^2 \times 73\cdot0875^3}{49\cdot3284^5 \times \cdot031264^2 \times \cdot00529}} - \sqrt[3]{\frac{\cdot0047238^2}{\cdot5423701}},$$

by logarithms.

631. Find, by the aid of logarithms, the value of

$$\sqrt[5]{\frac{7\cdot189546 \times 4764\cdot2^2 \times \cdot00326^5}{\cdot0004895361 \times 457^3 \times 5764\cdot387^2}}.$$

632. Find, by logarithms, the value of

$$\sqrt[5]{\frac{3\cdot14159 \times 4771\cdot213 \times 2\cdot718282^{\frac{1}{2}}}{30\cdot103^4 \times \cdot4342945^{\frac{1}{2}} \times 69\cdot897^4}}.$$

633. Find, by logarithms, the value of

$$\sqrt[7]{\frac{\cdot03271^2 \times 53\cdot42986 \times \cdot7754231^3}{32\cdot76894 \times \cdot000371^4}}.$$

634. Find, by logarithms, the value of

$$\sqrt[3]{\frac{732\cdot0561^2 \times \cdot0003572^4 \times 8979306}{4227\cdot984^3 \times 3\cdot457391 \times \cdot0026518^5}}.$$

635. Find the value of $\sqrt[3]{\left(\frac{7932 \times \cdot00657 \times \cdot8046392}{\cdot03274 \times \cdot6428}\right)}$, by logarithms; and find the values of x and y in the equations

$$a^x = b \text{ and } x^y = c.$$

636. Find the value of

$$\sqrt[7]{\left\{\frac{7812\cdot934^2 \times \sqrt[3]{(18\cdot5374)} \times \sqrt{(\cdot7526821)} \times \cdot6173958}{\sqrt{(59\cdot60827)} \times \sqrt[3]{(756\cdot0083)} \times 1726\cdot953 \times \sqrt{(\cdot0733)}}\right\}},$$

by the use of logarithms.

637. Find the value of

$$\sqrt[3]{\frac{7{\cdot}120635 \times \sqrt{({\cdot}13274)} \times {\cdot}05738921}{\sqrt{({\cdot}4346829)} \times 17{\cdot}3854 \times \sqrt{({\cdot}0096372)}}},$$

by the use of logarithms.

638. Find, by the aid of logarithms, the value of

$$\left\{\frac{3{\cdot}075526^2 \times 5771{\cdot}213^{\frac{1}{2}} \times {\cdot}0036984^{\frac{1}{5}} \times 7{\cdot}74}{7225851 \times 327{\cdot}9341^3 \times {\cdot}8697003^5}\right\}^{\frac{3}{5}}.$$

639. Find the value of

$$\sqrt[6]{\frac{7056421 \times 301{\cdot}572^3 \times 7{\cdot}830721^{\frac{1}{2}} \times {\cdot}54836}{{\cdot}416327^{\frac{1}{3}} \times 6284731^2}},$$

by logarithms.

640. Find the value of the following expression by logarithms:

$$\sqrt[3]{\frac{3{\cdot}576428 \times {\cdot}008630472 \times 598163{\cdot}2 \times \sqrt[5]{(21\tfrac{1}{4})}}{286{\cdot}9734 \times \sqrt[5]{({\cdot}0069847)} \times 2708637 \times \sqrt[5]{({\cdot}876)}}}.$$

641. Calculate by logarithms the value of the expression

$$\frac{a^2 . c . d^{\frac{1}{2}}}{e . f^{\frac{1}{3}}} \times \sqrt[7]{\frac{a . b^{\frac{1}{5}} . p^2 . r^{\frac{1}{3}} . s}{o^3 . k^2 . m . n}},$$

when $a = 574{\cdot}3268$, $f = {\cdot}03572$, $o = 21{\cdot}3641$,
$b = {\cdot}0372$, $p = 32{\cdot}6841$, $k = 3{\cdot}87964$,
$c = 73{\cdot}5486$, $r = 78561{\cdot}32$, $m = 76{\cdot}49573$,
$d = 1807{\cdot}463$, $s = 5736{\cdot}058$, $n = 12{\cdot}86097$,
$e = 179{\cdot}6284$.

642. Find, by the aid of logarithms, the value of

$$\sqrt[3]{\frac{{\cdot}0057146 \times 73{\cdot}24981 \times 2793{\cdot}468}{774{\cdot}2605 \times {\cdot}0000829 \times 3461{\cdot}724}}.$$

643. Find, by logarithms, the value of

$$\sqrt[3]{\frac{(36{\cdot}51942)^2 \times ({\cdot}005327481)^3 \times 863157{\cdot}6}{({\cdot}07136529)^2 \times (73{\cdot}69421)^3}}.$$

644. If a person be entitled to receive an annuity of £64 for ten years; what is the sum which he should receive in one present payment in lieu of the annuity, assuming 4 per cent. compound interest?

PROBABILITIES.

645. The probabilities of three Cadets *A*, *B*, and *C* qualifying at this examination, are respectively $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{7}{12}$; what is the probability of one at least of them qualifying?

646. A man makes six "outers," four "centres" and two "bull's-eyes" in twenty rounds rifle practice; what is the probability of his hitting the target every time in the next four rounds?

647. What are the odds in favour of throwing "aces" at least once in three throws with a pair of dice?

GEOMETRICAL DEDUCTIONS.

648. Prove that if the points of bisection of the sides of a triangle be joined, the triangle so formed will be equal to one-fourth of the whole triangle.

649. Divide a triangle into two parts which shall have the same ratio to one another as two of the sides of the triangle, by a straight line drawn through the angular point common to those sides.

650. It is required to cut off from a given triangle one-fourth part of it, by a straight line drawn parallel to one of its sides.

651. If a circle be described upon a side of a triangle as diameter, and its points of section with the other two sides be joined; the triangle so formed will be similar to the whole triangle.

652. If two straight lines be drawn from the same point, cutting a circle, and the alternate points of section be joined; the triangles so formed will be similar to one another.

653. If through the angles of an isosceles triangle which has each of the angles at the base double of the third angle, and is inscribed in a circle, straight lines be drawn touching the circle, an isosceles triangle will be formed which has each of the angles at the base one-third of the angle at the vertex.

654. From one of the angles C, at the base of an isosceles triangle ABC, of which the vertex is A, it is required to draw a straight line CD to meet AB in D, so that CD shall be a mean proportional between AC and BD.

655. If, at any point D, in the side AC of the triangle ABC, a straight line DE be drawn making an angle ADE equal to the angle ABC, and meeting AB in E, prove that if DB and EC be drawn, the angle ADB is equal to the angle AEC.

656. If from any point in the common tangent to two circles drawn through the point where they touch each other externally, as a centre, a circle be described cutting those circles; and straight lines be drawn from that centre through these points of intersection; then the other points of intersection of the straight lines with the circles will also be in the circumference of a circle.

657. If from the angle A of a triangle ABC, a straight line AD be drawn to bisect that angle, and to cut the opposite side in D; and from D, two straight lines be drawn respectively parallel to the remaining sides; the parallelogram thus formed will have all its sides equal, and each of them will be a mean proportional between the remaining segments of the two sides of the triangle.

658. If from the angles A and C, of the triangle ABC, perpendiculars, AD and CE, be let fall upon the opposite sides, prove that DE being joined, the angle CAD is equal to the angle CED; and that the rectangle CB, BD is equal to the rectangle AB, BE.

659. Find a mean proportional between two given straight lines. Also construct an arithmetic mean, and a harmonic mean between two given straight lines.

660. If two contiguous chords be drawn in a circle, and perpendiculars be let fall upon them from the centre, the angle contained by these perpendiculars will be equal to the angle in the remaining segment formed by joining the extremities of the chords.

661. Divide geometrically a given straight line into two parts, so that the square described on the greater part shall be double of the square described on the other part.

662. Describe a circle which shall touch a given circle, and also touch a given straight line in a given point.

663. Bisect a given triangle by a straight line drawn through a given point in any one of its sides.

664. If a straight line be drawn to bisect the angle formed by two straight lines, one of which is drawn to the focus from any point in the parabola, and the other perpendicular to the directrix, that line will be a tangent to the parabola at that point.

665. Describe a circle which shall touch a given straight line in a given point and also touch a given circle.

666. Prove that the three straight lines drawn from the angles of a triangle to the points of bisection of the opposite sides pass through the same point.

667. From a given point without a circle draw a straight line such that the part of it intercepted within the circle shall be equal to the side of the inscribed equilateral pentagon.

668. If two circles cut one another, all parallel straight lines drawn through the points of section and intercepted by the outer circumferences of the circles will be equal.

669. If a circle be described about a triangle, and a perpendicular from the centre upon one of the sides be produced to meet the circle, the line which joins this intersection with the opposite angle of the triangle will bisect that angle.

670. If two circles touch each other and also touch a straight line, the part of the straight line between the points of contact is a mean proportional between the diameters of the circles.

671. If any point be taken in the circumference of a circle, and lines be drawn from it to the three angles of an inscribed equilateral triangle, prove that the middle line so drawn is equal to the sum of the other two.

APPLICATION OF ALGEBRA TO GEOMETRY.

672. Given r and r_1, the radii of two concentric circles, and l the distance from their common centre of a point A (exterior to both of them); find the point K in the circumference of the outer circle, such that AK produced shall have its segment within that circle divided by the circumference of the other into three equal parts.

673. Given the segments of the hypothenuse of a right-angled triangle made by the straight line which bisects the right angle; determine the length of that line, the sides of the triangle, and the perpendicular from the right angle on the hypothenuse.

674. Show that the length of the perpendicular let fall from the opposite angle of a triangle whose sides are a, b, c upon the side c, is expressed by

$$\frac{2}{c}\sqrt{s(s-a)(s-b)(s-c)},$$

where $s = \frac{1}{2}(a+b+c)$.

675. Given the area of a triangle $=a^2$, and that of its inscribed square $=s^2$; find the perpendicular and the base of the triangle.

676. Find the point in the side of a square, through which, if a straight line be drawn to the opposite angle, the triangle so formed will be one-third of the square.

677. Given the ratio of the sides of a triangle, together with both the segments of the base made by a perpendicular from the opposite angle; find the sides.

678. Find the radius of the circle which circumscribes a triangle whose sides are a, b and c.

679. From a given point without a given circle draw a straight line which shall be divided in extreme and mean ratio by the circumference.

680. Given the base of an isosceles triangle, determine the sides when the diameter of the circumscribing circle is d.

681. In a triangle ABC, given $AC=CB$, $AB=b$, CD (the perpendicular on AB) $=a$, and DE a segment of the perpendicular $=\frac{a}{n}$: it is required to draw through E a straight line FEG, which shall bisect the triangle.

682. Given the two sides a and b of a triangle and the length of the line d, which bisects their contained angle, find the segments into which it divides the base.

683. One angle of a triangle, the perpendicular from that angle upon the opposite side, and one of the segments into which the perpendicular divides that side, being given, find the other segment.

684. The base of a triangle is 25, the perpendicular let fall upon it from the opposite angle is 12, and the rectangle contained by the sides is 300; find the sides. Explain the meaning of the 4 answers.

DESCRIPTIVE GEOMETRY.

Orthographic and Horizontal Projections.

685. Define the terms "line of level," "planes of projection," "trace," and "plan."

686. Show that the straight line which joins the projections of the same point is perpendicular to the line of level.

687. If a plane be perpendicular to one of the planes of projection, prove that its trace upon the other plane is perpendicular to the line of level.

688. Prove that the horizontal and vertical projections of a point are in the line drawn through either of them, perpendicular to the line of level.

689. State the principles upon which the projections of the intersection of two planes whose horizontal and vertical traces are given, may be determined.

690. Prove that the horizontal trace of the vertical projecting plane of a straight line is perpendicular to the line of level.

691. The plan and elevation of a straight line being given, find its traces.

692. The traces of a straight line being given, determine its plan and elevation.

693. Find the intersection of a given line with a given plane.

694. Find the point of intersection of three given planes.

695. The plan of a straight line in a given plane being given, find the elevation of that line.

696. Through a given point draw a plane parallel to two given straight lines.

697. From a given point draw a straight line perpendicular to a given plane, and find its true length.

698. Construct the angle of inclination of two given planes.

699. The true representation of any plane rectilineal figure upon a given plane, being given, find the plan and elevation of the figure.

700. If $(a_{16} b_{10})$ be the horizontal projection of a straight line; state clearly the principles on which you would determine its absolute length, and its inclination to the horizon.

701. The horizontal projections and indices of two points being given, state how the inclination to the horizontal plane of the straight line joining these points may be determined.

702. If the horizontal projections of the points a_{10}, b_{15} be twenty units of the scale of the plan distant from one another, at what distance from the former will be the horizontal trace of the line joining those points?

703. The scales of slope of two planes being given, determine the projection of the line of intersection of those planes.

704. The scale of slope of a plane being given; show how its trace upon the horizontal plane may be found.

705. Show that in a system of horizontal contours the increasing proximity of the lines indicates an increasing degree of slope; and *vice versâ.*

PLANE TRIGONOMETRY.

706. Define "the chord," "the tangent," and "the secant" of an arc; and show that $\text{chd}\ 2a = 2 \sin a$.

707. Express the "tangent" and "secant" of an arc in terms of its "sine."

708. Show that if a^0 be the angle in a segment of a circle whose radius is r, the base of the segment is $2r \sin a^0$.

709. The difference of two arcs of a circle is 20 grades, and their sum is 48 degrees; find the arcs.

710. Find, geometrically, the values of $\tan 30^0$, $\tan 45^0$, and $\tan 60^0$.

711. Find the angle whose circular measure is $\frac{2}{3}$.

712. Show that
$$\sin 75^0 = \frac{\surd(3)+1}{2\surd 2},$$
and
$$\sin 15^0 = \frac{\surd(3)-1}{2\surd 2}.$$

713. Give accurate line-definitions of the sine, tangent, secant, and cosine of an arc of a circle; and determine the numerical values of $\cos 30^0$, $\sin 45^0$, $\sec 45^0$, and $\tan 120^0$; the radius of the circle being unity in the first two examples, and 10 in the latter two.

714. Trace by a figure the changes in the cosine of an arc of a circle, as the arc varies from 0 to 360^0.

715. The radius of a circle being 16 feet, find the number of degrees, &c. in an arc of 6 feet, and also the circular measure of the subtending angle at the centre.

716. Find the values of $\tan 30^0$, $\tan 60^0$, $\tan 45^0$ and $\tan 7^0 30'$, and thence determine $\tan 15^0$ and $\tan 75^0$.

717. Find the length of the arc whose circular measure is 2.

718. Prove, geometrically, that
$$\sin a = \frac{1}{\operatorname{cosec} a}, \quad \tan a = \frac{1}{\cot a}, \quad \text{and} \quad \tan a = \frac{\sin a}{\cos a};$$
and thence show that
$$\sin a \,.\, \cos a = \frac{1}{\tan a + \cot a}.$$

719. In a right-angled triangle ABC, express the ratios

$$\frac{AB}{AC}, \frac{AB}{BC}, \frac{AC}{CB}, \frac{BC}{AB},$$

as trigonometrical functions of the angle A, when C is the right angle.

720. The length of an arc in inches is expressed numerically by five times its circular measure, and by ten times the reciprocal of its angular measure in degrees; find the length of the radius, and the number of degrees in the arc.

Prove the relations:

721. $\operatorname{cosec} a \,.\, \cos^2 a = \operatorname{cosec} a - \sin a.$

722. $\cot a \,.\, \operatorname{cosec} a = \dfrac{1}{\sec a - \cos a}.$ 723. $\cos 2\theta = \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}.$

724. $\dfrac{\cos a + \sin a}{\cos a - \sin a} = \tan 2a + \sec 2a.$

Investigate the following relations:

725. $\sin A + \sin B = 2 \,.\, \sin \tfrac{1}{2}(A + B) \cos \tfrac{1}{2}(A - B).$

726. $\cos 30^\circ - \cos 70^\circ = 2 \sin 50^\circ \,.\, \sin 20^\circ.$

727. $\tan \dfrac{A}{2} = \dfrac{1 - \cos A}{\sin A}.$

728. $\tan^2 a + \cot^2 a = 2 + 4 \cot^2 2a.$

Prove the relations:

729. $\tan \dfrac{A}{2} = \dfrac{\sin A}{1 + \cos A}.$

730. $\dfrac{\tan A + \tan B}{\cot A + \cot B} = \tan A \,.\, \tan B.$

731. $\cos (A + B) \,.\, \cos (A - B) = \cos^2 A - \sin^2 B.$

732. An arc of 40° on a circle whose radius is 6 inches is found to measure a inches; and this length applied to the cir-

cumference of another circle is found to cover an arc of 25°; what is the radius of the latter circle?

733. Give the algebraic signs of the sine, cosine, tangent and secant of each of the arcs 70°, 110°, 195°, and 280°; illustrating the result given by a geometrical figure.

734. Prove that $\sin 30^\circ = \frac{1}{2}$, and $\sin 45^\circ = \cos 45^\circ = \frac{1}{2}\sqrt{2}$ to radius unity; and thence find the values of $\tan 30^\circ$, $\sin 75^\circ$, and $\cos 25^\circ\ 30'$, to radius 10.

735. Prove that $\sin \alpha = -\cos\left(\frac{3\pi}{2} - \alpha\right)$

and $\tan \alpha = \sec \alpha \,.\, \operatorname{cosec} \alpha$.

Prove the relations:

736. $\sin\left(\frac{\pi}{2} + \alpha\right) = \cos \alpha$. 737. $\cos\left(\frac{\pi}{2} + \alpha\right) = -\sin \alpha$.

738. $\tan(\pi - \alpha) = -\tan \alpha$. 739. $\tan(\pi + \alpha) = \tan \alpha$.

740. $\cot \alpha + \tan \alpha = 2 \operatorname{cosec} 2\alpha$.

741. $\dfrac{1 - 2\sin^2 \alpha}{1 + \sin 2\alpha} = \dfrac{1 - \tan \alpha}{1 + \tan \alpha}$. 742. $\cos \theta = \dfrac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$.

743. $\sec 2\phi = \dfrac{\cot \phi + \tan \phi}{\cot \phi - \tan \phi}$.

744. $1 + \cos 2\theta \,.\, \cos 2\phi = 2(\sin^2 \theta \,.\, \sin^2 \phi + \cos^2 \theta \,.\, \cos^2 \phi)$.

745. $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{3}{5} = 45^\circ$.

746. $\sin 2\theta = \tan \theta \,.\, (1 + \cos 2\theta)$. 747. $\cos 2\alpha = \dfrac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha}$.

748. $\tan^2 \alpha + \cot^2 \alpha + 2 = \sec^2 \alpha \operatorname{cosec}^2 \alpha$.

749. $\dfrac{\cos \alpha + \cos \beta}{\cos \alpha - \cos \beta} = \cot \dfrac{\alpha + \beta}{2} \,.\, \cot \dfrac{\alpha - \beta}{2}$.

750. $\tan^2\frac{\theta}{2} = \frac{2\sin\theta - \sin 2\theta}{2\sin\theta + \sin 2\theta}$.

751. $\frac{\tan\alpha + \sec\alpha}{\cot\alpha + \operatorname{cosec}\alpha} = \tan\left(45 + \frac{\alpha}{2}\right)\tan\frac{\alpha}{2}$.

752. Prove that $\tan\left(\frac{\pi}{4} - \frac{i}{2}\right) = \sec i - \tan i$; and show that if

$$\cos(\alpha - \beta) \,.\, \cos(\theta + \phi) = \cos(\alpha + \beta)\cos(\theta - \phi),$$

then

$$\cot\alpha \,.\, \cot\beta = \cot\theta \,.\, \cot\phi.$$

Show that

753. $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$.

754. $\sec 2\alpha = 1 + \tan 2\alpha \,.\, \tan\alpha$.

755. $\tan^2\alpha - \sin^2\alpha = \tan^2\alpha \,.\, \sin^2\alpha$.

756. $\cos^{-1}\sqrt{\frac{2}{3}} - \cos^{-1}\frac{\sqrt{3} + \sqrt{2}}{2\sqrt{3}} = 60^\circ$.

757. $\sec\theta \pm \tan\theta = \cot\left(45^\circ \mp \frac{\theta}{2}\right)$. 758. $\sec^2\theta = \frac{2\sec 2\theta}{1 + \sec 2\theta}$.

759. $\sin\alpha = \frac{2}{\tan\frac{\alpha}{2} \,.\, \operatorname{cosec}^2\frac{\alpha}{2}}$.

760. $\cos^4\theta - \sin^4\theta = \cos 2\theta$.

761. $\frac{\sin\alpha}{1 - \cos\alpha} = \cot\frac{\alpha}{2}$.

762. $\frac{2\operatorname{cosec} 2\theta - \sec\theta}{2\operatorname{cosec} 2\theta + \sec\theta} = \cot^2\left(45^\circ + \frac{\theta}{2}\right)$.

763. $\cos\alpha = 1 - \frac{\sin^2\alpha}{\cos^2\frac{\alpha}{2}}$.

764. $2\sin(\alpha - \beta) \,.\, \cos\alpha = \sin(2\alpha - \beta) - \sin\beta$.

765. $2\operatorname{cosec} 2\theta = \sec\theta \,.\, \operatorname{cosec}\theta$.

766. *ABC* is a right-angled triangle, *B* being the right angle; prove by means of a figure, and Eucl. VI. 4, that

$$\cos A = \frac{AB}{AC}, \quad \tan A = \frac{BC}{AB}.$$

Prove that

767. $$\left(\sin\frac{\alpha+\beta}{2}+\cos\frac{\alpha+\beta}{2}\right).\left(\sin\frac{\alpha-\beta}{2}+\cos\frac{\alpha-\beta}{2}\right)$$
$$=\sin\alpha+\cos\beta.$$

768. $\tan^{-1}\dfrac{1}{\sqrt{2}}+\sin^{-1}\dfrac{1}{\sqrt{2}}=\tan^{-1}\dfrac{\sqrt{(2)}+1}{\sqrt{(2)}-1}$.

769. $\dfrac{1+\sin\alpha}{1-\sin\alpha}=\tan^2(45^\circ+\frac{1}{2}\alpha)$.

770. $\sin\theta=\dfrac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$.

771. $\tan 50^\circ+\cot 50^\circ=2\sec 10^\circ$.

772. $\tan^{-1}\dfrac{1}{3}+\tan^{-1}\dfrac{1}{5}+\tan^{-1}\dfrac{1}{7}+\tan^{-1}\dfrac{1}{8}=\dfrac{\varpi}{4}$.

773. Given $\operatorname{cosec}2\alpha-\sin 2\alpha=\tan\alpha$; find $\sin\alpha$.

774. Find the values of x and y in the simultaneous equation

$$2.\sin x=3\cos y \text{ and } 3\cot y=5\sec x.$$

775. If $\sin\theta=\sin 2\theta$, find $\cos\theta$; and find $\cos\phi$ when

$$\tan\phi=\operatorname{cosec}2\phi.$$

776. Show that, in any plane triangle,

$$\tan\frac{A-B}{2}=\tan(\phi-45^\circ).\tan\frac{A+B}{2},$$

where $\phi=\tan^{-1}\dfrac{a}{b}$.

777. If $\theta=\tan^{-1}1$, and $\phi=\tan^{-1}3$, find the values of $\theta+\phi$ and $\theta-\phi$, and thence show that $3\sin 2\theta=5\sin 2\phi$; interpret the negative signs wherever they appear in your result.

778. Find x from the equation

$$\frac{\cos(\beta+x)}{\cos(\alpha-x)}=\frac{m\sin\beta}{n\sin\alpha}.$$

779. Find the value of θ in the equation

$$\tan(45^\circ+\theta)=3\tan(45^\circ-\theta).$$

780. If

$$\cos(\alpha-\beta-\theta)\cos(\alpha+\beta)+\cos(\alpha+\beta+\theta)\cos(\alpha-\beta)=0,$$

find θ.

781. Show that if

$$\sin\alpha+\sin\gamma = 2\sin\beta\,.\cos(\beta-\alpha),$$

then α, β and γ are in arithmetical progression.

782. If $\tan(a+\theta)=n \operatorname{cosec} 2a-\cot 2a$; find θ.

783. If $\tan\beta=\mu$; find $\sin 2\beta$.

784. Given

$\sin\theta\,.\cos\phi=\sin\phi\,(2\cos\theta+7\sin\phi)$, and $\sin\theta=3\sin\phi$; find $\tan\phi$.

785. Find the value of the tangent of the arc

$$\tan^{-1}(a+b)+\tan^{-1}(a-b).$$

786. Given $\sin(A+B)=\frac{3}{10}\sec(A-B)$,

and $\sin(A-B)=\frac{1}{10}\sec(A+B)$;

find A and B.

787. A and B are two arcs of a circle, radius unity; find them, if

$$\frac{\sin A}{\sin B}=\surd 2,\ \text{and}\ \frac{\tan A}{\tan B}=\surd 3.$$

788. Show that, in a plane triangle, ABC:

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c},$$

and $\cos\frac{A}{2}=\sqrt{\frac{s\,.(s-a)}{b\,.c}}$ when $s=\frac{a+b+c}{2}$.

789. The sides of the triangle ABC being denoted by a, b, c, show that

$$a^2 = b^2 + c^2 - 2bc \cos A,$$

$$\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}.$$

Also, find the least and greatest angles of the triangle ABC, to the nearest second, by means of the formula for the tangent of half an angle, when

$$AB = 2264, \quad BC = 1854, \quad AC = 2016.$$

790. Two sides of a plane triangle are 10 and 20 and the included angle is 60°; find the remaining angles and side without the use of tables.

791. In a plane triangle given $a = 584{\cdot}7328$, $b = 367{\cdot}4001$, and $B = 37^\circ\ 42'\ 15''$; find the remaining angles and sides.

792. A, B, C; a, b, c being the angles and sides opposite to them in a plane triangle, prove the relations

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc},$$

and thence prove that

$$\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s\,.\,(s-a)}};$$

and $$a = (b-c)\sec\theta,$$

if $$\theta = \tan^{-1}\left\{\frac{2\sin\frac{1}{2}C}{b-c}\,.\,\sqrt{(bc)}\right\},$$

and also $$c = (a+b)\cos\phi,$$

if $$\sin\phi = \frac{2\cos\frac{C}{2}}{a+b}\sqrt{(ab)}.$$

Prove also that the perpendicular let fall from C upon c may be expressed by

$$\frac{a^2\sin B + b^2\sin A}{a+b}.$$

793. The three sides of a plane triangle are 3050, 7854 and 5398; find its angles.

794. Wishing to know the height of the crest of the parapet of an inaccessible fortress above the spot on which I stood, I measured its angle of elevation above the horizontal plane through that point, and retreating 350 yards up a slope inclined 10° to the horizon, I measured the corresponding angle of elevation: what was the height of the observed point above the horizontal plane passing through the first station, supposing the measured angles of elevation to have been respectively 15° 27′ 40″ and 12° 32′ 10″?

795. Wanted to know the distance between the flanked angles of two adjacent bastions C and D, of a besieged fortress; I measured a base line $AB = 250$ yards, and then found that the angles subtended at each extremity of the base by the line CD, were $CAD = 28° 37' 40''$, $CBD = 25° 18' 50''$, and that the angles CAB and ABD were respectively 96° 10′ 20″ and 92° 17′ 10″; what was the distance CD?

796. Wanting to find the breadth of a river having a straight course; I measured a distance of 500 yards along its bank, and then found that the angle between the point from which I started and an object immediately opposite on the other bank was 30°; what was the breadth of the river?

797. At what distance from the foot of the escarp must the lower end of a scaling ladder, 40 feet long, be placed, in order that the other end may just reach the top of the escarp which is inclined at an angle of 85° to the horizon; the vertical depth of the ditch being 35 feet?

798. Show that if r be the radius of the earth, h the height of the eye above the sea, and D the depression or "dip" of the sea horizon below the plane tangential to the surface of the earth at the place of observation; then $\tan D = \frac{\sqrt{\{(2r+h)h\}}}{r}$, and, prac-

tically, h being very small compared with r, $\tan D = \sqrt{\frac{2h}{r}}$; or if h be taken in yards, and r be assumed $= 3960$ miles, then

$$\tan D = \frac{\sqrt{(2h)}}{2640}.$$

799. The top-gallant-mast truck, 120 feet above the water-line of a man-of-war coming into port at the rate of 10 miles an hour, was first seen on the horizon at $8^h\,45^m$ A.M. by a person swimming near the water's edge; and at $10^h\,6^m$ A.M. she cast anchor: find an approximate value for the radius of the earth.

800. The angle subtended at a certain point S by the line joining the flanked angles of two alternate bastions A and C of a fortress was found to be $38^\circ\,17'\,40''$, and the angle between A and the flanked angle B of the intermediate bastion was $26^\circ\,45'\,10''$: and by a plan of the place it was found that the distances of these three points were $AB = BC = 360$ yards and $AC = 600$ yards; what was the distance of S from B?

801. Show that if, in a regular fortification, the length of the curtain be c, that of the flank of the bastion f, the distance between the angles of the shoulder of two adjacent bastions d, and the angle which the lines of defence make with the curtain a; then, when the lines of defence are perpendicular to the flanks,

$$f = \frac{c}{\operatorname{cosec} a - 2\sin a} \text{ and } d = \frac{c}{\cos 2a}.$$

802. At a point P, near the top of Woolwich Common, and in a line with the flagstaff at the mortar battery F and the east chimney in the dockyard E, the horizontal angular distance of these from the west chimney W was observed to be $16^\circ\,55'\,25''$; find the distance of P from each of the three objects, supposing their mutual distances to be $FE = 1108$, $FW = 1120$ yards, and $EW = 645$ yards.

803. At each extremity of a base $AB = 758$ yards, the angles between the other extremity and two remarkable objects C and

D were observed, viz. $CAB = 103^\circ\ 50'\ 41''$, $DAB = 53^\circ\ 17'\ 24''$, $DBA = 85^\circ\ 47'\ 30''$, and $CBA = 46^\circ\ 13'\ 27''$; find CD.

804. Having measured a base $AB = 5038{\cdot}127$ feet, I took at each station the angles subtended by the other and each of two remarkable objects C and D, viz. $CAB = 74^\circ 16' 30''$, $DAB = 23^\circ 17' 52''$, $DBA = 87^\circ\ 21'\ 13''$, and $CBA = 41^\circ\ 13'\ 20''$; find the distances CD, AD and AC.

805. At a station A, I found that the angle of elevation of the top of a tower which I knew to be 200 feet above the horizontal plane passing through A, was $52^\circ\ 21'\ 30''$, and at another station B, in the vertical plane through the tower and A, I found the angle of elevation to be $28^\circ\ 15'\ 41''$, and the angular distance between A and the top of the tower to be $19^\circ\ 36'\ 20''$. What was the direct distance from A to B, and their difference of height?

806. Wishing to determine the distance between two buildings A and C (each of which, as well as every part of the line joining them, is inaccessible to me), and also the distance of a particular station B from the line AC, I measure a base $BD = 650$ yards in a perpendicular direction towards AC; I observe also the angles

$$CBD = 42^\circ\ 27'\ 15'',\ CDB = 114^\circ\ 16'\ 30'' \text{ and } ADB = 126^\circ\ 38'\ 20'':$$

what are the required distances?

807. The elevation of a tower standing on a horizontal plane is observed, and, at a station p feet nearer to it in a direct line, the elevation is found to be the complement of the former. On advancing q feet nearer still, the elevation is found to be double the first; show that the horizontal distance of the steeple from the last station is $\frac{p}{2}$ and its height

$$\sqrt{\left\{(p+q)^2 - \frac{p^2}{4}\right\}}.$$

6—2

808. Find the horizontal distance between the flanked angles of two bastions C and D of a besieged fortress, from the following horizontal angles measured at the extremities of the horizontal base AB, 800 yards in length: viz.

$$CAB = 97^\circ\, 32'\, 10''; \ DAB = 53^\circ\, 57'\, 20''; \ CBA = 32^\circ\, 2'\, 10'',$$
$$\text{and } DBA = 68^\circ\, 1'\, 15''.$$

809. From the top of a cliff 108 feet high, the angles of depression of the top and bottom of a cliff which forms the opposite bank of a river are observed to be 30° and 60° respectively; find the height of the opposite cliff, and the breadth of the river.

810. From the bottom of a martello tower 50 feet high, the angle of depression of a ship at anchor was found to be $25^\circ\, 17'\, 20''$, and, from the top, $31^\circ\, 12'\, 30''$; find the direct distance of the ship from the top of the tower, and the height of the cliff on which the tower is built, above the sea-level.

811. From the top A of a tower AB, standing upon the summit of a hill, the slope of which, BCD, is inclined to the horizon at an angle α, the depressions of two objects C and D were observed to be β and γ respectively. Find the distance between the objects; the height of the tower being h.

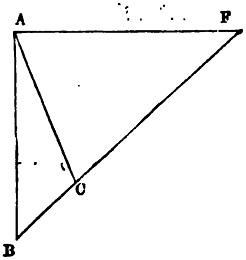
812. From the top of a hill the angles of depression of two objects in the plain at its base were observed to be 45° and 30°, and the horizontal angle between them was also 30°; find the height of the hill in terms of the distance between the objects.

813. Wishing to know the breadth of a river from A to B, I measured from A a distance of 150 yards, to a point C on the prolongation of the line BA, and then measured 200 yards to a point D on a line at right angles to BAC; from this point the angle BDA was found to be $37^\circ\, 18'\, 30''$. Find the breadth of the river.

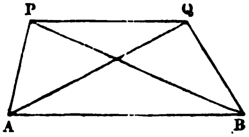
814. Two conspicuous headlands are observed from the deck of a ship, sailing due east, to bear 63° 21′ 10″ and 24° 17′ 30″, respectively, to the northward of the ship's course; and, after sailing 8 miles, the corresponding angles were observed to be, 151° 16′ 20″ and 97° 12′ 15″. Find the distance of the headlands from one another, and from the ship at the first observation.

815. A river flows between two towers, one of which is 40 feet high: from its summit the angle of elevation of the top of the other is found to be 2° 15′ 30″; and from its base the corresponding angle is 10° 18′ 15″: find the height of the other tower, and the breadth of the river.

816. To determine the distance of a battery at *A* from a fort *F*, a distance *AB* = 200 yards was measured in a direction at right angles to that of *AF*. Walking from *B* in a straight line towards *F*, a pole was placed at a convenient station *C*; the distance *AC* was then measured and found to be 178 yards, and the angle *BAC* was 27° 50′; find the distance of the fort *F* from either station *A* or *C*.



817. In a besieged city two conspicuous forts *P* and *Q* were visible from two batteries at *A* and *B* outside the city; the distance *AB* between the batteries was 1080 yards, and at *A* and *B* the following angles were observed, viz.



$$BAP = 80^\circ\ 10'\ 15'', \quad ABQ = 85^\circ\ 23'\ 41'',$$

$$BAQ = 42^\circ\ 17'\ 29'', \quad ABP = 38^\circ\ 51'\ 26'';$$

determine the distance *PQ* between the forts, and if *QP* and *BA* were produced to meet, find at what angle they would intersect.

818. From the extremities, A and B, of a base-line 3070 yards long, the horizontal angles subtended by two distant objects C and D were observed, viz.

$$DAC = 41^\circ\ 24'\ 30'', \quad CBD = 33^\circ\ 47'\ 15'',$$
$$CAB = 48^\circ\ 35'\ 30'', \quad DBA = 27^\circ\ 16'\ 20'';$$

it is required to find the horizontal distance DC.

819. The distances between the points A, B, C in a fortress are known, and at another point P in the same plane as A, B, C, the angles APC and BPC are observed: find the distance of P from each of the points A, B, C; supposing $AB = 400$ yards, $AC = 600$ yards, $BC = 300$ yards, the angle $APC = 36^\circ\ 40'$ and $BPC = 11^\circ\ 20'$, the points C and P being on the same side of the direct line from A to B.

820. The triangle RAB is in a horizontal plane, and T is an elevated object 103·517 feet vertically above R; the angle $TAR = 13^\circ\ 23'\ 20''$, $TBR = 17^\circ\ 41'\ 30''$, and $ARB = 41^\circ\ 19'\ 40''$; find AB.

821. From the extremities A and B of a wall 500 yards long, running north and south, the distance between two objects C and D subtends equal angles of 30°, and it is found that C is due east of A and north-east of B. Find the distance between C and D, D being observed on the right of C from both ends of the wall. *Without the use of Tables.*

822. From the top C of a cliff 600 feet high, the angle of elevation of a balloon B was observed to be $47^\circ\ 22'$, and the angle of depression of its shadow S upon the sea was $61^\circ\ 10'$; find the height of the balloon, the altitude of the sun being $65^\circ\ 31'$, and B, S and C being in the same vertical plane.

823. The angle of elevation of the top, C, of a tower on a hill observed at a point A is $13^\circ\ 17'\ 20''$, and at a point B (not in the vertical plane passing through C and A), the angle of elevation is $22^\circ\ 35'\ 15''$, the angles CBA and CAB are observed to be

65° 14′ 30″ and 47° 32′ 10″ respectively, and the difference of level of *C* and *A* is known to be 500 feet; find the distance *AB*, and the difference of level of *A* and *B*.

824. Two sides *AC*, *BC*, of an equilateral triangle, subtend angles of 30° and 45° at a point *D*; find the distances *DA*, *DB*, *DC*, supposing *DC* to fall between *A* and *B*, *D* and *C* to be on opposite sides of *AB*, and the side of the triangle to be 10.

825. The angle subtended by a diagonal of a square redoubt from a point in the prolongation of the other diagonal was found to be 21° 32′ 10″, and at a point on the same line, 200 yards nearer to the redoubt, the corresponding angle was 28° 17′ 15″; find the side of the redoubt.

826. A circular reservoir subtends at a certain point an angle of 34° 22′ 18″, and advancing 150 yards directly towards its centre, I find it subtends an angle of 72° 18′ 30″; find its diameter.

827. From the top of a tower 113·786 feet high the angles of depression of the top and bottom of a column standing on the same horizontal plane as the tower were found to be, 32° 15′ 20″ and 68° 54′ 33″ respectively; find the height and distance of the column.

828. The distance between two horizontal parallel telegraph wires running N.W. and S.E., and vertically above one another, is 4 feet, and at noon the perpendicular distance between their shadows on a horizontal plane is 6 feet; find the altitude (*i.e.* the angle of elevation) of the sun.

MENSURATION.

829. The lengths of the perpendiculars let fall from points in an irregularly curved line of fence upon a straight line of 5·86 chains, at equal distances from each other, are found to be 93, 84,

72, 68, 43, 54, 37, 29, and 23 links; find the area included between the extreme perpendiculars which fall upon the ends of the straight line.

830. The sides of a quadrilateral, taken consecutively, are 2416, 1712, 1948 and 2848; the angle between the first two is 30°, and that between the last two 150°; find the area of the figure.

831. Find an expression for the area of a triangle whose sides are a, b and c.

832. Find the area of a triangular field whose sides are 7·32 chains, 4·57 chains, and 5·48 chains.

833. Find the number of acres in a triangular field whose sides are 10·42 chains, 8·74 chains, and 12·63 chains.

834. Three sides of a triangle are 6, $6+\sqrt{2}$, and $6-\sqrt{2}$; find its area.

835. The area of a right-angled triangle is 84·5 square feet, and one of its sides is 39 inches; find its hypothenuse.

836. The area of a triangular field is 14 acres; find its sides, which are known to be in the ratio of 3, 5, and 7.

837. A quadrilateral field $ABCD$ has its sides $AB=6$ chains, $BC=8$ ch., $CD=8$ ch., $AD=9$ ch., and its diagonal, $BD=12$ ch.; find its area in acres.

838. A mahogany plank 24 feet in length, is 18 inches wide at one end and 15 at the other; the plank is cut across at a distance of 3 feet 6 inches from the broader end: how many square feet are cut off the plank, and how many are in the whole plank?

839. The floor of a room is a regular octagon, the distance between any parallel sides being 20 feet; find the area in square yards.

840. The base of an isosceles triangle is 20, and its area is $\frac{100}{\sqrt{3}}$; find its angles.

841. The parallel sides of a trapezoidal field of 25 acres are 10 chains and 15 chains respectively: find the perpendicular distance across the field.

842. A trapezoidal field of which the parallel sides are 579 links and 854 links, and the perpendicular distance between them 723 links, is let at £4. 10*s.* per acre; what income does it produce?

843. The sides of a triangle are $2+\sqrt{2}$, $2-\sqrt{2}$, and 3; find its area.

844. Find the area of a regular polygon in terms of its side s.

845. The sides of a right-angled triangle are 3, 4, and 5; find the area of the space contained by the segments of the sides 3 and 4, and the arc of the inscribed circle included between these segments.

846. Express the area of a regular plane figure in terms of a the length of the side, and n the number of sides: and apply it to the determination of the area of the equilateral triangle, the square, and the regular pentagon, whose sides are each 10 units in length.

847. Deduce the expressions for the areas of the regular polygons of n sides circumscribing and inscribed within the circle whose radius is r; and thence show that they have to one another the ratio of 1 to $\cos^2\dfrac{180^\circ}{n}$.

848. Show that the area of a regular polygon of n sides is equal to $\dfrac{na^2}{4}.\cot\dfrac{180}{n}$; where a is the length of its side.

849. Find the area of an isosceles triangle, each of whose equal sides is 50, and each of whose equal angles is 75°.

850. What must be the diameter of a carriage-wheel in order that it may make 500 turns in a mile?

851. A regiment, advancing in open column of companies, is wheeled by successive companies to the left; show that the distance, in paces, lost by each company during the wheel, is two-fifths of the number of files in the company, supposing the ratio of files to paces to be 10 to 7, and that the right file of each company takes exactly the full pace.

852. The driving-wheel of a locomotive engine being 6 feet in diameter, determine the number of strokes made per minute by each piston, when the train is running at the rate of 30 miles an hour; two strokes of each piston causing one revolution of the driving-wheel.

853. Prove that the area of the largest circle which can be cut from a regular hexagon is three times the area of the circle described on one of the sides as a diameter.

854. What is the area of the sector of a circle whose arc of 24^0 measures 10 feet?

855. Ten persons dine at a circular dining-table; what is the area of the table-cloth, supposing each person to occupy 2 feet of the circumference of the table, and that the cloth overlaps 15 inches on each side? Find also the area unoccupied when a dinner plate of 10 inches diameter is placed before each person.

856. Find the area of the remaining portion of a circle whose radius is 20 inches, when a segment having an arc of 25^0, has been cut off from it.

857. Find the area of the segment of a circle whose height is one-half of the radius, when the radius of the circle is 1 foot.

858. Find the area of a segment of a circle whose base is 54 and height 10.

859. A regular hexagon is inscribed in a circle whose radius is 10 inches, and another is circumscribed about it; find the area of the latter, and show that the area between the boundaries of

the hexagons is equal to one-third of the area of the inscribed figure.

860. If a regular hexagon, a square, and an equilateral triangle be inscribed in a circle, the square described upon the side of the triangle is equal to the sum of the squares described upon one side of each of the other two figures.

861. Show that the area of a regular polygon inscribed in a circle is a mean proportional between the areas of two polygons of half the number of sides inscribed within and circumscribed about the same circle.

862. The exterior diameter of the outer ring of a circular target is 5 feet, and it is divided into 6 concentric rings of equal breadth, alternately white and black, the outer ring being white; find the number of square feet of white paint in the target.

863. Find the area of the sector of a circle whose arc of 20° is 18 inches long.

864. Find the area of a circular segment whose height is 7 inches and its base 3 feet.

865. The angular points of an irregular pentagon are each at a distance of 100 yards from a certain point within it; and at this point the sides taken in order subtend angles of 45°, 60°, 90°, 45°, and 120°; find the area of the pentagon and the length of each of its sides, without using tables.

866. If a regular hexagon be placed within an equilateral triangle, so that three of its sides are upon the sides of the triangle, show, analytically, that the areas of the figures will be as their perimeters, and that the areas of their circumscribing circles will be as 1 : 3.

867. *A* and *B* are two points in the circumference of a circular pond of water, and a dyke from *A* to *B* is to be made so as to cut off the smaller part of the pond. If the circumference of

the whole pond be 100 yards, and the length of the dyke AB be 18 yards, how many square yards will the surface of the pond now occupy?

868. The paving of a circular court cost £50, at the rate of *3s. 4d.* per square yard; what is its circumference in feet?

869. Show that the ratio of a square to its circumscribing circle is $2 : \varpi$.

870. Prove that the areas of two sectors of circles are equal when their angles are inversely proportional to the squares of their radii.

871. Having a cord 20 yards long, and wishing to know the area of a circular reservoir, of which only a portion was accessible, I found that when the cord was stretched between two points on its margin, the perpendicular distance from the point of bisection of the cord to the nearest point of the circumference was 1·716 yard. What was the area of the reservoir?

872. If a be the length of an arc of a circle (radius r), and α the number of degrees in it, find the area of the corresponding segment.

873. A circular building is to be erected on a triangular plot of ground, the sides of which are 40, 30, and 50 yards; find the radius of the circle so that the unoccupied external area may be one-tenth of an acre.

874. If, in a wire rope, the diameter of each component wire be d inch, find the number of wires in each square inch of section of the rope; and show thence, that, if the circumference of the rope be C inches, the bearing area of the section will be $\frac{C^2}{8\sqrt{3}}$.

875. Divide the surface of a sphere into four zones of equal surface; and find the angular breadth of each zone.

876. Find the surface exposed to the force of the exploding power in a 13-inch shell whose thickness is 1·85 inch.

877. The length and breadth of the rectangular base of a wedge are a and b inches respectively, and the length of its edge is c inches. If V be its volume, and h the perpendicular distance of any point in the edge from the base of the wedge, prove that

$$V = \frac{1}{6} hb . (2a + c);$$

whether c be greater or less than a.

878. A cylinder and a hemisphere have equal circles for their bases; find the form of the cylinder in order that its volume may be one-half of that of the hemisphere.

879. A brass gun was struck by a spherical shot: a section across the middle of the wound being a segment of a circle, the base of the segment was found to be 6 inches and its depth 1 inch; find the weight of the shot which made the wound, the 9-lb. shot being 4 inches in diameter.

880. Show that if d be the diameter of a round shot which weighs p pounds, R and r the radii of the exterior and interior spherical surfaces of a shell, then the weight of the shell, in pounds, will be expressed by

$$\frac{R^3 - r^3}{d^3} . 8p.$$

881. If a cone, the diameter of whose base is equal to its slant height, be enveloped in a sphere, and the sphere be enveloped in a cylinder, show that the respective volumes are as 9 : 32 : 48, and that their complete surfaces are as 18 : 32 : 48.

882. A sphere of lead, 2 inches in diameter, is placed within an inverted, hollow, regular hexagonal pyramid, 3 inches in the side and 5 inches deep; find the quantity of water which can be poured into the pyramid, and also the quantity when the sphere is just covered. The axis of the pyramid being, in both cases, vertical.

883. It is required to construct a cylindrical gallon measure having a hemispherical bottom, the total depth being 12 inches: the imperial gallon being 277·274 cubic inches.

884. A cylindrical vessel 5 inches high and 10 inches in radius has three heavy spheres placed within it which rest at the bottom, touching each other, the bottom, and the curve surface of the cylinder; find the number of cubic inches of water which can be poured into the cylinder.

885. A sphere is placed within a triangular equilateral pyramid, or regular tetrahedron, and is found to touch all its sides; find the volume of the sphere, the side of the tetrahedron being 1 foot.

886. A tank or cistern for holding water is in the form of the frustum of a pyramid, the length and breadth of its rectangular bottom are 10 and 6 feet, and it measures 15 feet by 9 at top; how many gallons of water will it hold, supposing its depth to be 4 ft. 6 in.?

887. Find the capacity of a cylindrical pontoon having hemispherical ends, its extreme length being 22 feet, and the length of the cylinder 19 feet.

888. Find the number of cubic feet of air in a conical tent 10 feet high and 14 feet in diameter.

889. A 13-inch shell weighs nearly 200 lbs., the thickness of the shell being 2 inches; find the thickness of the 36-inch shell which weighs 26 cwt.

890. Find the number of cubic feet of air in a rectangular hut, with a sloping roof; the sides of the base being 80 feet and 25 feet, the height to the eaves 10 feet, and to the ridge of the roof 15 feet.

891. Find the quantity of gas contained, after firing, in the bore of an 8-in. gun at the instant the centre of the shot passes

the plane of the muzzle, the total length of the bore, which is terminated by a hemisphere, being 8 ft. 7 in.

892. A hemisphere, radius $2r$, has a cylindrical hole, radius r, bored through it perpendicularly to the plane of its base; the axis of the cylinder coinciding with a radius of the hemisphere. Find the remaining volume.

893. What quantity of paint would be required to paint the 8-inch shells in a square pile of 15 courses, supposing that 1 lb. of paint will cover 100 square feet?

894. The mound on the field of Waterloo is 200 feet high, and is in the form of a cone, having the inclination of its sides to the horizon 30°; find its content in cubic yards, and its surface in acres.

895. The diameter of the 9 lb. solid shot being 4 inches; find the diameter of the 12 lb. and 24 lb. shot; and the thickness of the 13-inch shell which weighs 196 lbs.

896. A cylindrical pontoon 2 feet in diameter, having conical ends, is so far sunk under water that there is a mean pressure of 2·3 lbs. upon each square inch of its surface; find the total pressure, the extreme length of the pontoon being 14 feet, and the height of each cone being equal to the diameter of its base.

897. The sides of a circular reservoir are inclined at an angle of 30° to the horizon, and the diameter of the horizontal bottom is 70 feet; find the number of gallons contained in it when the water is 10 feet deep.

898. Find the solidity of the frustum of a cone, the diameters of whose ends are 5 and 2, and the height between them 4.

899. What must be the thickness of an 8-inch shell in order that it may weigh 54 lbs. when unloaded; the 9 lb. shot being four inches in diameter?

900. Find the weight in grains of a bullet in the form of a cylinder and cone upon a common base ·6 inch in diameter; the

altitude of the cone being one-fourth of that of the cylinder, the total length of the bullet being 1·25 inch, and the weight of a cubic inch of lead 6·57 oz.

901. Show that the capacity of a tent, considered as a prism of n sides surmounted by a pyramid, whose heights are respectively h and h_1 and length of side a, is

$$(3h+h_1).\frac{n.a^2}{12}.\cot\frac{180^\circ}{n}.$$

902. What is the content of a cylindrical pontoon having hemispherical ends, its extreme length being 20 feet and diameter 3 feet? And what is its weight, supposing its thickness to be one-sixteenth of an inch, and a cubic foot of its substance to weigh 4 cwt.?

903. Find the content of the bore of a gun 5 feet long and 4·5 inches in diameter, the end at the breech being a hemisphere of the same diameter.

904. The 9 lb. iron shot is 4 inches in diameter, and a cubic foot of water weighs 1000 ounces: find the weight of a 13-inch shell when filled with water, supposing its thickness to be 1·8 inch.

905. Find the weight of a 36-inch shell, 4 inches thick; supposing the 9 lb. shot to be accurately 4 inches in diameter.

906. The present 13-inch shell which is 1·85 inch thick, weighs 196 lbs.; what would be its weight if the thickness were diminished to one inch?

907. Show that the volumes of spherical shells of the same thickness are proportional to those of conic frusta, of equal heights, which have the diameters of their ends proportional to the external and internal diameters of the corresponding shells.

908. Show that if a gun or mortar have a conical chamber the sides of which are inclined to the axis of the gun at an angle

of 30°, the available space for the charge of powder, when the shot is in contact with the sides of the chamber, is one-eighth of the volume of the shot.

909. A heavy sphere is just immersed in a conical glass full of water; find the quantity of water in the glass, which is 4 inches deep, and 6 inches in diameter.

910. Find the volume remaining of a sphere of 16 inches radius, after a circular hole 8 inches in radius has been bored directly through its centre.

911. Assuming a cask, 41 inches long, (measured from head to head, externally, over the side) to be formed of two conic frustums having a common base; find the weight of water contained in it when the diameter of the head is 21 inches and the circumference at the bung is 7 feet, both measured externally, and the thickness of the wood is ·5 inch, assuming the cubic foot of water to weigh 1000 oz.

912. Find the weight of a 14-inch shell made of lead 2 inches thick; the weight of the 13-in. iron shell which is 1·8 in. thick being 196 lbs., and a cubic foot of lead being to a cubic foot of iron, as 100 to 64.

913. The slant height of a frustum, cut from a pentagonal pyramid whose slant height is 10 inches, is found to be 7·5 inches, and each side of the base is 4 inches; find the content of the frustum.

914. What must be the thickness of a 36-in. shell, in order that it may weigh 1 ton; supposing a 13-in. shell to weigh 200lbs. when 2 inches thick?

915. Divide a cone into three equal parts by planes drawn parallel to its base.

SPHERICAL TRIGONOMETRY.

916. State "Napier's Rules," and apply them to the determination of formulæ for the solution of a right-angled spherical triangle in which A and b are given, and C is the right angle.

917. Prove that in every right-angled spherical triangle, whose right angle is c,

$$\cos c = \cos a \,.\, \cos b, \qquad \text{and } \cos A = \tan b \,.\, \cot c.$$

918. The side a and the hypothenuse c of a right-angled spherical triangle ABC being given, write down the formulæ of solution for the angles and remaining side.

919. Prove that in every spherical triangle,

$$\cos c = \cos a \,.\, \cos b + \sin a \,.\, \sin b \,.\, \cos C;$$

and show that, if $\tan\theta = \tan a \,.\, \cos C$, this reduces to

$$\cos c = \cos a \,.\, \cos(b - \theta) \,.\, \sec\theta.$$

920. Show, that, in a quadrantal triangle, $\cos C = -\cot a \,.\, \cot b$, when c is the quadrantal side.

921. Assuming that in an oblique-angled spherical triangle A, B, C:

$$\tan\frac{1}{2}(A+B) = \frac{\cos\frac{1}{2}(a-b)}{\cos\frac{1}{2}(a+b)} \cdot \cot\frac{C}{2},$$

prove that

$$\tan\frac{1}{2}(a+b) = \frac{\cos\frac{1}{2}(A-B)}{\cos\frac{1}{2}(A+B)} \cdot \tan\frac{c}{2}.$$

922. Find the area of a spherical triangle whose angles are $79^\circ\ 10'\ 20''$, $57^\circ\ 43'\ 2''$ and $43^\circ\ 7'\ 18''$, on the surface of the earth whose radius is, approximately, 8000 miles.

923. Prove, from first principles, that, in a right-angled spherical triangle *ABC*, having the right angle at *C*:

$$\cos c = \cos a \,.\, \cos b.$$

924. By the application of "Napier's Rules," write down the values of the sides and of the remaining angle of a right-angled spherical triangle in terms of the hypothenuse *c*, and one of the angles, *B*.

925. Prove that, in an oblique-angled spherical triangle,

$$\cos A = \frac{\cos b \,.\, \cos c - \cos a}{\sin a \,.\, \sin b};$$

and thence determine the value of $\sin \frac{1}{2} A$ in terms of the sides.

926. Show, from first principles, that in a right-angled spherical triangle *ABC*, having the right angle at *C*,

$$\cos c = \cot A \,.\, \cot B,$$

and

$$\sin a = \sin A \,.\, \sin c.$$

927. The spherical triangle *ABC* has one of its angles *ACB* a right angle, and the sides opposite to *A*, *B*, *C* are *a*, *b*, *c* respectively; prove the following properties:

(1) $\tan b = \tan B \sin a$; (2) $\cos B = \cos b \sin A$.

928. Show by "Napier's Rules," that the altitude of a regular tetrahedron whose side is *s*, is $\frac{s}{3}\sqrt{6}$.

929. The sides and angles of a polar spherical triangle are the supplements of the angles and sides respectively of its primitive triangle.

7—2

930. Show that in a right-angled spherical triangle

$$\sin(c-a) \cdot \sin(c+a) = \cos^2 a \cdot \sin^2 b,$$

c being the hypothenuse.

931. In a spherical triangle, given

$$a = 148^\circ\ 25'\ 34'',\ b = 149^\circ\ 31'\ 48'',\ \text{and}\ C = 109^\circ\ 57'\ 57'';$$

find the values of A and B.

932. Investigate Napier's "First Analogies," and show how the "Second Analogies" may be determined from them. State to what cases of solution of spherical triangles they are applicable.

933. Show that in a right-angled spherical triangle ABC,

$$\sin a \cdot \tan\frac{1}{2}A - \sin b \cdot \tan\frac{1}{2}B = \sin(a-b),$$

C being the right angle.

934. Show, from first principles, that, in an oblique-angled spherical triangle,

$$\tan\frac{A}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin s \cdot \sin(s-a)}}.$$

935. Show that in a spherical triangle

$$\cos c = \cos a \cdot \cos b + \sin a \cdot \sin b \cdot \cos C;$$

show what this becomes in the case of a right-angled triangle of which c is the hypothenuse; and obtain the same result by the application of "Napier's Rules."

936. The angles at the vertex of a triangular pyramid are $37^\circ\ 15'\ 45''$, $53^\circ\ 22'\ 17''$ and $48^\circ\ 33'\ 20''$; find the inclinations of its planes to one another.

937. The plane angles at the vertex of a triangular pyramid are $33^\circ\ 14'\ 20''$, $42^\circ\ 25'\ 50''$, and $50^\circ\ 17'\ 10''$; find the inclinations of its sides to one another.

938. A pair of compasses, opened so that the legs include an angle $2a$, have the points placed upon a table so that the plane of the compasses is inclined at an angle β to the plane of the table. Show, by "Napier's Rules," that, if θ be the inclination of each leg to the plane of the table,

$$\sin \theta = \sin \beta . \cos a.$$

939. Solve completely the spherical triangle in which

$$A = 34^\circ\, 47'\, 40'',\ a = 65^\circ\, 22'\, 30'',\ \text{and}\ b = 57^\circ\, 32'\, 10''.$$

940. The elevation of the top of a tower, seen from a point in a level straight road, is 10°, and the horizontal angle between the tower and the road is 60°; find, *by Spherical Trigonometry*, the greatest elevation the top of the tower can have from any point in the road.

941. A, B, C are three points upon a plane inclined at an angle of $34^\circ\, 16'\, 20''$ to the horizon; the line AB is horizontal, and AC is inclined $17^\circ\, 52'\, 5''$ to the horizon: find the angle CAB.

942. A, B, C being three places on the surface of the earth the distances of which from one another are known, and E the position of a balloon above the earth; the angles AEB, BEC, CEA were measured at the instant when the balloon was vertically over the line AB. Show how, by the solution of two spherical triangles and three plane triangles, the height of the balloon may be determined, and give the necessary working formulæ.

943. Two of the plane angles forming a trihedral solid angle are 30° and 60° degrees respectively, and their planes are perpendicular to one another; find the remaining angle and the inclination of its plane to those of the other angles.

944. The altitude of a regular pentagonal pyramid is to the slant height as $\sqrt{3} : 2$; find the inclination of its sides to one another, by the solution of a spherical triangle.

945. If a cube be cut by a plane which passes through the three diagonals of its adjacent faces, find the inclination of the

plane to those faces, by the solution of a right-angled spherical triangle.

946. Two straight lines are drawn from a point in the intersection of two planes, one in each plane, making angles of 30° and 60° respectively with the intersection; find the angle between the lines when the planes are perpendicular to one another.

947. If A and B, two of the angles of a spherical triangle, be right angles, show that the area of the triangle is

$$\frac{C}{720} \times (\text{surface of sphere}).$$

948. Two sides of a triangle on the surface of the earth are 20 miles and 40 miles respectively, and the angle included between them is $57^\circ\ 18'\ 10''$; find the remaining angles, and thence, the area of the triangle, the radius of the earth being assumed 4000 miles.

949. Show that if E'' be the spherical excess in a triangle of which the area in feet is A, and R be the radius of the earth in miles, $E'' = \dfrac{45A}{\pi \, . \, (44R)^2}$.

950. Find the area of the triangle ABC upon a sphere whose radius is 60 inches, where $A = 54^\circ\ 13'\ 20''$, $b = 32^\circ\ 27'\ 14''$, and $c = 45^\circ\ 50'\ 30''$.

951. Find the angles, the spherical excess, and the area of an equilateral spherical triangle, described upon a sphere whose radius is 20 feet, each side of the triangle being 60°.

952. The angles of a spherical triangle upon a sphere whose radius is 4 inches, are 70°, 59°, and 81°; find its area.

953. The sides of a spherical triangle are $121^\circ\ 15'\ 45''$, $57^\circ\ 23'\ 15''$, and their included angle is $75^\circ\ 25'\ 24''$; find the area of the triangle upon the surface of a sphere of 16 ft. 8 in. diameter.

PRACTICAL ASTRONOMY.

954. Define the terms "altitude," "azimuth," "right ascension," "declination," "ecliptic," and "first point of Aries."

955. The meridional altitude of a star, observed on the south side of the zenith, is $57^\circ\ 18'\ 6''$, and the latitude of the place $37^\circ\ 14'\ 6''$ N.; find the north polar distance of the star.

956. The meridional altitudes of a circumpolar star in the southern hemisphere were observed to be $19^\circ\ 32'\ 7''$ on the southern, and $72^\circ\ 12'\ 4''$ on the northern side of the zenith; find the latitude of the place and the declination of the star.

957. By an observer situated on the equator of the earth, the meridional altitude of a star was observed to be $27^\circ\ 13'\ 24''$. What was the declination of that star?

958. The declination of a star is $-21^\circ\ 37'\ 15''$, and the latitude of the place of observation $51^\circ\ 28'\ 39''$ N.; find the meridional zenith distance.

959. The meridional altitudes of a southern circumpolar star were observed to be $81^\circ\ 16'\ 4''$ and $13^\circ\ 15'\ 10''$; find its declination and the latitude of the place.

960. At midnight, March 12, 1853, the R. A. of the moon was $27^\circ\ 39'\ 19''{\cdot}2$, and her declination $6^\circ\ 56'\ 59''{\cdot}7$ N.; find her longitude and latitude, the obliquity being $23^\circ\ 27'\ 32''$.

961. At midnight, February 6, 1853, the longitude of the moon was $302^\circ\ 46'\ 29''{\cdot}3$, and her latitude was $2^\circ\ 56'\ 44''{\cdot}8$ S.; find her R.A. and declination, the obliquity being $23^\circ\ 27'\ 31''$.

962. At noon, January 18, 1853, the R.A. of the moon was $40^\circ\ 54'\ 24''{\cdot}75$, and her declination $11^\circ\ 53'\ 49''{\cdot}7$ N.; find her longitude and latitude, the obliquity of the ecliptic being $23^\circ\ 27'\ 31''$.

963. At noon, February 9, 1853, the R.A. of the Moon was $339^\circ\ 47'\ 7''\cdot65$, and her declination $13^\circ\ 39'\ 9''$ S.; find her longitude and latitude, with the same obliquity.

964. The Equation of Time being $+ 14^m\ 53^s$, and the R.A. of the sun $15^h\ 27^m\ 34^s$; find the Sidereal Time corresponding to $7^h\ 33^m\ 12^s$ mean time.

965. The Mean Time being $21^h\ 13^m\ 27^s$, the Equation of Time $- 9^m\ 47^s$, and the Sidereal Time $16^h\ 53^m\ 21^s$; find the R.A. of the sun.

966. The R.A. of the Sun being $5^h\ 14^m\ 2^s$, the Sidereal Time $15^h\ 37^m\ 10^s$, and the Equation of Time $+ 12^m\ 53^s$; find the Mean Time.

967. Find the Sidereal Time of sunset at the autumnal equinox, and thence the Sidereal Time at the following mean noon; the Equation of Time being $+ 7^m\ 20^s$ at sunset.

968. Find the Equation of Time when the Sidereal clock marks $15^h\ 10^m\ 4^s$, and the Mean Time clock $4^h\ 7^m\ 23^s$, the R.A. of the sun being $10^h\ 58^m\ 4^s$ at that instant.

969. The Sidereal Time of sunrise being $20^h\ 14^m\ 37^s$, and the R.A. of the sun $3^h\ 5^m\ 53^s$; find the Equation of Time at the following Apparent Noon, supposing the Mean Time of sunrise to be $17^h\ 0^m\ 0^s$, and that the Equation of Time at the previous noon was $+ 8^m\ 20^s$.

970. Find the mean solar interval corresponding to the sidereal interval $8^h\ 43^m\ 27^s$.

971. Find the sidereal interval corresponding to the mean solar interval $17^h\ 54^m\ 30^s$.

972. The transit of a star on the equator was observed at $7^h\ 15^m\ 43^s$ Mean Time; find the Mean Time of its setting.

973. The Sidereal Time at Apparent Noon was $21^h\ 14^m\ 33^s$; find the Sidereal Time at $6^h\ 0^m\ 0^s$ Apparent Time.

974. The Mean Time of the transit of the first point of Aries was $13^h\ 56^m\ 4^s$; find the Mean Time of transit of a star whose R.A. was $6^h\ 14^m\ 37^s$.

975. The star Regulus was observed to pass the meridian when a chronometer regulated to Greenwich Mean Time marked $15^h\ 27^m\ 32^s$; what is the longitude of the place of observation, the Equation of Time being $+5^m\ 8^s$, and the R.A. of the sun $17^h\ 12^m\ 3^s$?

976. At what time does the Sun set at Edinburgh, in latitude $55^\circ\ 57'\ 0''$ N., when its declination is $+14^\circ\ 46'\ 0''$?

977. The azimuth of the Sun at setting was found to be N. $54^\circ\ 15'\ 0''$ W., when its declination was $+17^\circ\ 32'\ 10''$; find the latitude of the place and the Apparent Time of sunset.

978. The meridional altitudes of a circumpolar star were observed at a place in the southern hemisphere to be $10^\circ\ 17'\ 54''$ and $72^\circ\ 25'\ 18''$; find the latitude of the place and the declination of the star.

979. The true zenith distance of α Pegasi was observed at $19^h\ 27^m\ 50^s$ Sidereal Time to be $80^\circ\ 14'\ 10''$; find the latitude of the place, the R.A. and declination of the star being $22^h\ 57^m\ 26^s$ and $+14^\circ\ 24'\ 55''$.

980. The sidereal interval between the transits of Arcturus across the prime vertical was $2^h\ 17^m\ 36^s$; find the latitude of the place, and the Sidereal Time of the star's setting, the R.A. and declination of Arcturus being $14^h\ 8^m\ 58^s$ and $+19^\circ\ 56'\ 56''$.

981. The altitudes of a southern circumpolar star when on the meridian were $14^\circ\ 27'\ 30''$ and $73^\circ\ 14'\ 10''$; find the declination of the star, and the latitude of the place.

982. The interval which elapsed between the transits of Arcturus across the prime vertical was observed to be $3^h\ 4^m\ 15^s$; find the latitude of the place, the declination of Arcturus being $+19^\circ\ 57'\ 40''$.

983. At a place in South America the altitude and azimuth of the Sun's centre at $23^h\ 30^m\ 12^s$ Greenwich Mean Time were, *Alt.* 21° 25′ 40″, *Az.* 69° 18′ 20″. Find the latitude and longitude of the place; the Equation of Time being $7^m\ 42^s$, to be added to apparent time, and the declination of the Sun's centre being $-17°\ 37'\ 29''{\cdot}5$.

984. The R.A. and declination of a certain star being r and δ, and its angular distance from the moon when on the equator d, show that

$$\text{moon's R.A.} = r \pm a,$$

where a is derived from the equation

$$\cos a = \cos d \,.\, \sec \delta.$$

985. At $13^h\ 17^m\ 24^s$, Greenwich Mean Time, the altitude of the Sun's centre was found to be 35° 12′ 4″ and its azimuth S. 42° 38′ 10″ W.; find the latitude and longitude of the place of observation; the equation of time being $4^m\ 15^s$ to be added to apparent time, and the declination of the Sun's centre 12° 13′ 42″ S.

986. The sidereal interval between the transits of α Ursæ Majoris across the prime vertical was observed to be $2^h\ 53^m\ 10^s$; what was the latitude of the place of observation, the declination of the star being $+62°\ 32'\ 36''$?

987. At $16^h\ 52^m\ 13^s$, Greenwich Mean Time, the star Sirius was observed to pass the meridian; the R.A. of the sun was $21^h\ 14^m\ 38^s$; the equation of time $-14^m\ 18^s$: what was the longitude of the place?

988. At $2^h\ 15^m\ 21^s{\cdot}5$, Greenwich Mean Time, the transit of the star α Lyræ was observed; and it was found that the R.A. of the sun was, at that instant, $7^h\ 23^m\ 16^s{\cdot}2$, and the equation of time $-5^m\ 8^s{\cdot}6$: what is the longitude of the place at which the observation was made?

989. The transit of the Sun's centre was observed at $12^h\ 13^m\ 4^s{\cdot}2$, Greenwich Mean Time, when the equation of time was $+10^m\ 14^s{\cdot}7$; find the longitude of the place.

990. The transit of β Leonis was observed at $22^{h}\ 17^{m}\ 53^{s}\cdot 7$, Greenwich Mean Time, when the Equation of Time was $+\ 3^{m}\ 38^{s}\cdot 8$, and the R.A. of the sun $2^{h}\ 57^{m}\ 14^{s}\cdot 1$; what is the longitude?

991. At $18^{h}\ 42^{m}\ 7^{s}$, Local Mean Time, the First Satellite of Jupiter was observed to disappear in the shadow of the planet; and, upon reference to the almanac, this phenomenon was found to be predicted as taking place at $11^{h}\ 51^{m}\ 26^{s}$, Greenwich Mean Time: what was the longitude of the place of observation?

992. The difference of the times of transit of δ Piscium and the Moon's bright limb was observed to be $0^{h}\ 27^{m}\ 21^{s}$. At her Greenwich transit the almanac states that their R.A.'s were δ Piscium $0^{h}\ 41^{m}\ 2^{s}$, Moon's bright limb $1^{h}\ 4^{m}\ 56^{s}\cdot 33$, and the change of the Moon's R.A. for one hour of longitude was 114 seconds; what is the longitude of the place, the star being to the westward of the Moon?

993. The difference of the times of transit of θ Virginis and the Moon's bright limb, was observed to be $12^{m}\ 15^{s}$; and the R.A. at Greenwich transit were, θ Virginis $13^{h}\ 2^{m}\ 20^{s}\cdot 97$, the Moon's bright limb $13^{h}\ 32^{m}\ 5^{s}\cdot 39$, and the change of the Moon's R.A. for one hour of longitude 136·69 seconds; what was the longitude of the place, the star being to the westward?

994. The star γ Draconis was observed, when due east, to have an apparent altitude of $64^{\circ}\ 31'\ 10''$; find the latitude of the place of observation, and the Sidereal Time, the R.A. and declination of the star being $17^{h}\ 53^{m}\ 12^{s}$ and $51^{\circ}\ 30'\ 29''$.

995. Regulus was observed to be due west at $14^{h}\ 12^{m}\ 27^{s}$, Sidereal Time; find its altitude at that time, and the latitude of the place, the R.A. and declination of the star being $10^{h}\ 0^{m}\ 32^{s}$ and $12^{\circ}\ 41'\ 1''$.

996. The R.A. of Capella being $5^{h}\ 5^{m}\ 50^{s}$, its altitude was observed at $23^{h}\ 5^{m}\ 50^{s}$ Sidereal Time, to be $27^{\circ}\ 42'\ 10''$; what was the azimuth of the star, and the latitude of the place? the declination of the star being $45^{\circ}\ 50'\ 33''$.

997. At $18^{h}\ 0^{m}\ 0^{s}$, Apparent Time, the altitude of the sun's lower limb was observed to be $17^{\circ}\ 16'\ 39''$, and the azimuthal

angle of his east (or *left-hand*) limb from a terrestrial object to the south 110° 15′ 27″; and his declination and semidiameter were found in the Almanac to be 20° 32′ 5″ N. and 16′ 12″: what is the latitude of the place, and the azimuth of the object, east or west of south?

998. Find the Mean Time of the true sunset in latitude 49° 13′ 27″ N. when the Sun's declination is + 18° 56′ 39″, and the Equation of Time $+ 3^{m}\ 54^{s}$; and find the azimuth of his centre at setting. Find also these values in the same latitude south of the equator.

999. The sidereal interval which elapsed between the transits of γ Draconis over the prime vertical was observed to be $1^{h}\ 17^{m}\ 13^{s}\cdot2$; find the latitude of the place, the declination of γ Draconis being + 51° 30′ 52″.

1000. The mean solar interval between the observed equal altitudes of Antares was $4^{h}\ 17^{m}\ 33^{s}$; find the latitude of the place, supposing the observed altitudes to be 54° 17′ 27″, the declination of Antares being 26° 6′ 4″.

1001. The mean solar interval between two observed equal altitudes of the Sun was $3^{h}\ 27^{m}\ 16^{s}$; the Greenwich Mean Time of the first observation was $18^{h}\ 7^{m}\ 26^{s}$; the Equation of Time was $- 7^{m}\ 43^{s}$; the declination at first observation was + 10° 15′ 7″, and the approximate latitude 27° 4′ N.; find the difference between the time at Greenwich and the time at the place, supposing the diminution of declination in the interval between the observations to have been 2′ 57″ = 177″.

1002. The apparent altitude of a star, whose declination was + 33° 14′ 0″, was observed to be 47° 22′ 54″, when its azimuth was 106° 15′ 0″; find the time at which it would pass the meridian, the Mean Time of the observation being $14^{h}\ 15^{m}\ 21^{s}$.

1003. The apparent altitude of a star, whose declination was + 15° 7′ 43″, was observed to be 62° 17′ 36″, when its azimuth was 75° 15′ 10″; what was the latitude of the place of observation?

1004. At $4^h\ 23^m$, Woolwich Mean Time, on May 19, 1852, the Sun was observed to cast no shadow in front or back of the Royal Military Academy; in what direction does the building face, its latitude being $51^\circ\ 28'\ 28''$ N., the Sun's declination $19^\circ\ 44'\ 21''$ N., and the Equation of Time $+3^m\ 49^s$?

1005. At a certain time, when the R. A. and declination of Sirius were $6^h\ 38^m\ 36^s$ and $-16^\circ\ 30'\ 49''$, and of Procyon $7^h\ 31^m\ 31^s$ and $+5^\circ\ 36'\ 10''$, it was observed that the true setting of Sirius took place $3^h\ 42^m\ 10^s$ (sidereal) before that of Procyon. What was the latitude of the place?

1006. The altitude of Regulus was observed to be $16^\circ\ 31'\ 5''$ at $9^h\ 15^m\ 0^s$, and at $11^h\ 43^m\ 8^s$ it was again observed and found to be $40^\circ\ 11'\ 27''$; what was the latitude of the place of observation, the declination of Regulus being $+12^\circ\ 41'\ 30''$?

1007. At $17^h\ 51^m\ 6^s$, Greenwich Mean Time, the altitude of the Sun's lower limb was observed, after it had passed the meridian, to be $62^\circ\ 58'\ 54''$, and the azimuth of the east limb $112^\circ\ 32'\ 0''$; the declination of the Sun's centre was $+15^\circ\ 27'\ 40''$, its semi-diameter $15'\ 55''$, and the Equation of Time $+3^m\ 7^s$; find the latitude and longitude of the place of observation.

1008. In south latitude $30^\circ\ 17'\ 10''$, the altitude of β Leonis, before coming to the meridian, was observed to be $17^\circ\ 45'\ 9''$, and its azimuthal distance from a distant terrestrial object to the westward of it was $60^\circ\ 26'\ 23''$; what was the angular distance of that object from the meridian, the declination of β Leonis being $+15^\circ\ 24'\ 14''$?

1009. At a place in Canada, the altitude and azimuth of α Lyræ, before passing the meridian, were observed, at $5^h\ 14^m\ 10^s{\cdot}6$ Greenwich Mean Time, to be, altitude $15^\circ\ 30'\ 59''$, azimuth $47^\circ\ 42'\ 40''$. Find the latitude and longitude of the place of observation; and thence determine its distance from Quebec (in latitude $46^\circ\ 49'$ N., longitude $71^\circ\ 16'$ W.), and also the bearing of Quebec. The R. A. and declination of α Lyræ being $18^h\ 31^m\ 55^s$ and $38^\circ\ 39'\ 14''$; the

R. A. of the sun $11^h\ 6^m\ 18^s\cdot5$, the Equation of Time $+2^m\ 24^s$; and the radius of the earth 3956 miles.

1010. The observed altitude of the Moon's lower limb was $34^\circ\ 17'\ 22''$, when that of α Arietis was $42^\circ\ 13'\ 47''$ (to the east of the meridian), and their measured distance $21^\circ\ 3'\ 18''$; the Moon's bright limb being towards the star. The Moon's horizontal parallax was $54'\ 15''$ and her semidiameter $14'\ 49''$; find the longitude of the place of observation, supposing the distances nearest to the above, tabulated in the almanac, to be, at 3^h Greenwich Mean Time, $20^\circ\ 22'\ 21''$; at 6^h Greenwich mean time, $21^\circ\ 27'\ 54''$; and that the Local Mean Time of the observation was $15^h\ 27^m\ 31^s$.

1011. The apparent altitude of the Sun's lower limb was $34^\circ\ 5'\ 44''$, and of the Moon's lower limb $56^\circ\ 56'\ 29''$, when the apparent distance between their nearest limbs was $35^\circ\ 16'\ 40''$. Find the true distance between their centres; the Moon's horizontal parallax being $54'\ 15''$, and that of the Sun $0'\ 8''$.

1012. The apparent altitude of a star was $20^\circ\ 13'\ 26''$, and that of the Moon's upper limb $31^\circ\ 32'\ 16''$, when the apparent distance of the star from the Moon's bright limb (which was on the side next the star) was $72^\circ\ 27'\ 22''$. The semidiameter of the Moon was $14'\ 54''$, and her horizontal parallax $54'\ 26''$. Find the true distance of the star from the Moon's centre.

1013. The apparent distance of the nearest limbs of the Sun and Moon was observed to be $80^\circ\ 29'\ 40''$, when the altitude of the Sun's lower limb was $72^\circ\ 9'\ 50''$, and that of the Moon's upper limb $18^\circ\ 46'\ 38''$: find the true distance of their centres, the semidiameter of the Sun being $16'\ 10''$, that of the Moon $16'\ 38''$, and her horizontal parallax $60'\ 58''$; that of the Sun being as in question 1010.

1014. The measured distance of a star from the Moon's bright limb, which was away from the star, was $18^\circ\ 37'\ 58''$, when the zenith distance of the star was $46^\circ\ 33'\ 10''$, and that of the Moon's lower limb $58^\circ\ 47'\ 23''$: what was the true distance, the semidiameter of the Moon being 16′ 23″, and her horizontal parallax 60′ 21″?

1015. The distance between a certain star and the Moon's centre was observed to be 27° 35′ 0″, when the altitude of the star was 29° 47′ 0″, and that of the Moon's centre 57° 22′ 0″; what was the true distance, the horizontal parallax of the Moon being 60′ 3″?

1016. The apparent altitude of the Moon's centre was observed to be 16° 26′ 0″, when that of Venus was 29° 41′ 0″, and their measured distance 98° 15′ 31″; the Moon's horizontal parallax was 60′ 35″ and that of Venus 20″; find the true distance.

1017. The measured distance of Jupiter from the Moon's centre was 120° 18′ 46″, when the altitude of Jupiter was observed to be 8° 26′ 0″; and that of the Moon's centre 19° 24′ 0″; the Moon's horizontal parallax being 57′ 14″ and that of Jupiter 1″·5; find the true distance.

CO-ORDINATE GEOMETRY.

1018. Find the equation to the straight line which passes through the points $(5\,;\,-7)$ and $(-4\,;\,3)$; find the tangent of the angle which it makes with the axis of x, and the points where it cuts the two rectangular axes.

1019. Find the distance of the point $(3, -5)$ from the line whose equation is

$$2x - 8y + 7 = 0,$$

and the angle which this line makes with the axis of x.

1020. Find the equation of the line which joins the points P and Q; P being the intersection of the lines

$$y = 7x + 3 \text{ and } 3y - 5x = 2\,;$$

and Q being the intersection of the lines

$$5y + 4x = 12 \text{ and } 6y + 8x = 4.$$

1021. Find the area of the triangle whose angular points are $(3; -2)$, $(5; 4)$ and $(-7; 3)$.

1022. Find the equation to the straight line which passes through the points $(5; -7)$ and $(-1; 4)$.

1023. The equations of the three sides of a triangle are

$$10x+5y=4,\ 2y-3x=6 \text{ and } y=0;$$

find the co-ordinates of its angular points.

1024. The co-ordinates of the angular points of a triangle are for A, $(3, -2)$, for B, $(7, 1)$, and for C, $(5, 9)$; find the length of the perpendicular let fall upon AB from C.

1025. Find the equation of the straight line which passes through the point $(5, -3)$, and makes an angle of 30° with the axis of x.

1026. Find the perpendicular distance of the intersection of the lines $2y-4x=10$ and $3y+9x=-21$ from the line $y-3x=15$.

1027. Find the intersection of the straight lines whose equations are $2y+5x-3=0$ and $7y-3x+10=0$.

1028. Find the equation to the straight line which passes through the point $(7, -5)$ and makes an angle of 45° with the line whose equation is $2y-6x=3$.

1029. The equation of a straight line is $5x+10y-7=0$; find its angles of inclination to the axes of x and y respectively; and also the points where it cuts these axes.

1030. If $y=ax+b$ be the equation to a straight line, show that the length of the perpendicular let fall upon it from the point (h, k) is

$$\frac{k-ah-b}{\sqrt{(1+a^2)}};$$

and show the form which this takes when the equation to a line is of the form

$$Ax+By+C=0.$$

1031. Find the equation of the straight line which passes through the point $(3, -2)$, and which is perpendicular to the straight line which passes through the points $(-5, 7)$ and $(2, 5)$.

1032. Find the length of the perpendicular drawn from the point $(8; 4)$ to the line whose equation is $y = 2x - 16$.

1033. Find the angle which the line $3y + 6x - 2 = 0$ makes with the line $2y - 10x + 7 = 0$.

1034. Find the angle included between the lines whose equations are

$$2x + 5 = 3y \text{ and } 4y + 3x = 0;$$

and draw a figure indicating roughly the position of the lines, and mark, on the figure, the particular angle you have obtained.

1035. Show, by oblique co-ordinates, that the three straight lines drawn from the angles of a triangle to the points of bisection of the opposite sides pass through the same point.

1036. Find the co-ordinates of the point of intersection of the three lines drawn perpendicular to the sides of a triangle at their middle points, and show that that intersection is equidistant from the angular points of the triangle.

1037. Show that the straight line which passes through the points (h, k), (h_1, k_1) intersects the straight line which passes through the points $(k, -h)$, $(k_1, -h_1)$ at right angles; and find the point of their intersection when $h = 5$, $k = 3$, $h_1 = -4$, and $k_1 = 2$.

1038. Find the equation to the line which passes through the intersection of the lines $x - 2y - a = 0$, and $x + 3y - 2a = 0$, and is parallel to the line $3x + 4y = 0$.

1039. Find the equation of the straight line which cuts the straight line passing through the points $(7, -1)$ and $(-3, 5)$, at an angle of 45°, and which also cuts the axis of x at a distance 5 from the origin.

1040. The equations of three straight lines are:

$$y = 5x - 7 \quad\ldots\ldots\ldots\ldots\ldots\ldots(1),$$
$$y = 7x - 5 \quad\ldots\ldots\ldots\ldots\ldots\ldots(2),$$
$$y = -4x + 2 \quad\ldots\ldots\ldots\ldots\ldots\ldots(3);$$

find the length of the portion of (3) intercepted between (1) and (2).

1041. Find the distance from $(-3, 7)$ to $(6, -5)$ when the axes of reference are at right angles to one another; and also when they are inclined at an angle of 60°.

1042. Find the point of intersection of the line $y = 5x - 7$, with the line $5y + x - 3 = 0$; and find the angle at which they intersect.

1043. A straight line whose equation is $y + x - 5 = 0$ cuts two others whose equations are $5y - 6x - 3 = 0$ and $y - x + 3 = 0$; find the length of the intercepted portion of the first line.

1044. Show that the measure of the angle contained by the two lines $y = ax + b$ and $y = a_1x + b_1$ is expressed by

$$\tan^{-1}\frac{a - a_1}{1 + aa_1}.$$

1045. Find the angle included by the lines $y = \frac{5}{2}x$, and $3y + 6x - 8 = 0$; and determine the condition that the lines $y = ax + b$, and $y = a'x + b'$ may be perpendicular to each other.

1046. Find the points at which the line $7y - 21x + 28 = 0$ cuts the co-ordinate axes, and the angle which it makes with the line which passes through the points $(3, -2)$ and $(-4, 7)$.

1047. The co-ordinates of the angular points of a triangle are $(3, 0)$, $(0, 3\sqrt{3})$ and $(6, 3\sqrt{3})$: find (1) the angles of the triangle, and (2) its area.

1048. Determine in inches the area of the triangle whose angles are at the points $(7, 2)$, $(4, -5)$ and $(-3, 1)$, measured on the half-inch scale.

1049. Find the distance from the point $(2, -7)$ to the straight line which passes through the points $(-4, 1)$ and $(3, 2)$.

1050. Find the distance of the point of intersection of the two lines $7x - 5y = 1$ and $4x + 2y = 20$ from the origin.

1051. Find the area of the triangle of which $(-1, 2)$, $(4, 4)$ and $(6, -3)$ are the angular points.

1052. Find the general equation to the circle:

(1) When the origin is at its centre.
(2) When the origin is at its circumference.
(3) When the origin is at any point within or without it.

1053. Construct the circle whose equation is

$$x^2 + y^2 - 2x + 6y = 3;$$

and determine the equation of that diameter of it which passes through the origin of co-ordinates.

1054. A circle passes through the origin of rectangular co-ordinates, and through the point $(3; 7)$; its centre is on the axis of x: find its equation.

1055. Find the radii and centres of the circles

(1) $6x^2 - 2y(7 - 3y) = 0$,
(2) $3x^2 - 6x + 3y^2 + 9y - 12 = 0$.

1056. Determine generally the points of intersection of a straight line and circle, when the origin is at the centre; and thence deduce the necessary relation between a, r and b when the straight line becomes a tangent to the circle.

1057. Find the radius and the co-ordinates of the centre of the circle whose equation is

$$7x^2 + 3y^2 - 4y - (1 - 2x)^2 = 0,$$

and find the points in which it cuts the axis of x.

1058. Find the locus of the point to which if straight lines be drawn from the angular points of a given triangle, the sum of their squares will be a constant quantity s^2.

1059. Find the equation of the circle which passes through the three points $(7, 5)$, $(-2, 4)$ and $(3, -3)$.

1060. Find the equation to the circle which passes through the points $(7, 4)$, $(-5, 3)$ and $(1, 7)$.

1061. Show that the general equation of the circle becomes

$$x^2 + y^2 - 2ry = 0$$

when the origin of the rectangular co-ordinates is a point in the circumference, and the centre is on the axis of y; r being the radius of the circle.

1062. The equation of a circle is

$$(y^2 + x^2)(1 + a^2)^{\frac{1}{2}} - 2b(x + ay) = 0;$$

find its radius.

1063. Find the equation of a circle referred to oblique axes; and find the centres and radii of the circles

$$y^2 + x^2 + 2y - 6x = 3,$$

$$\text{and} \quad 3y^2 + x\frac{6x-1}{2} + 6(y-2) = 5.$$

1064. Find the equation to the tangent to a circle at a point (h, k) in the circumference; thence deduce the equation to the normal, and show that it always passes through the centre.

1065. The equations of three circles A, B and C are

$$x^2 + y^2 = 3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (A),$$

$$x^2 + y^2 - 6x - 10y + 25 = 0 \quad \ldots\ldots\ldots\ldots \quad (B),$$

$$x^2 + y^2 - 4(4x + y) + 52 = 0 \quad \ldots\ldots\ldots\ldots \quad (C).$$

Show that the line joining the centres of A and B is perpendicular to that which joins the centres of B and C.

1066. Find the equation to the common chord of the two circles

$$x^2 + y^2 = 25 \text{ and } x^2 + y^2 + 6x - 8y = 0.$$

1067. The equation to a circle is

$$x^2+y^2-6x-12y+41=0;$$

find its radius, and also the equation to the tangent which passes through the origin of co-ordinates.

1068. Find the angle contained by those diameters of the circles

$$x^2+y^2+x+2y+1=0, \text{ and } x^2+y^2+2x+y+1=0,$$

which pass through the origin of co-ordinates.

1069. Determine the equation of the circle which has its centre at the point $(1, -3)$, and which touches the straight line

$$y-2x+4=0.$$

1070. Find the points in which the line $y=5x+2$ intersects the circle

$$y^2+x^2-4y-13x=9;$$

and the length of the part within the circle.

1071. Determine the points in which the line $y=3x+2$ cuts the circle

$$y^2+x^2-4x+4y=7.$$

1072. Find the angle at which a tangent to the circle

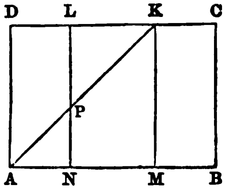
$$x^2+y^2-16=0$$

at the point whose abscissa is -3, cuts the axis of x.

1073. $ABCD$ is a rectangle; MK and NL are parallel to AD;

$$AN=BM,$$

and AK is joined; find the locus of P.



1074. Investigate the general equation of the conic sections, and show how it may be reduced to the form

$$y^2=\frac{b^2}{a^2}(a^2-x^2),$$

which is the equation to the ellipse.

1075. Define a parabola, and from the definition find its equation when the origin is a point in the curve and the axis of x is parallel to the directrix; the abscissa of the focus being a, and the distance of the focus from the directrix being $2m$.

1076. Investigate the equation to the ellipse, considered as the locus of a point in a straight line of given length, which has its extremities in two fixed straight lines at right angles to one another.

1077. Give a general definition of the curves termed conic sections; deduce their general equation, and show under what conditions it represents each of the three particular curves.

1078. Determine the major and minor axes, and the eccentricity of the ellipse

$$x^2 - 5 = \frac{5}{3}y^2.$$

1079. A straight line moves so as to have one extremity in, and to be always perpendicular to another straight line whose length is $2r$; find the locus of the other extremity, supposing the square of the first line to be always equal to the rectangle of the segments into which it divides the other.

1080. Find the locus of the point which is equidistant from the axis of y, and from the point $(6, 0)$.

1081. The legs of a pair of compasses, standing vertically upon a horizontal table, are gradually extended so that the hinge descends vertically; find the locus of the point of bisection of each leg.

1082. If (h, k), (h_1, k_1) be two points in a circle whose equation is $x^2 + y^2 = r^2$, show that the equation to the straight line joining those points may be reduced to the form $hx + ky = r^2$, when $h = h_1$ and $k = k_1$; and apply this principle to the determination of the equation of the tangent to a parabola.

1083. Prove that the locus of the intersection of two straight lines at right angles to one another, which pass through two fixed points, is a circle; find its radius, and the position of its centre.

1084. If a straight line of given length move so that its extremities are always upon two straight lines at right angles to one another, show that any point in it will describe an ellipse whose semi-axes are the segments of the line; and explain the particular cases in which (1) the point is at the bisection of the line, and (2) at either extremity.

1085. The parabola being the locus of a point which is equidistant from a given point and from a given straight line, find its equation when the given point is the origin of co-ordinates and the axis of y is parallel to the given line; and show, from this equation, that the distance between the points where the curve cuts that axis is equal to twice the distance from the given point to the given straight line.

1086. A tangent to an ellipse is inclined at an angle of 45° to the axes; find its point of contact with the curve.

1087. Find the point in which the line which joins the focus and the extremity of the axis minor of an ellipse meets the curve.

1088. Show, by means of its equation, $y^2 = 4mx$, that the parabola is a curve which cuts the axis of x in but one point, and that every point in the curve is on the same side of the axis of y.

1089. In the parabola $y^2 = 4ax$, find the length of the normal which passes through the extremity of the latus-rectum, intercepted between the two branches of the curve.

1090. The major axis of an ellipse and the axis of a parabola are in one straight line, and the vertex of the parabola is at the centre of the ellipse; find the points of intersection of the curves, the parameter of the parabola being equal to the minor axis of the ellipse.

1091. Assuming the area of a parabola to be two-thirds of the rectangle of the same base and altitude, show that parabolic areas of the same parabola and having parallel bases are as the cubes of their bases; and bisect a given parabolic area by a line drawn parallel to its base.

1092. Show that the locus of the point whose distances from a given point on the axis of x, and from a given straight line parallel to the axis of y, are always in the ratio $e : 1$, is the equation

$$y^2 + (1 - e^2)\, x^2 - 2m\,(1 + e)\, x = 0,$$

where m is the distance of the given point from the origin: and investigate the several forms which this equation may be made to assume when the given ratio is greater than, equal to, or less than unity.

1093. A parabola and an ellipse have the same vertex; and the axis of the parabola coincides with the major axis of the ellipse; find the points in which the parabola cuts the ellipse when the parameter of the former is equal to half the minor axis of the latter.

1094. Find the equation of the parabola which passes through the points $(0, 0)$, $(3, 2)$ and $(3, -2)$.

1095. Prove that the perpendicular from the focus of a parabola upon a tangent intersects that tangent in the tangent at the vertex.

1096. Find the locus of the point whose distances from two given points are always in a given ratio to one another.

1097. Show that a parabola having the same vertex and axis as an ellipse will cut the ellipse in two points if its latus-rectum be less than $\dfrac{2b^2}{a}$; and that it will not cut it at all if the latus-rectum be greater than that quantity.

1098. Find the curve from any point of which two normals being drawn to a given parabola, they will be at right angles to one another.

STATICS.

1099. Define "relative rest," "relative motion," and "equilibrium."

1100. Assuming that the resultant of two forces which are represented in magnitude and direction by the adjacent sides of a parallelogram is represented in *direction* by its diagonal; prove that such resultant is also represented in *magnitude* by that diagonal.

1101. Deduce the general expression for the resultant of two forces f_1 and f_2 whose directions make an angle a with one another; and also for the angle θ which the direction of this resultant makes with the direction of the force f_1.

1102. Assuming the parallelogram of forces, show that, when three forces are in equilibrium, they will be to one another as the sides of a triangle formed by drawing straight lines parallel to their directions.

1103. A balloon which could just raise a weight of 10 cwt. is held to the ground by a rope which makes an angle of 75° with the horizon; find the tension of the rope and the force of the wind upon the balloon.

1104. Two weights P and Q act by a string over two fixed pulleys at A and B, and support a third weight W, at C, between them; find the ratios of P, Q and W, when AB is horizontal, the angles of the triangle ABC are in arithmetic progression, and C is a right angle.

1105. Two forces in the ratio of $2 : 5$, and whose resultant is a mean proportional between them, make an angle θ with one another: find $\cos\theta$.

1106. Three forces represented by the weights 50 lbs., 40 lbs. and 30 lbs., act, in one plane, upon a point; what angles does the force 50 lbs. make with the other two when the three are in equilibrio?

1107. What force must a man exert in a horizontal direction to draw a weight of 3 cwt. four feet out of the perpendicular, supposing it to be suspended from a point twenty feet above that at which he applies his strength?

1108. Two forces which act at two adjacent angles A and B of a parallelogram $ABCD$ are proportional to the corresponding diagonals CEA, DEB, and act in those directions respectively; find the direction, the relative magnitude, and the position of a third force which will keep the figure at rest. Show that the effect upon the figure will be the same whether it be acted on by forces proportional to, and in the direction of, EB and EA, or DB and BA, or $\frac{1}{2}DA$ and $\frac{1}{2}CB$,—all the forces acting at E.

1109. A parallelogram $ABCD$ is acted upon by forces in the directions and proportional to AB, BD, and DC; find the direction and proportional magnitude of a fourth force which will produce equilibrium.

1110. Three forces, P, Q, and R, all acting in the same plane upon a particle, keep it at rest; supposing the values of P and Q, and the directions of Q and R to be given; find the value of R, and the direction of P.

1111. Four forces of 5 lbs., 6 lbs., 8 lbs. and 11 lbs. make angles of 30°, 120°, 225° and 300° respectively, with a fixed straight line: find the magnitude and direction of their resultant with reference to that line.

1112. Three tacks are fixed in a vertical wall and form an isosceles triangle ABC, in which $AB = AC = 50$, $BC = 30$ and AB is horizontal: a string is passed *over* the tacks A and B, and *under* the tack C; and, at its extremities, two equal weights of 20 lbs. are suspended: find the magnitude and direction of the pressure upon C.

1113. Two strings, AC and BD, are tied to two pegs, A and B, in a horizontal line, the string BD passing through a smooth ring at C: find the position of C, and the tension of AC when a weight of 6 lbs. is suspended at D, and $AC = \frac{3}{4} AB$.

1114. Two forces P and Q make an angle, a, with one another; find an expression for the magnitude of their resultant R, and the angle which it makes with the force Q.

1115. Three forces, 5, 6, and 7, all in the same plane, and making equal angles with one another, act upon a point; what force, acting in the same plane, will keep the point at rest; and what angle must it make with the force 6?

1116. Three forces f_1, f_2, f_3 make angles of 30° and 45° with one another taken in the above order; find the value of f_3 in terms of f_1 and f_2.

1117. A weight of 8 cwt. suspended by a rope from the top floor of a warehouse, on one side of a street 50 feet wide, is required to be drawn across by a rope from a floor at the same height on the other side of the street, attached to a point in the first rope 35 feet below the point of suspension; what will be the tension of the ropes when the weight is vertically over the centre of the street?

1118. A, B, and C are three tacks at the angular points of a vertical equilateral triangle; AB is inclined at an angle of 15° to the horizon and C is vertically below AB: a weight of 60 lbs. is suspended by a rope fixed at A, passing under C, over B, and then over C; find the direction and amount of the pressure upon C.

1119. A peg or tack A has four cords attached to it, at the ends of which four men pull, each with a force of 100 lbs., and in directions which are all in the same vertical plane and make equal angles of 30° with one another; find the magnitude and direction of the strain upon A when a gun weighing 18 cwt. is hung upon it, and when the angles which the outer cords make with the horizon are equal.

1120. Three tacks A, B and C in a vertical plane are so situated that AB is horizontal, the angle ABC is 60° and the angle BAC 45°; and a cord connecting two equal weights is passed over A and B, under C, and back again over B: find the pressures upon the tacks, and their directions.

1121. (α) If the angle between the directions of two forces be increased, their resultant will be diminished. And (β) if three forces in a plane acting upon a point be proportional to the sides of a triangle constructed upon their directions taken in order, each of them will be equal and opposite to the resultant of the other two. But (γ) if an angle of this triangle be increased, the opposite side will be increased, and therefore the resultant will be *increased.* Demonstrate the statements (α) and (β), and reconcile the apparent anomaly involved in them as indicated in (γ).

1122. Show that, when any number of forces f_1, f_2, f_3, &c. acting at a point, have their directions all in the same plane, the angle which their resultant makes with any fixed line through the same point, may be expressed by

$$\tan^{-1}\frac{f_1 \sin \alpha_1 + f_2 \sin \alpha_2 + f_3 \sin \alpha_3 + \&c.}{f_1 \cos \alpha_1 + f_2 \cos \alpha_2 + f_3 \cos \alpha_3 + \&c.},$$

when α_1, α_2, α_3, &c. are the angles which the several forces make respectively with that fixed line.

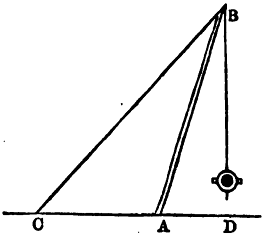
1123. Prove that if R and S be two commensurable forces acting at a point, their resultant will act in the direction of the diagonal of the parallelogram constructed upon the two lines, drawn in the directions of R and S respectively, and proportional to them in magnitude.

1124. Assuming the principle of the "parallelogram of forces," show that when three forces are in equilibrium, they are to one another as the sides of a triangle formed by drawing straight lines making any, the same angle with their respective directions.

1125. A cord whose length is 10 feet is fastened at *A* and *B*, two points in a horizontal line, and distant 6 feet from each other; find the tension of the cord when a weight of 20 lbs. is suspended to a ring which runs freely upon the cord.

1126. A string, 2 feet long, is fastened to two tacks in the same horizontal line and 18 inches apart, and, at a point six inches from one extremity, a weight of 10 lbs. is fixed; find the tensions of the two parts of the string, and the portions of the weight supported upon each tack.

1127. A 9-pounder gun weighing 13 cwt. 2 qrs. is suspended at the extremity *B* of a spar *AB*, which is supported by a stay *BC*, fixed at *C*; find the tension of the stay and the pressure on the spar when $AB = 13$ feet, $AC = 10$ feet, and *AD*, the horizontal distance of the gun from *A*, $= 5$ feet.



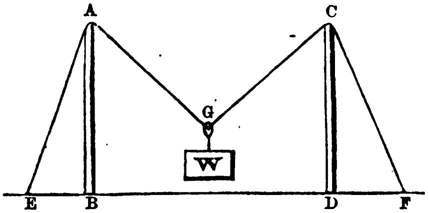
1128. Three pegs, *A*, *B*, and *C* are fixed in a vertical board at the angles of an equilateral triangle, the side *AB* being horizontal, and the angle *C* downwards: two strings having weights *P* and *Q* attached to them, are tied to *A* and *B* respectively; the first string is then passed over *B*, under *C*, and over *A*; and the other is passed over *C*. Find the strains on *A*, *B*, and *C*, and their several directions.

1129. A heavy rod, 10 feet long, has a cord 20 feet long attached to its ends, and passing over a smooth peg; find the position in which it will rest, if its centre of gravity divide its length in the ratio of 2 : 3.

1130. A rope runs through a ring to which is suspended a weight of 1 cwt.; and two men pull at the ends of the rope: find its tension, supposing the hands of each man to be 3 feet above the ring, and the length of the rope 12 feet.

1131. A string 14 inches long, is fastened at its extremities to two tacks, *A* and *B*, 10 inches apart, in a horizontal line; and 6 inches from *A* a weight of 20 ozs. is suspended: find the tensions of the two portions of the string, and the portion of the weight supported by each tack.

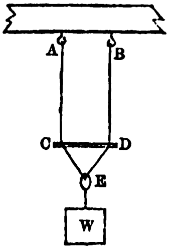
1132. A gun weighing 9 cwt. is to be raised from the ditch of a fortress by means of a rope fastened to the top of the escarp and running through a ring to which the gun is hung, and then passing up to the top of the counterscarp: find the tension of the rope when the ring is 16 feet below the top of the counterscarp, the difference of relief of the escarp and counterscarp being 10 feet, and the horizontal distance between their summits 80 feet.



1133. *AB* and *CD* are two equal vertical poles which support a weight *W* by means of a rope *AGC* running through a ring at *G*; these poles are prevented from falling by stays *AE* and *CF*, and it is required to find the strains or tensions of these stays, and the pressures on the poles *AB* and *CD*, when $AGC = 2a$, $EAB = FCD = \theta$.

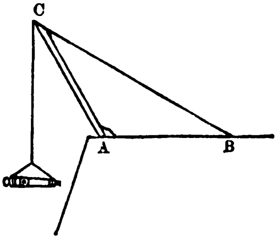
1134. A rectangular ferry-boat 20 feet long and 10 feet wide is moored in the middle of a river by a rope 100 yards long fastened to the centre of the boat; find how far the force of the stream will urge the boat laterally, if its keel be kept at an inclination 30° to the direction of the current, and the force of the stream be supposed uniform.

1135. A uniform iron rod weighing 14 lbs. is supported at its ends by two strings which will bear no greater strain than 10 lbs. and 8 lbs. respectively; find the directions of the strings when the greatest weight possible is suspended from the rod; and find its point of suspension.



1136. A rod CD is suspended in a horizontal position by two parallel cords AC and BD, and a weight W is suspended by another cord CED, running through a smooth ring at E, and fixed at C and D: find the tensions of AC, CE, ED, and DB and the compression of CD, when the angle $CED = 60°$. State what would take place if the cord $ACEDB$ were continuous and allowed to run smoothly through eyes at C and D.

1137. A canal boat is drawn along the middle of a straight canal by two men hauling upon ropes 50 feet and 80 feet long respectively; find the ratio of the strains upon them, supposing the canal to be 30 feet wide.



1138. A gun, to be lowered over the parapet of a fortress, is suspended from the extremity C of a spar AC, 10 feet long, held in its position by a stay BC 15 feet in length, the distance from A to B being 10 feet; find the tension of BC and the thrust upon AC, when AB is horizontal, and the weight of the gun 33 cwt.

1139. Define "the centre of parallel forces," and show that in the case of two such forces acting upon a rigid body, their "centre" divides the straight line which joins their points of application in the inverse ratio of the forces.

1140. Five equal bodies are placed so that their centres of gravity are at the angular points of a regular hexagon; find the position of their common centre of gravity, when referred to the unoccupied angle.

1141. If three parallel forces be proportional respectively to a, b, c, the sides opposite to the angles of the triangle at which they act; show that the distance of their centre from the angle A is

$$\sqrt{\frac{bc\,.\,(s-a)}{s}}; \qquad \text{where } s = \frac{a+b+c}{2}.$$

1142. The triangle formed by joining the points of bisection of two of the sides of a triangle with its centre of gravity and with one another, is one-twelfth of the area of the triangle.

1143. Three parallel forces act at the circumference of a graduated circle, at the points marked 30°, 135° and 240°, and their magnitudes are to one another as the cosines of these respective arcs; find the distance of the centre of these forces from the centre of the circle, and the angle which the line joining these centres makes with the first force.

1144. Five equal parallel forces act at five of the angles of a regular hexagon; find the distance and direction of their centre from the remaining angle.

1145. The interior diameter of a 13-inch shell being 9·3 inches, find the position of the centre of gravity of one of unequal thickness, in which the centres of the interior and exterior spherical surfaces are 1 inch apart; neglecting the fuze-hole.

1146. The opposite points of bisection of the sides of a parallelogram are joined, and one of the parallelograms thus formed is cut from the figure: find the position of the centre of gravity of the remaining surface.

1147. If through the centre of gravity of a triangle two straight lines be drawn parallel to two adjacent sides and limited by the third side; the triangle so produced will be equal to one-ninth of the original triangle.

1148. Three weights of 1, 2 and 3 lbs. are placed at the angles of an equilateral triangle; find the distance of their common centre of gravity from each of them when the side of the triangle is 2 feet.

1149. Find the centre of gravity of a solid composed of a cylinder and cone on the same base and of equal altitudes:

1st When their densities are equal;

2nd When the density of the cone is double that of the cylinder.

And find their relative altitudes when the centre of gravity is in the centre of their common base.

1150. Find the centre of gravity of a cylinder having a cylindrical bore; the cylinder being 10 feet long and 10 inches in diameter; the bore 8 feet long and 4 inches in diameter.

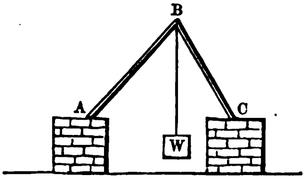
1151. Find the position of the centre of gravity of a gun whose content is C and length of bore b; the bore being cylindrical (circular section, radius r) with a conical chamber whose length is one-twelfth of b: supposing the distance of the centre of gravity of the unbored mass to be at a distance m from the muzzle.

1152. Prove that the centre of gravity of a pyramid is in the line joining its vertex and the centre of gravity of its base, and that it divides this line in the ratio of 1 to 3.

1153. An equilateral triangle is suspended at a point in one of its sides one-third of its length from the adjacent angle; find the inclination of its other sides to the vertical.

1154. A beam whose centre of gravity divides its length in the ratio of $a : b$, is placed in a smooth hemispherical bowl, and subtends an arc 2α at the centre of the sphere; find the inclination of the beam to the horizon.

1155. AB and BC are two beams united by a hinge at B, and resting on walls at A and C; W is a weight suspended from B, and the weights of the uniform beams are P and Q. Find the horizontal thrusts at A and C, supposing $W = P = Q = 100$ lbs., $ABW = 45^\circ$ and $CBW = 30^\circ$.



1156. Prove that the centre of gravity of a triangle divides each of the lines drawn from the angular points of the triangle to the points of bisection of the opposite sides in the ratio of 2 : 1.

1157. A square lamina has one of its angles cut off by a straight line which bisects two of its adjacent sides; find the centre of gravity of the remainder.

1158. Prove that the centre of gravity of a triangle remains unaltered, if the triangle formed by joining the points of bisection of its sides be removed.

1159. A rectangle is divided into four triangles by its diagonals, and one of these triangles is cut out from the rectangle; find the position of the centre of gravity of the remaining surface.

1160. A square and an equilateral triangle having a common side and of uniform thickness, are rigidly connected, and a weight equal to the weight of the triangle is suspended at the vertex of the triangle; find the position in which the figure must be placed so that it may balance on a horizontal edge parallel to the common side.

1161. Find the centre of gravity of the figure formed by describing an equilateral triangle upon one of the sides of a square.

1162. A cylinder 10 feet long has a cone attached to one end of it by its base, which is of the same circular section as the cylinder; find the height of the cone, so that the whole mass may balance about a point 8 feet from the other end of the cylinder.

1163. A 13-inch shell weighing 200 lbs., slung on a pole 8 feet long, has to be carried some distance by two men, one of whom, having just come out of hospital, is required to support only two-thirds the weight the other man carries; where must the shell be slung in order to produce this effect? Find also the weight supported by the weak man when the shell is suspended 6 feet from his end of the pole.

1164. A cone and hemisphere of the same material and having the same base are fixed together, and it is found that they can be made to roll in a straight line from the top to the bottom of an inclined plane; find their relative dimensions.

1165. A straight uniform rod, 20 inches long, balances upon a fulcrum 8 inches from one end, when weights of 25 oz. and 10 oz. are suspended at its extremities; find the weight of the rod.

1166. If from one point of the angular points of any regular plane figure straight lines be drawn to the other angles, show that the centres of gravity of the triangles thus formed lie in the circumference of a circle, the radius of which is two-thirds of that of the circle inscribed within the figure.

1167. If at the angular points of a material plane triangle, weights A, B, and C be placed; find the position of the centre of gravity of the whole system, the weight of the triangle being W; and show from the result that, when the weights A, B, and C are equal, the common centre of gravity is that of the triangle.

1168. Find the position of the centre of gravity of a hollow cylinder, the radii of whose interior and exterior surfaces are r and R, and their axes at a distance a from one another.

1169. Find the position in which a cylinder having a conical cavity (the axis and the base of which are coincident with those of the cylinder, and its altitude and the diameter of its base one-half of those of the cylinder) will rest when suspended freely at a point in the circumference of its base.

1170. If from a sphere 16 inches in diameter a cone having the diameter of its base equal to its slant height be turned out, find the position of the centre of gravity of the remaining solid.

1171. Find the centre of gravity of a solid composed of a cylinder and cone having a common base, the length of the cylinder being 1 foot and the height of the cone 6 inches.

1172. A circular marble slab weighing 20 lbs. is supported as a table upon three vertical legs at its circumference, their distances subtending angles of 75° and 135° at the centre; find the pressure on each leg.

1173. A cone and a hemisphere have a common base, and their common centre of gravity is in the centre of that base; find the height of the cone, the centre of gravity of the hemisphere being three-eighths of its radius from the centre of its base; and show that the compound solid will rest in any position on the hemisphere when placed on a horizontal plane, but that there is no position in which it will rest on the hemisphere when the plane is inclined to the horizon; assuming always that the plane and hemisphere are sufficiently rough just to prevent sliding.

1174. The centre of gravity of a hemisphere is at a point distant three-eighths of the radius from the centre of its base; find the position in which the solid will rest if suspended from a point in the circumference of its base; and find a point in its convex surface from which if it be suspended, the plane of its base will be inclined to the same amount as before on the other side of the vertical.

1175. A conical vessel full of water has mercury poured into it until the mercury rises to one-fourth of the side of the cone measured from the vertex. What is the position of the common centre of gravity of the mercury and water, the specific gravity of the mercury being m?

1176. If the arm BF of a straight or bent lever be of such length and thickness as to cause the other arm FA to rest horizontal when a moveable weight w is suspended at H (between F and A); show that the distances from H at which the constant weight w must be placed in order to balance successive weights suspended at B, will be proportional to those weights.

1177. A weight is placed upon a horizontal table which has three vertical legs A, B, and C; the portion of the weight supported by A is 8 lbs., that by B is 5 lbs., and that by C is 9 lbs.; find the weight and its position upon the table, the distances between the legs being 2 feet, 4 feet, and 5 feet respectively.

1178. A uniform bent lever, when supported at the angle, rests with the shorter arm horizontal; but if this arm were twice as long it would rest with the other horizontal; find the lengths of the arms and the angle between them, when the whole length of the lever is 30 inches.

1179. A piece of timber, 25 feet long, balances upon an edge at a distance of 10 feet from one end, and, when the edge is shifted to 12 feet, it requires 56 lbs. to be placed upon the other end to make it balance; what is the weight of the piece of timber?

1180. Four weights, 2, 6, 14, and 10, are placed at equal distances on a straight lever. Determine the fulcrum when the lever is 21 inches long and the weights 2 and 10 are placed at its extremities; supposing the lever to be without weight.

1181. A uniform bar of iron 10 feet long projects 6 feet over the edge of a wharf, there being a weight placed upon the other end; and it is found that when this is diminished to 3 cwt. the bar is just on the point of falling over; find its weight.

1182. A uniform lever, whose weight is 8 lbs. and length 3 feet, has a weight of 20 lbs. suspended from one end, and 14 lbs. from the other; find the position of the fulcrum when there is equilibrium.

1183. The ratio of the arms of a bent uniform lever, inclined to one another at an angle of 120°, is 5 to 4; find their inclination to the horizon, when weights of 20 lbs. and 16 lbs. respectively are suspended from their extremities; the fulcrum being at the angle. Find also the point on the longer arm upon which the lever will balance so as to have that arm horizontal; the lengths of the arms being 30 inches and 24 inches, and no weights being suspended.

1184. To the extremities of a smooth circular arc of β degrees without weight, two weights P and Q are suspended; find the position of the fulcrum over which the arc must be suspended, concavity downwards, to produce equilibrium.

1185. The arms of a bent uniform rod are as 3 to 5, and they are at right angles to one another; find the position in which it will rest when suspended by the angle.

1186. A uniform rod $2a$ feet long, and $2p$ pounds in weight, is balanced upon its middle point; how much must it be lengthened in order that it may remain horizontal when a weight W

is suspended from one of its ends, the fulcrum remaining unaltered?

1187. A uniform rod of unknown length, weighing $1\frac{1}{2}$ lb. per linear foot, rests on a fulcrum 4 feet from one end; find what weight suspended from that end will keep it at rest, the pressure on the fulcrum being 75 lbs.

1188. A weight of 10 lbs. suspended at one extremity of a horizontal lever, 3 feet long, at 9 inches from the fulcrum, is balanced by a force P acting at the other extremity and making an angle of 60° with the lever, which weighs 2 lbs. Find P when the centre of gravity of the lever is 7 inches from the point of application of the weight.

1189. Find the tension of each of two ropes which, passing round the circumference of a cylindrical pontoon and fastened to the upper extremity of a plane, support upon it the pontoon which weighs 4 cwt.; the ropes being parallel to the plane, which is inclined to the horizon at an angle of 30°.

1190. In the system of n pulleys in which all the strings are fastened to the weight, find the strain upon the axle of the pulley which is fixed; assuming the weights of the pulleys to be each equal to w.

1191. The weights of the several moveable pulleys in the system where each string is attached to the weight, are 1, 2, 3, and 4 oz. respectively, commencing at that over which the power first acts; find the weight supported by the force of 1 lb.

1192. A gun, in which the distance of the muzzle from the axis of the trunnions is t, is found to balance upon its trunnions when a weight w is suspended from the muzzle; find the weight of the gun, supposing g to be the distance of its centre of gravity from the muzzle.

1193. A bent lever whose arms are a and b, inclined at an angle α to one another, has the weights P and Q respectively suspended at their extremities; prove that the inclination of the arm b to the horizon is $\cot^{-1}.\dfrac{P.a.\sin\alpha}{Qb - Pa.\cos\alpha}$.

1194. A force f is required to lift the trail of a field-gun when a man whose weight is M rests his whole weight upon the muzzle of the gun; the length of the trail measured from the axle of the wheels being t, and the horizontal distance of the centre of gravity of the carriage g, that of the gun G (both behind the axle), and that of the man m, all measured from the axle; find the weight of the gun, the weight of the carriage being C, the angle which the line drawn from the end of the trail to the axle makes with the horizon a, and the gun horizontal.

1195. A gun is just raised from the ground at one end by a cord attached to the neck behind the breech and passing over a fixed pulley vertically over the neck, having a weight w suspended on the other side of the pulley; and it is found that when the same arrangement is made at the muzzle, a weight w just raises that end; find the weight of the gun, and the distance of the centre of gravity from the muzzle, the length from the muzzle to the neck being l feet, and the length from the muzzle to the base-ring l_1 feet.

1196. A weight of 7 cwt. rests upon a plane inclined at an angle of 30° to the horizon; what force, acting parallel to the horizon, will just prevent its sliding down the plane when the coefficient of friction is $\frac{1}{4}$; and what is the least force which, acting parallel to the plane, will draw it up?

1197. Show that, if a be the inclination of a plane, i the angle which the supporting force P makes with the face of the plane, W the weight supported, and R the pressure upon the plane, the following relations subsist when there is equilibrium:

$$P = W.\frac{\sin a}{\cos i}; \qquad W = R.\frac{\cos i}{\cos (a+i)}; \qquad R = P.\frac{\cos (a+i)}{\sin a}.$$

1198. In the single moveable pulley, show that, if the strings make an angle a with the horizon,

$$P = \frac{W}{2}.\operatorname{cosec} a.$$

1199. Find the weight which can be supported by a power of 15 lbs. by means of a system of pulleys, in which each string is attached to the weight, there being four moveable pulleys, each weighing 4 lbs.

1200. A force P, making an angle i with the plane whose angle of inclination is a, acts upon a body which rests upon the plane, and whose weight is W; supposing that P is not sufficient to support the body upon the plane, find an expression for an additional force Q which, making an angle γ with the plane, will just produce equilibrium.

1201. A gun, weighing with its carriage 11 cwt., is to be drawn up a plane inclined to the horizon at an angle of 20°; find the force which must be exerted to draw it up by ropes making an angle of 10° with the plane; and the pressure on the plane, supposing the amount of friction to be inconsiderable.

1202. What force is necessary to support a 13-inch shell (weight 196 lbs.) upon a plane inclined to the horizon at an angle of 30°, the force acting horizontally?

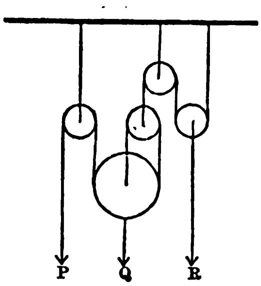
1203. A shell is found to remain at rest on a plane inclined to the horizon at an angle of 15° when a horizontal force h is applied at its surface in a direction passing through its centre; find the weight of the shell.

1204. A gun and carriage weighing 12 cwt. are to be drawn up a plane inclined to the horizon at an angle of 30°, by four ropes fastened to the naves of the four wheels, and making angles of 15° with the face of the plane; find the tension of each rope when the gun just begins to move up the plane.

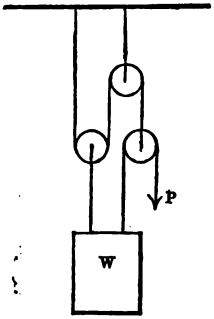
1205. Find the power which must be applied to a system of pulleys in which each string is fixed to the weight, in order to raise a weight of 2 cwt., supposing that there are four strings, and that each pulley weighs 2 lbs.

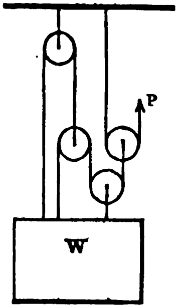
1206. What weight will be supported by a power of 12 lbs. acting by means of a system of pulleys in which each cord is fastened to the weight, when there are five moveable pulleys, each weighing $1\frac{1}{2}$ lb.?

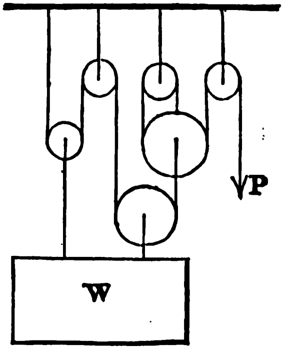
1207. Find the weights which must be suspended at Q and R to establish an equilibrium in the annexed system of pulleys when a force P acts vertically at P, the weights of the pulleys being neglected.



1208. Determine the ratio of the power to the weight in each of the following systems of pulleys:







1209. Five turns of a screw working in a fixed collar have the effect of raising the end of the screw one inch; what would be the effective force at the end of the screw in the direction of its axis, if a force of 10 lbs. were applied at the extremity of a lever 5 feet long working in the head of the screw?

1210. The length of each arm of the elevating screw of a gun is 6 inches, and it is found that one turn of the screw has the effect of elevating the piece $41'$; what power is required to turn the screw, supposing the whole vertical resistance to be overcome to be 3 cwt., and the distance from the axis of the trunnions to the centre of the screw to be 3 ft. 6 in.?

1211. Find the inclination to the horizon of the thread of a screw, which, with a force of 5 lbs. acting at an arm of 2 feet, can support a weight of 300 lbs. on a cylinder of 3 inches radius.

1212. A locomotive having run off the line, it is proposed to raise one side of it by means of a machine consisting of a winch which turns an endless screw, which works in the teeth of a wheel having a pinion attached; the teeth of this pinion are of the same dimensions as those of the wheel, and work in a vertical rack, at the extremity of which is the point of support: what effect would such a machine produce if a force of 56 lbs. were applied to the winch, the various dimensions being:

Arm of the winch = 15 inches; No. of teeth in wheel = 100;
Distance between the threads = 1 inch; No. of teeth in pinion = 5?

1213. Ten men heave, with a force of 50 lb. each, at as many capstan-bars, 10 feet from the axis of the capstan; the cylinder round which the hawser winds is 2 feet in diameter, and the hawser four inches; find the tension of the hawser.

1214. Supposing that, in question 1201, the power is to be applied by a capstan and system of pulleys in which the same string passes round all the pulleys, all the strings being parallel to the face of the plane; find the power to be applied with the following data:

Length of handspike 6 feet; diameter of axle 16 inches; number of sheaves in each block 4.

1215. An endless screw works in the teeth of a wheel, round the axle of which a rope is wound, which then passes *continuously* round a system of pulleys having four sheaves in the lower block; find the weight which a power of 10 lbs., applied to the arm of the screw, which is 1 foot long, will support, supposing the distance between the threads of the screw to be 1 inch, the radius of the wheel 1 foot, and that of the axle 3 inches.

1216. A pinion of 12 teeth works in a wheel of 96 teeth, upon the axle of which, 6 inches in diameter, a rope winds, the other end of which is coiled round a wheel 12 feet in diameter, and fastened to its circumference: upon the axle of this wheel, 18 in. in diameter, another rope is coiled which, passing down a plane inclined to the horizon at an angle of 45°, is fastened to the truck to be drawn up the plane: what number expresses the mechanical advantage of this arrangement, if the first pinion be worked by a winch 2 feet long?

1217. A cone whose height is h and the radius of its base r, rests with its curved surface upon an inclined plane, and is prevented from sliding; find an expression for the tangent of the greatest inclination of the plane, so that the same surfaces may continue in contact.

1218. Find the strain upon a tent-peg fixed into the ground, when the effective force of the wind upon the top of the pole in the direction of the horizontal line drawn in the plane of the pole and the peg is 120 lbs., and the angle which the tent-rope makes with the horizon is 45°.

1219. An isosceles right-angled triangle is suspended by one of its acute angles; find the angle which its hypothenuse makes with the horizon.

1220. Find the cylinder of greatest length, the radius of whose base is r, which can be made to rest with its base inclined to the horizon at an angle α.

1221. A uniform beam, whose length is $2a$, rests with one end fixed upon a smooth horizontal plane by a hinge, and, at a distance c from this end, it rests upon a smooth sphere which is supported upon the same plane; determine the horizontal force necessary to maintain the sphere in its position.

1222. A beam, loaded with a weight W at its middle point, rests upon two supports in the same horizontal line, and at a distance l from one another; find the moment of strain when the deflection is α.

1223. Two weights P and Q balance one another upon the surface of a sphere, being attached to one another by a string which passes over the highest point of the sphere. Find the position of equilibrium when $P = 2Q$.

1224. A beam AB moveable, in a vertical plane, about A, has its other extremity B supported by a weight P acting over a pulley vertically above B; find the weight of the beam when its centre of gravity divides its axis in the ratio of $5 : 7$, from A towards B.

1225. A ladder AB resting with its foot against the base of a vertical wall AC, is to be partially raised by means of a rope passing over a pulley at C, and attached to the ladder at a point D, which is a feet distant from A: what angle will the ladder make with the wall when a weight Q is attached to the rope, supposing the centre of gravity of the ladder to be b feet from A, and the wall to be c feet high?

1226. A mortar, whose weight is W, being suspended from a uniform beam whose length is $2l$ and weight w, at a distance b from its lower end, which rests on the ground at a distance d from a vertical wall, against which the other extremity of the beam rests: it is required to find the greatest tension of a rope, which, being fixed to the upper end of the beam, is hauled upon at a point whose horizontal distance from the foot of the wall is a.

1227. A rectangle whose sides are a and b, is suspended from a point in the side a, which is at a distance c from one of the adjacent angles; find the position in which it will rest, that is, the angle which the side a makes with the horizon.

1228. In the system of pulleys in which each string is attached to the weight, find the tension of the nth string, in terms of p the power and w the weight of each pulley.

1229. Three turns of a screw acting vertically have the effect of raising a weight 2 inches; find the pressure which a power of 10 lbs. acting at the extremity of an arm 10 inches long will produce at the opposite end of the screw.

1230. Five turns of a screw carry the head forward 1 inch; find the power necessary to support a pressure of 1 ton, the length of the lever being 2 feet.

1231. A power of 5 lbs. acting at an arm 10 inches long supports a weight of 150 lbs. upon a screw an inch and a half in diameter; find the inclination of the thread of the screw.

1232. Find the distance between the threads of a screw which is worked by an arm l, when the power applied at its extremity is an nth part of the pressure on the screw.

1233. A man applies a force of 80 lbs. at the end of a handspike 6 feet long, working in an axle 8 inches in diameter, about which is coiled a rope which runs through a pulley of two sheaves at the lower block. What weight can the man support?

1234. If the distance of the centre of gravity of a gun from the axis of the trunnions be g, the radius of the trunnion r, the circumference of the elevating screw c, the linear pitch h, the length of its arms R, the distance of the head of the screw from the axis of the trunnions l, and the coefficient of friction tan 30°; find the forces necessary to elevate and to depress the gun, its weight being represented by W.

1235. In a system of wheels and pinions, the numbers of teeth in the successive wheels are 120 and 96, and the numbers in the pinions are respectively 20 and 6; and, upon the axle of the last wheel, which is 6 inches in diameter, a rope is wound, which, passing round a system of 3 moveable and 3 fixed pulleys of the first kind acting parallel to a plane inclined to the horizon at an angle of 30°, draws a weight up the plane. What must be the length of the lever attached to the first pinion, in order that a power of 20 lbs. may draw a weight of 25 tons up the plane, excluding friction?

1236. An endless screw works in the teeth of a wheel, attached to which is a pinion of ten teeth, which again work in the 74 teeth of another wheel, upon the axle of which, 7 in. in diameter, is a rope proceeding from a system of pulleys of the third kind, in which each rope is fixed to the weight: find the weight which can be supported by a power of 1 lb., supposing that the diameter of the first wheel is 30 inches, that there are 4 moveable pulleys, each weighing 6 lbs., that the length of the arm which works the screw is 2 feet, and the distance between the threads half an inch.

1237. In a train of wheels and pinions the numbers of teeth in the wheels are 80, 60, 50, and 30 respectively, and in the pinions 10, 6, 5, and 4 respectively; the first pinion is turned by a winch 10 inches long, and the last wheel has an axle of 6 inches diameter, round which a cord is bound, and to which a weight W is suspended: find the force P which, acting at the winch, will produce equilibrium, disregarding friction.

1238. In the testing of chain cables, a combination of 4 levers is so arranged that a lb. weight in the scale-pan of the last lever indicates a strain of one ton upon the chain, which is attached to the first; find the ratio of the arms, on the supposition that it is the same in all.

1239. In a traversing crane consisting of five pinions and wheels, the winch which works the first pinion is 2 feet long, and on the axle of the last wheel a rope is wound, which, passing continuously round a series of pulleys of the first kind, is fastened to the lower block; find the weight which could be supported by a power of 20 lbs. acting at the winch, supposing each pinion to contain 6 teeth and each wheel 54 teeth, the radius of the axle being 4 inches, and the number of sheaves in the lower block 4.

1240. A gun, whose weight is W, is attached to a point 4 feet distant from one end of a uniform beam 12 feet long, which rests upon a smooth horizontal plane, while the other end rests against a smooth vertical plane; find the horizontal force necessary to prevent the beam from sliding, when it makes an angle of 60° with the horizon, and the weight of the beam is w.

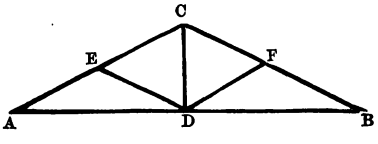
1241. A drawbridge whose length is l and weight W, revolves upon a horizontal hinge and is lowered by two side chains fastened to its further end and passing over two pulleys fixed in the masonry of the gate at a height h above the roadway; find the strain upon each chain and upon the hinge when each chain makes an angle of 30° and the plane of the bridge an angle of 60° with the vertical.

1242. A moveable beam is fixed to an upright wall by a hinge which allows it free motion in a vertical plane; at a point in the wall vertically above the hinge, and at a height equal to the length of the beam, is a fixed pulley, over which is passed a rope which is fastened to the beam at its extremity; at the other extremity of the rope is attached a weight w, which hangs freely on the other side of the wall: find the position of equilibrium when the weight of the beam is W, and its centre of gravity divides its length in the ratio of $a : b$.

1243. In a roof consisting of two equal beams, inclined at an angle α to the horizon, the weight of each of which, with the covering, is w, and which support a weight W on the ridge; show that the horizontal thrust is $\dfrac{W+w}{2\tan\alpha}$.

1244. Find the horizontal thrust or tension of the tie-beam of a roof constructed of three equal rafters, of which the middle one is horizontal, and the others equally inclined to the horizon, at an angle α.

1245. In the isosceles truss ABC, in which the struts ED and DF are parallel to BC and AC; find the whole weight supported by the king-post, and the longitudinal strain upon the tie-beam, when weights of 2 cwt. are supported at E and F, and 5 cwt. at C; the weights of AC and CB being each 1 cwt., of ED and DF and of the king-post 56 lbs., and the angle CAB 30°.



1246. In an equilibrated arch of voussoirs, show that their weights are proportional to the differences of the tangents of the angles which their joints successively make with the vertical, and that the horizontal thrust is $\frac{W}{2} \cdot \cot \frac{a}{2}$, where W is the weight of the half-arch, and a the angle at which the faces of the piers are inclined to one another.

1247. A rectangular door, having its sides a and b and weight W, is hung by two hinges in a vertical line at distances one-sixth of its height from the top and bottom; find the direction and magnitude of the strain upon each hinge, supposing each to support one-half of the weight of the door.

1248. A ladder weighing 4 cwt. is placed against the side of a house, with which it makes an angle of 30°; find the horizontal force necessary to prevent its slipping along the ground, supposing its centre of gravity to divide its length in the ratio of 3 to 5.

1249. A sphere, 36 inches in diameter and weighing 29 cwt., is supported upon a horizontal circular ring 12 inches in internal diameter; find the least force which will roll the sphere over.

1250. A weight is supported upon a plane inclined to the horizon at an angle a, by a force which is exactly equal to the pressure upon the plane; find the direction of this force.

1251. A uniform rod AB, 10 feet long, weighing 2 lbs., is fixed by a hinge at A to a vertical wall, and at a point C, 8 feet vertically above A, a weight of 10 lbs. is fastened by a string which passes over the end B of the rod which is supposed to be perfectly smooth; find the position in which AB will rest.

1252. A man rolls a cylindrical barrel (weight b) up a plane inclined at an angle a to the horizon, by the vertical action of his weight W when standing upon the barrel; show that there will be equilibrium when his feet are at a point θ° from the highest point in the barrel, if $\sin\theta = \frac{W+b}{W} \cdot \sin a$.

1253. The elevation of an inclined plane is 30°, and a weight W of 30 lbs. is sustained upon it by a power P of 20 lbs. acting over a pulley 10 feet vertically distant from the top of the plane; find the distance of the weight from the top of the plane, when P and W are in equilibrium.

1254. A weight of 56 lbs. is supported upon an inclined plane by three forces, viz. 7 lbs. acting parallel to the plane, 28 lbs. acting parallel to the horizon, and 10 lbs. acting (independently of the reaction of the plane) perpendicular to the face of the plane; find its inclination and the pressure upon its surface.

1255. A homogeneous sphere of weight W is moveable about a fixed point in its surface, and is also partly supported upon a plane inclined at an angle α to the horizon; find the pressure on the plane when the radius passing through the fixed point is inclined at an angle β to the vertical.

1256. Show that if μ be the coefficient of friction between two surfaces, λ the limiting angle of quiescence, R the oblique resistance, and N the normal pressure,

$$\mu = \tan \lambda, \quad \text{and} \quad R = N \,.\, \sqrt{(1 + \mu^2)}.$$

1257. It is found that a force of 5 cwt., acting at an angle of 45°, is required to draw a block of stone over a rough horizontal surface; find the weight of the block, supposing the coefficient of friction to be ·62.

1258. A cone is placed with its base upon a rough horizontal plane, and a string is attached to its vertex; find the dimensions of the cone when, a horizontal force being applied to the string, the cone will turn over when it is just upon the point of sliding along the plane.

1259. A post stands at a distance of 3 feet from a rough vertical wall, and a rough beam, 12 feet long, is placed over the post with one end resting against the wall; find the equations of equilibrium necessary for determining the greatest inclination of the beam to the wall when the coefficient of friction for the post and wall is μ.

1260. Find the force necessary to draw a block of stone up a plane, down which it would descend by the action of its own weight, supposing the force to be inclined to the horizon at an angle equal to twice that of the plane.

1261. A cone, 4 inches in diameter and 20 inches high, stands symmetrically upon a cylinder 6 inches in diameter and 14 inches high which rests upon a plane: the coefficient of friction between the cone and cylinder is ·5, and between the cylinder and plane is ·4. What will take place if the inclination of the plane be gradually increased?

1262. In the inclined plane, show that the ratio of P to W is that of $\sin(i \pm e)$ to $\cos(a \mp e)$, where i is the inclination of the plane, a the inclination of the force P to the face of the plane, and e the limiting angle of friction. And show how this may be applied to the case of the screw, where h is the linear pitch of the screw, c the circumference of the cylinder, C the circumference described by the point of application of the force P, and μ the coefficient of friction.

1263. Find the inclination of a plane down which a block of stone will just slide by the action of its own weight, the coefficient of friction being ·63.

1264. What force, acting parallel to the plane, will be requisite to draw a square block of timber up a plane inclined at an angle of 60°, the coefficient of friction being ·63, and the weight of the block 5 cwt.?

1265. Find the force (acting parallel to the plane) necessary to draw a 10 in. howitzer, weighing with carriage and limber 124 cwt., up a plane inclined to the horizon at an angle of 30°; 1st, assuming that there is no friction, and 2nd, that the friction is one-tenth of the normal pressure.

1266. Find the least force which, acting parallel to the plane, is necessary to drag a weight of 10 cwt. up a plane inclined to the horizon at an angle of 15°, the limiting angle of resistance being 30°.

1267. A right cone whose height is equal to three times the diameter of its base, stands with its base upon an inclined plane; determine whether it will slide or fall over when the inclination of the plane is gradually increased, if the coefficient of friction be ·7.

1268. Find the least force necessary to draw a weight of 500 lbs. up a plane inclined at an angle of 15° to the horizon, the limiting angle being 30°.

1269. The weight of a body standing upon a plane of 60° inclination is eleven-tenths of the force necessary to draw it up the plane by a cord parallel to the plane; find the coefficient of friction between the body and the plane.

1270. A rough sphere is placed upon a rough inclined plane, the limiting angle of friction being e; what is the inference if it neither roll nor slide? State under what conditions it can be made to slide without rolling, and *vice versa.*

1271. Two forces, P acting parallel to the plane and Q making an angle a with the plane, just support a body upon a rough plane inclined to the horizon at an angle i, and when Q and P are interchanged, the body is on the point of moving up the plane; find the values of P and Q.

1272. Two planes of equal slope, 60°, but of unequal height, are placed back to back, and two rough particles of equal weight, connected by a fine string which passes over a smooth pulley at the top of the highest plane, rest upon the planes, one upon each; find the angle which the string makes with the lower plane, when the particle upon the other is on the point of moving up the plane, the limiting angle being 15°.

1273. If the force which, acting parallel to a plane, is necessary to draw a body up the plane be twice the force which, acting in the same direction, will allow it to slide down; what is the relation between the inclination of the plane and the coefficient of friction?

1274. If the force which, acting parallel to a plane inclined i to the horizon, is just sufficient to draw a body up, be n times the force which will just allow it to slide down the plane, show that $\tan i \,.\, \cot \epsilon = \frac{n+1}{n-1}$, ϵ being the limiting angle.

1275. If P be the force which, acting parallel to a plane of known inclination, would just draw a body up the plane, and P' the force which, acting in the same direction, would just allow the same body to slide down the plane; find the coefficient of friction.

1276. Two rough planes, rigidly connected at their intersection, are suspended upon a horizontal axle passing through it, and, upon them, two rough, heavy particles of equal weight, joined to one another by an inextensible string passing freely over the top are placed; find the angle through which the system may be made to move round the axle without displacing the weights from their position on the planes.

1277. A cylinder 6 in. in diameter and 14 in. high, begins to slide at the same time that it begins to topple over when the inclination of the plane on which it stands is gradually increased; find the coefficient of friction.

1278. A rectangular block of stone rests at an angle θ with one of its edges on a horizontal plane and another against a vertical wall; find the least value of θ consistent with equilibrium, the sides of the vertical section of the block being a and b, and the coefficients of friction μ and μ'.

1279. A heavy beam rests at an angle of 60°, with one end against a smooth vertical plane and with the other upon a smooth horizontal plane; find the value and direction of the oblique pressure of the beam against an obstacle in the horizontal plane which prevents its sliding, when the weight of the beam is W, and its centre of gravity is at a point one-third of its length from the lower end.

1280. A beam 21 feet long rests with its ends upon a horizontal and a vertical plane respectively; find how far the lower end may be drawn out from the wall before the beam will slip;

the coefficient of friction being $\frac{1}{4}$ and $\frac{1}{6}$ respectively, and the centre of gravity, which is nearest to the lower end, dividing the beam in the ratio of $3 : 4$.

1281. If a beam AB rests with its ends upon rough horizontal and vertical planes respectively, and BE be drawn in the direction of the oblique resistance of the vertical plane to meet the vertical DGE drawn through the centre of gravity of the beam and intersecting the horizontal plane in D, show that the ratio of AD to DE is the coefficient of friction at A.

1282. A uniform rectangular board rests with one of its sides upon a rough horizontal plane and parallel to the base of a rough vertical wall, against which the opposite side of the board rests; find the least angle which the plane of the board can make with the horizon, the coefficients of friction being μ and μ'; and prove that this angle is the same whichever side of the rectangle be the base. State also the relation between the coefficients of friction when the inclination is 45°.

1283. A beam is to be placed against a vertical wall, and it is required that its inclination to the horizontal floor on which its lower end rests shall not be greater than 30°; find the position of its centre of gravity, so that it may not slip at that angle when the coefficient of friction at each end is ·5. And show that although a beam may be made to stand at an inclination to the horizon when the floor is rough and the wall smooth, it cannot be made to stand in any but a vertical position when the floor is smooth and the wall rough.

1284. A hemisphere rests with a point in the circumference of its base upon a rough horizontal plane, and a point in its convex surface in contact with a rough vertical wall; find the coefficient of friction when the body is just supported with its base in a plane parallel to the wall.

1285. A ladder which is divided by its centre of gravity into segments of 10 feet and 20 feet, is placed against the side of a house at an angle of 30°; find the highest round upon which a weight of 4 cwt. can be suspended; the weight of the ladder being 3 cwt., and the coefficient of friction between the ladder and wall, and also between the ladder and horizontal pavement which supports its heavier end, being $\frac{1}{\sqrt{3}}$.

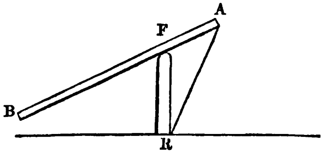
1286. A uniform beam *AB* rests upon a rough horizontal plane *AC* and against a rough vertical wall *CB*; show that it will stand when the coefficient of friction, being the same at both ends, is not less than

$$\frac{BA-BC}{AC}.$$

1287. A rough body is supported upon a plane, inclined to the horizon at an angle α, by a string attached to a peg in the surface of the plane; find the least angle which the string can make with the horizontal line drawn through the peg.

1288. Two uniform and equal rough rods, joined at an angle of 120°, are set astride over a rough horizontal cylinder in a plane perpendicular to its axis; find the least angle which one of them can make with the vertical, the coefficient of friction being μ, the length of the rods $2a$, the radius of the cylinder r, and the plane of the rods being perpendicular to the axis of the cylinder.

1289. A uniform beam *AB*, 12 feet long, weighing 100 lbs., is placed upon a vertical post *FR*, the extremity *A* being fastened to the bottom of the post by a rope *RA*, and it is found that when the beam is just about to slide off the post in the direction *AB*, the angles *FAR* and *FRA* are 45° and 15° respectively, and $AF = 4$ feet: find the friction at *F*.



1290. A flat bar weighing 10 lbs. is to be balanced at its middle point upon a horizontal edge, and its centre of gravity is 12 inches from one end and 18 inches from the other: find the greatest force which can be applied obliquely at the lighter end without causing the bar to slip; the coefficient of friction being $\frac{1}{\sqrt{3}}$.

1291. If a flat uniform bar be supported upon an edge at its middle point, and a weight w be suspended at one end of it, find the greatest angle the direction of any force which will keep the bar horizontal can make with the vertical; μ being the coefficient of friction between the bar and edge, and W the weight of the bar.

1292. A uniform beam length $2a$ rests against a smooth immoveable hemisphere, of radius r, placed, base downwards, upon a rough horizontal plane upon which one end of the beam is supported. Show that if θ be the greatest inclination of the beam,

$$\cot(2\theta + e) = \cos e - \frac{2r}{a} \sin e,$$

the coefficient of friction being $\tan e$.

1293. Show that, in the wheel and axle, when a force P, acting at the circumference of the wheel, supports a weight Q upon the axle,

$$P \,.\, (R \mp \rho \sin \epsilon) = Q\,(r \pm \rho \sin \epsilon) \pm W\rho \sin \epsilon,$$

where R, r and ρ are the radii of the wheel, the axle and their common axis respectively, and ϵ is the limiting angle of resistance.

1294. A vertical water-wheel weighing 5 cwt. has a cylindrical bearing of 3 inches radius; find the pressure upon the circumference which will just overcome the statical friction of the axle, the diameter of the wheel being 15 feet, and the coefficient of friction being ·2.

1295. A balance having equal and uniform arms 10 inches long, turns upon a cylindrical axle 1 inch in diameter; find by how much a 20 lb. weight in one scale may be exceeded by the weight in the other, before the friction is overcome; the weight of each scale being 1 lb., of each arm $2\frac{1}{2}$ lbs., and the coefficient of friction tan 30°.

1296. A uniform lever 2 feet long, weighing 1 lb., is balanced upon an axle whose radius is half an inch, when weights of 10 oz. and 14 oz. are suspended at its extremities, the former weight being about to preponderate; find the length of the arms, if the coefficient of friction be ·4.

1297. Show that if P be the force which, acting at the circumference of a horizontal wheel radius R, supported upon the plane end of its cylindrical axle, radius r, just overcomes the friction, $P = \frac{2\mu r W}{3R}$; where W is the normal pressure on the bearing.

1298. State Guldin's Theorem for determining the surface of a solid of revolution, and apply it to find the centre of gravity of an arc of a semicircle.

1299. The area of a parabola is two-thirds of that of its circumscribing rectangle, and the volume of a paraboloid is one-half of the volume of a cylinder of the same base and altitude, all the sections of the solid through its axis being equal parabolas. Find the distance of the centre of gravity of any given semiparabola from its axis.

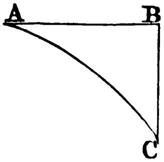
1300. The properties of Guldin being assumed, find the position of the centre of gravity of the arc of a circle in terms of the arc, its chord, and the radius of the circle.

1301. Find, by Guldin's Theorem, the volume generated by the revolution of a parabola about the tangent at its vertex; the curve being limited by its latus rectum.

1302. Find the centre of gravity of the semi-ellipse cut off by the major axis; and thence determine the volume of the solid generated by its revolution round that axis.

1303. A sphere is divided into eight equal parts by three planes perpendicular to one another; find, by integration, the position of the centre of gravity of one of these parts.

1304. Find the centre of gravity of the parabolic spandril ABC; A being the vertex of the parabola, and its axis being parallel to BC.



1305. In a common isosceles trussed roof in which the two struts are parallel to the opposite rafters, find the tension of the king-post, supposing the weight of the tie-beam to be 1 cwt., of the roof 10 cwt., and of the king-post and of each strut 56 lbs.; and state what would be the effect upon the king-post if the tie-beam were to give way, supposing the parts of the truss to be rigidly connected.

1306. The radius of curvature of a deflected rectangular beam being $\frac{Ecb^3}{12P(a-x)}$, where E is the modulus of elasticity, b the depth in the plane of curvature, c the breadth and a the length of the beam; show that the amount of deflection at its extremity is represented by $\frac{4Pa^3}{Ecb^3}$.

1307. A beam of fir 8 inches square and 10 feet between its horizontal supports, has a $\frac{2}{3}$-inch iron wire securely fastened to its middle point, by means of which increased weights may be suspended until the beam or the wire give way; determine which will first take place, the tabular constant for the breaking strain of fir being 1100 lbs., and the tensile strength of iron wire 36 tons per square inch.

1308. Find the maximum pressure on a vertical revetment wall 20 feet high, the natural slope of the earth being 30°, and a cubic foot of earth weighing 140 lbs.; supposing the friction against the wall to be the same as that of the earth.

PRACTICAL MECHANICS.

1309. What allowance per mile must be made in the length of rails upon a railway for a change of temperature of 100°, the modulus of expansion of wrought iron being ·00000642?

1310. The length of a base line being measured with glass rods was found to be 23702 ft., but the temperature of the rods was observed to be 10° above the standard; what was the true length of the base; the modulus of expansion for glass being ·00000431?

1311. Two steel six-foot rods are found to differ from each other by ·0035 of an inch, when the difference of their temperatures is $8^{\circ}\!\cdot\!5$; at what temperatures will they be accurately the same in length supposing the temperature of the longer one to be kept unaltered at 60°, the modulus being ·00000636?

1312. The bore of the tube of a thermometer is ·014 in. and the quantity of mercury in the thermometer is one-third of a cubic inch; how many inches of scale would there be between 32° and 212°; the modulus of cubic expansion of mercury in glass being ·00008696?

1313. The weight of an empty thermometer tube is 250 grains, and when filled with mercury it is 1327 grains; find the diameter of the tube when the mercury rises 6·2 inches for a change of 50°.

1314. A steel bar 3 feet long is riveted at one end to a brass bar of the same weight, so that their free ends are towards the same parts. Find the length of the brass bar so that the distance of the centre of gravity of the mass from the free end of the steel bar may be unaltered by change of temperature, the modulus for brass being ·00001052.

1315. Supposing the ends of the horizontal bar of an iron railing to be immoveably fixed in two vertical walls; find the thrust exerted upon the walls when the temperature rises 30° F.; the section of the bar being three square inches, the modulus of elasticity 29000000, and of expansion ·0000642.

1316. The four iron wire stays of a mast are each one square inch in section and are inclined to the mast at an angle of 30°; find the increase of vertical pressure upon the mast due to a change of 40° F.

1317. The specific gravity of ice (water at 40° F. being the standard) is ·91, find its modulus of expansion, supposing the expansion in volume to be three times the linear expansion; and show this supposition to be very nearly correct.

1318. The strain upon a hempen cable being 10 tons, what must be its dimensions that it may bear this safely, and what additional strain will be produced by its own weight if it be 50 fathoms long?

1319. Show that if w be the weight of a unit of length of a bar of uniform section and length l, the extension X produced by its own weight will be

$$X = \frac{wl^2}{2m},$$

where m is the modulus of elasticity.

1320. The bearing area of a wire rope of which the circumference is C, being $\frac{C^2}{8\sqrt{3}}$ (see Qu. 874), find the dimensions of an iron wire rope which will bear *with safety* a strain of 10 tons.

1321. Find the length of copper wire rope which it will be just possible to suspend by one end.

1322. What must be the dimensions of a square cast-iron column to support safely, a mass of brick work 12 feet high, one foot thick and six feet long?

1323. It is proposed to substitute sandstone columns for some of granite originally planned, what must be the change in dimensions?

1324. Define the "unit of work," and find the number of units expended in raising a weight of 1 ton of building materials to a height of 40 feet.

1325. Find the time required to raise a block of granite 4 feet long, 3 feet broad, and 2½ feet thick, to a height of 20 feet, by two men working at the winch of a traversing crane, the number of units of work by each man per minute thus working being 2600, and the modulus of the machine ·8.

1326. How many gallons of water per minute can be pumped up from the bottom of a shaft 200 fathoms deep by an engine working at a pressure of 15 lbs. to the square inch; the diameter of the piston being 40 inches, the length of stroke six feet, the number of strokes 10 per minute, and the modulus ·65 ?

1327. Supposing the water in the last question to be carried over a wheel of 25 feet diameter which works a pump the water from which also passes over the wheel, find the whole quantity raised, the modulus of the wheel being ·55.

1328. A stream of water 10 feet broad and 5 feet deep, flowing at the rate of 37 feet per minute, is conducted over a water-wheel 15 feet in diameter, which works a pump by which the water from the stream is raised to a height of 20 feet; what is the quantity of water raised per hour, the modulus being ·65 ?

1329. Find the horse power of the wheel in question 1328, and compare it with that of the engine in question 1326.

1330. The diameter of the piston of an engine is 30 inches, the mean pressure of the steam is 10·5 lbs. per square inch, the length of the stroke is 12 feet, and the number of strokes per minute is 10; find how many cubic feet of water per hour the engine will raise from a depth of 200 fathoms, the modulus of the engine being ·6.

1331. In what time would two locomotives of 100 horse power each, convey a battery of field artillery from Woolwich to Brighton (60 miles), the engines, trucks, and carriages weighing W tons, and the men, guns, horses, and material A tons, supposing a tons to be conveyed in the trucks and carriages at each trip, and the resistances to be 10 lbs. per ton ?

1332. The piston of an engine is 3 feet in diameter, the length of stroke 5 feet, the number of strokes per minute 12, the pressure per square inch 15 lbs., and the modulus of the engine is ·75; find the quantity of water which can be pumped from a depth of 30 fathoms.

1333. The piston of an engine is 48 in. in diameter; the pressure upon it 15 lbs. per sq. in.: the length of the stroke 5 feet, and the number of strokes 10 per minute; find the horse *power* of the engine.

1334. If the length of stroke be l, the number of strokes per minute n, and the pressure p; what must be the diameter of the piston in order that the horse power of the engine may be K?

1335. What must be the horse power of an engine which will raise a tilt hammer weighing $1\frac{1}{2}$ ton 25 times a minute, the lift being 2 feet?

1336. A locomotive has to move a train of 12 carriages, each weighing, with the passengers, 4 tons, at the rate of 50 miles an hour; what must be its horse power if the resistances be 10 lbs. per ton and the weight of the engine and tender 16 tons?

1337. If the weight of a train be 30 tons and that of the engine and tender 15 tons; find the radius of the driving wheel in order that with a pressure of 15 lbs., pistons of 14 in. diameter, a stroke of 3 feet, and a modulus of ·75, the engine may work smoothly, and its full power be available, the resistances being estimated at 10 lbs. per ton.

1338. Find an expression for the diameter of the piston of a locomotive in terms of r the radius of the driving wheel, p the pressure per sq. in., l the length of stroke, R the total resistance in lbs. and m the modulus.

1339. Show that the length of stroke of a locomotive should be equal to the product of the total resistance, and the radius of the driving wheel, divided by the product of the modulus and the pressure on the piston, and multiplied by π.

1340. A quantity of ballast is to be conveyed in trucks to a distance of 10 miles, by a train of 8 trucks, each weighing $2\frac{1}{2}$ tons, and capable of carrying $7\frac{1}{2}$ tons of ballast. The engine is of 20 horse power, and weighs with its tender 12 tons; and the resistances are 15 lbs. per ton. What quantity of ballast can be delivered in 72 hours, allowing half an hour for discharge and loading at each trip?

1341. How many bushels of coal will be consumed in 24 hours by an engine of 50 horse power, if its duty be 55 millions?

1342. How many bushels of coal will be consumed by an engine of 100 horse power and 70 millions duty, in raising 200000 cubic feet of water from a depth of 150 fathoms?

1343. An engine of 65 millions duty has to pump water up three separate shafts of 70, 100, and 120 fathoms deep respectively, viz. 10 cubic feet per minute from the first, 20 cubic feet from the second, 50 cubic feet from the third. How many bushels of coal will the engine consume in 12 hours?

1344. If the depths of a number of shafts of a mine be f_1, f_2, f_3, f_4 &c. feet, and the number of cubic feet of water pumped from them per minute be w_1, w_2, w_3, w_4 &c. respectively, find the horse power of the engine; and its duty, supposing it to consume B bushels of coal in 24 hours.

1345. Find the number of units of work required to wind up a rope of 4 in. circumference 800 feet long?

1346. What is the number of units of work required to wind the same rope, supposing it to be laid horizontally on the ground, the coefficient of friction being ·70?

1347. Two boxes are worked up and down a shaft 150 feet deep by a connecting rope over a drum. The load each box will contain is 3 cwt. of coal. The engine has a piston of 8 inches diameter, the pressure of steam is 10 lbs., the length of the stroke 3 feet, the number of strokes 20 per minute, and the modulus ·52. Find the quantity of coal raised per hour.

1348. Find the quantity of coal raised per hour by the same engine, supposing that only one box be used, that the time occupied in emptying the box added to the time of descent will be equal to the time of ascent of the loaded box, and that no time is lost in unhooking the empty box and hooking the full one at the bottom of the shaft; the rope being 3 inches in circumference and the weight of each box 56 lbs.

1349. Find the number of units of work expended in distributing 5000 tons of ballast equally over a line of railway, supposing 18 tons to cover 10 yards length of the line, and the resistances to be 12 lbs. per ton.

1350. Find the number of units of work required to hoist the main-sail of a ship 50 feet by 30 feet to a height of 40 feet, supposing the sail-cloth to weigh 4 lbs. per square yard, and the yard to weigh 5 cwt.

DYNAMICS.

1351. Define the terms "uniform motion," "variable motion," "velocity," "moving force," and "accelerating force."

1352. State the three "laws of motion," and give illustrations of their truth from familiar facts.

1353. State the second law of motion, and mention experimental facts which would lead to its assumption. What is the nature of the final evidence which is considered conclusive as to the truth of this law?

1354. Define "variable velocity;" explain how it is produced by the action of a constant force; state how this connexion is made use of in the dynamical measurement of such a force; and find the numerical representative of the force of gravity, supposing the minute and the yard to be the units of time and space respectively.

1355. Find the unit of space when the accelerating force of gravity is 14, and the unit of time 5 seconds.

1356. Show that, when a body moves under the influence of a constant force f, the space s passed over in the time t will be represented by the formula

$$s = \frac{1}{2}ft^2.$$

1357. An engine starts a train with a pressure which continues uniform for 5 minutes, when it is found that the train is moving at the rate of 23 miles an hour. At what velocity would the train be moving at the end of 15 minutes, if the pressure continued uniform during that time?

1358. A balloon, rising with a uniform velocity of 30 feet per second, is carried by the wind over a horizontal distance of one mile in $1^m. 28^s.$; find the angle which the line of its motion makes with the horizon, and the horizontal distance from an object at which a heavy body must be let fall from the balloon, when at a height of half a mile, so as to strike the object. Does the body fall in a straight line?

1359. A wherry pulled at the rate of five miles an hour, is required to cross a river half a mile wide, to a point which is a quarter of a mile lower down the stream than the point of starting: what angle must the direction of the boat's head make with the bank, supposing the stream to run at the rate of 3 miles an hour; and how long will it be in crossing?

1360. A body weighing 322 lbs. is urged forward on a perfectly smooth horizontal plane by a constant pressure of 10 lbs.; and another body weighing 483 lbs. is moved forward in the opposite direction by a pressure of 75 lbs. Find the point at which they will meet, supposing them to have been 500 feet distant from one another at the commencement of their motion.

1361. A railway train and its locomotive of 40 horse power weigh 40 tons when empty, and 100 tons when loaded. Find the velocities in the two cases, supposing the engine to work with full power in each case, and that the resistance is 8 lbs. per ton.

1362. Find the numerical value of the force of gravity when the minute and the foot are assumed as the units of time and distance.

1363. Two railway trains, running in the same direction on parallel lines of rails, at the rate of 20 miles an hour and 30 miles an hour respectively, arrive, at the same instant, at a junction where the lines diverge at an angle of 30°; find the relative velocity of the two trains after passing the junction.

1364. Find the pressure, in tons, which will bring each of the trains in the last question to rest, in three minutes, supposing them each to weigh 50 tons.

1365. A pressure of 2 tons acts upon a railway train weighing 150 tons for 10 minutes; find the momentum the train has acquired.

1366. A railway train weighing 30 tons, detached from the engine, is moving against the wind at the rate of 20 miles an hour; how far will it run before coming to rest, supposing the resistance of the wind and the effect of friction to be equivalent to a pressure of 2 cwt.?

1367. If a 68-lb. shot be propelled from the mouth of a gun with a velocity of 322 feet per second; find the statical pressure urging the shot forward while in the gun, supposing it to be uniform and the shot to take ·05 second to traverse the bore.

1368. Distinguish between the statical and dynamical measures of a force, and find the pressure, in ounces, of a force, whose dynamical measure is 10, upon a body which weighs 644 oz. at the surface of the earth: find also the weight of this body, supposing it to be removed to a distance from the surface equal to one-fourth of the radius of the earth.

1369. Show that the force of gravity is represented numerically by 79000 very nearly when the units of distance and time are a mile and an hour respectively.

1370. Two bodies, A and B, of equal weights w, and a third body C of weight p, are attached to one another in alphabetical order by strings a inches in length, and C hangs freely over the edge of a table upon which A and B are placed in contact with one another; find the velocity with which A commences its motion, excluding friction.

1371. The steam from a locomotive makes an horizontal angle of 60° with the line of rails when the train is running at 10 miles an hour, and an angle of 30° when the train runs at 20 miles an hour; find the direction and velocity of the wind.

1372. A spherical shot is rolling directly across the deck of a ship with a velocity of 10 feet per second; find the point at which it would strike the side, supposing the ship, which is steaming at 10 miles an hour, to be suddenly arrested in its course when the shot is 30 feet from the side; and find the velocity with which it would strike.

1373. A railway train 150 feet long passes a station in 5 seconds, and another train two-thirds of the length of the former passes the same point in 4 seconds; and it is observed that the line of steam from the former is half the length of that from the latter; find the velocity of the wind, which was blowing in the direction of the line of rails.

1374. A dragoon is galloping at the rate of 14 miles an hour past a sentry, who does not perceive him until he is at the nearest point to his post, 200 yards distant; what allowance must the sentry make in taking his aim, supposing the velocity of his rifle-bullet to be 1000 feet per second?

1375. A person travelling westward at the rate of 4 miles an hour, observes that the wind seems to blow directly from the south; and that, on doubling his speed, it appears to blow from the south-west: find the velocity of the wind and its direction.

1376. A body is thrown, with a velocity of 25 feet per second, horizontally from the window of a railway carriage moving at the rate of 30 miles an hour, and in a direction making an angle of 30° with the rear of the train; find the direction of the vertical plane in which it moves, and the perpendicular distance from the rail at which it strikes the ground, supposing the window to be 8 feet from the ground.

1377. A steamer whose course was N.E. had the smoke from its funnel passing off in a line to the westward from the steamer, the wind being due S.; find the velocity of the wind, supposing that of the steamer to have been 12 miles an hour, and bearing in mind that the motion of the steamer has no effect upon the particles of smoke after they have issued from the funnel.

1378. A boatman sees from the bank of a river, when at a distance of half a mile, a vessel which is sailing through the water at the rate of 5 miles an hour, and coming up with the tide which is running at the rate of 3 miles an hour; and, as he wishes to board her, he immediately puts off. In what direction must he keep the head of his boat, supposing that he can pull at the rate of 5 miles an hour, and that the vessel maintains her distance of one-fourth of a mile from the bank of the river?

1379. A boy attempts to drop a stone into the funnel of a locomotive as it passes under a railway-bridge at the rate of 40 miles an hour; where will it strike the train, supposing that it is dropped when the funnel is immediately underneath, and that the boy's hand is 48·3 feet above the roofs of the carriages?

1380. The captain of a steam-frigate, steaming parallel to the coast at a distance of a quarter of a mile, wishes to fire a broadside at a certain point in an enemy's battery at the water's edge, when directly opposite to it; how many feet, and on which side of the object must the guns be laid, supposing the steamer to be running at the rate of 14 miles an hour, and the horizontal velocity of the shot to be 800 feet per second?

1381. Two spheres A and B, of which the mass of A is double that of B, but the velocity of B double that of A, move in opposite directions; find their velocities after impact, e being the elasticity of the spheres.

1382. Two non-elastic bodies, moving in opposite directions, impinge directly upon one another with velocities of 20 feet, and 15 feet per second; what will be their common velocity after impact, supposing the ratio of their masses to be $5 : 3$?

1383. A, B, and C are three ivory balls whose elasticity is $\frac{7}{8}$: A $(=9$ oz.) impinges directly on B $(=6$ oz.) which is at rest, with a velocity of 10 feet per second; and then B impinges directly upon C, at rest; find the weight of C, in order that it may move on with a velocity of 15 feet per second.

1384. Two spheres, A and B, moving with equal velocities of 9 feet per second in directions inclined 60° to one another, impinge when they are at equal distances from their respective starting-points. Determine their motions after impact; their common elasticity being $\frac{2}{3}$, their volumes equal, but the mass of A double the mass of B.

1385. Two spheres, A and B, moving in opposite directions, impinge directly upon one another with velocities of 5 and 2 respectively; find the ratio of their masses in order that A may be reduced to rest after impact, and find the velocity of B after impact, their common elasticity being $\frac{2}{3}$.

1386. Two bodies, $A=16$ and $B=24$, moving in lines which are inclined 30° to the common tangent at impact, with velocities of 20 and 16 respectively, strike one another obliquely: find their distance from one another 4 seconds after impact, their common elasticity being $\frac{7}{8}$.

1387. Two spheres, A and B, impinge upon one another with velocities of 10 feet and 8 feet respectively, in directions which make angles of 60° and 150° with the line of centres at impact. Find their velocities and the directions of their motions after impact, the mass of A being double that of B, and their modulus of elasticity being $\frac{1}{3}$.

1388. Two perfectly elastic spheres, of masses m and m', approach each other obliquely with velocities V and V', and in directions which make angles α and β with the line of centres at the time of impact; determine the motion after impact, when

$$V : V' :: B.\cos\beta : A.\cos\alpha.$$

1389. Two spheres, A and B, having equal velocities, impinge directly upon one another; find the ratio of their masses, so that, after impact, the motion of B may be reversed, and its velocity doubled.

1390. If two planes inclined at an angle α to one another be placed vertically upon a horizontal board, find the angle at which an elastic sphere rolling along the board must strike one of the planes in order that after three successive impacts it may retrace a portion of its course.

1391. Show that if a body be projected from the angle A of a plane triangle ABC so as to strike the side CB at a point D, then, if its course after reflexion at D be parallel to AB,

$$\tan DBA = \frac{(1+\epsilon)\cot B}{1-\epsilon\cot^2 B}.$$

1392. Three equal spheres, A, B, and C, are at the angular points of a triangle ABC (the angle C being greater than 60°): what will be the resulting velocity of C, and the direction of its motion, if A and B move uniformly along the sides AC and BC with velocities proportional to those sides?

1393. A body A impinges upon B at rest, with a velocity of 10 feet per second, and B, in consequence, strikes against a perfectly hard plane inclined at an angle of 45° to B's course; find the position of B four seconds after impact by A, supposing it to take place 15 feet from the plane, the modulus of elasticity being $\frac{3}{4}$, and A's mass being $\frac{3}{2}$ of that of B.

1394. Two bodies, A and B, move in the same direction with velocities of 50 and 30 respectively; find how long after impact they will be 64 feet from each other, their common elasticity being $\cdot 8$; and find the ratio of their masses, so that, if possible, A may remain at rest after impact.

1395. A billiard ball is struck from one corner A of a billiard table $ABCD$, and after striking three of its sides falls into the pocket at B; show that the alternate sides of its course are parallel, and find the distance of the first point of impact from B, when $AB = a$ and $BC = b$.

1396. Three spheres A, B and C, of the same material, are placed in the same straight line, upon a perfectly smooth horizontal table, B and C being 4 feet apart; A is then struck so as to impinge upon B with a velocity of 9 feet: find the position of *C* 77·5 seconds after the commencement of *B*'s motion, the radii of the spheres being as 1 : 2 : 4, and the elasticity represented by $\frac{3}{5}$.

1397. *A* and *B*, two elastic spheres moving in opposite directions, with velocities 9 feet and 6 feet per second, respectively, strike one another; find their distance from one another 3 seconds after impact, their common elasticity being $\frac{2}{3}$, and the weight of *A* two-thirds the weight of *B*.

1398. A sphere 3 inches in diameter impinges directly with a velocity 10 upon another sphere of the same material 2 in. in diameter, moving in the same direction with a velocity 4; find the position of each body with reference to the place of impact three seconds afterwards.

1399. A stone is dropped from the top of a vertical shaft of a mine, and is heard to strike the bottom in *t* seconds; find an expression for the depth of the shaft, supposing the velocity of sound to be *a* feet per second.

1400. A stone is let fall from the top of a cliff, and after $3\frac{1}{2}$ seconds it is heard to strike the base; find the height of the cliff, supposing sound to travel at the rate of 1120 feet per second.

1401. Two bodies are projected vertically upwards, at the same instant, with velocities of 100 feet and 50 feet respectively, and they are observed to pass one another at the end of 3 seconds; find the difference of level of the points from which they were projected.

1402. A body is let fall from the top of a vertical cliff 579·6 feet high; with what velocity must another body be projected from the same point, vertically upwards, at the same instant, in order that it may strike the base of the cliff four seconds after the first body?

1403. A body runs down a plane inclined at an angle of 30°, and then falls over a vertical cliff 150 feet high; find the vertical velocity with which it strikes the ground, the length of the plane being 50 feet.

1404. A sphere, 2 inches in diameter, is let fall from a height of 80·5 feet, and, one second afterwards, another, 1 inch in diameter, is projected vertically downwards from the same point with a velocity of 64·4 feet per second; find the time in which each will reach the ground, the coefficient of elasticity being $\frac{3}{4}$.

1405. An arrow shot vertically upwards just reaches the top of a tower; find the height of the tower, supposing the time of flight from the bottom to the top to be $2\frac{1}{2}$ seconds.

1406. A body is projected with a velocity of 60 feet per second up an inclined plane which makes an angle of 30° with the horizon; what will be its position at the end of ten seconds?

1407. An elastic ball is let fall from a height of 10 feet; find the height to which it will rise at the 3rd rebound, supposing the elasticity to be $\frac{2}{3}$: and deduce a general expression for the whole space passed over by the ball before it comes to rest.

1408. A railway train is moving with a velocity of 30 miles per hour, and the steam is shut off from the engine at the top of an incline of 1 in 322; find the rate at which the train would be moving after it had passed over one mile of the incline, supposing that there were no friction.

1409. Two bodies, A and B, are both projected vertically upwards at the same instant with velocities of 25 feet and 200 feet respectively, A from the top, and B from the bottom of a vertical cliff 300 feet high. Find the point at which they will meet, and the directions of their motions at that instant.

1410. Show that if a body be acted on by a constant force f it will, in the interval between the end of the t^{th} and of the $(t+n)^{\text{th}}$ seconds, describe a space σ such that

$$\sigma = \frac{n}{2}(2t+n)f.$$

1411. Two bodies are projected at the same instant, one vertically downwards from the top, and the other vertically upwards from the bottom of a vertical cliff, with velocities of 50 feet and 100 feet respectively, and they are observed to strike one another at the end of 2·5 seconds: find the height of the cliff.

1412. Find the time in which a body, projected vertically, with a velocity of 322 feet per second, will reach a height of 1500 feet; and how long it will continue to ascend.

1413. A body is projected vertically upwards from the base of a tower with a velocity of 128·8 feet ($4g$), and is just two seconds in reaching the top; find the height of the tower; the height to which the body will rise; and the time when it again arrives at the top in descending.

1414. With what velocity must a body be projected vertically upwards in order that it may reach the height of 200 feet in 3 seconds; how high will it rise; and after how many seconds will it again reach the former point?

1415. A body is projected vertically upwards with a velocity of 100 feet per second: how high will it rise in 2 seconds, and what will be the space passed through by it in the last second before it comes to the ground?

1416. How long will a sphere take to roll down a plane inclined to the horizon at an angle of 30°, its greatest height being 200 feet: and what would be the velocity acquired?

1417. A body is let fall from the top of a cliff 600 feet high at the same instant that another is projected vertically upwards from its base with a velocity of 125 feet; find the point at which they meet.

1418. A railway train running at the rate of 15 miles an hour, comes to a downward incline of 1 in 70, which is one mile in length; find the velocity of the train at the bottom of the *incline*, and the time it takes to pass over the lower half of it.

1419. Two given weights are connected by an inextensible string which passes over a smooth pulley; find the motion of the weights, and the tension of the string.

1420. Two bodies, A and B, of equal masses, are connected by an inextensible string which passes over a smooth pulley at the highest point of a smooth plane inclined at an angle of 30° to the horizontal plane on which it rests, A resting on the inclined plane, and B on the horizontal plane. Find the length of the string so that, if A be allowed to run down from the top of the plane, it may just reach the bottom.

1421. Two weights, P and $2P$, connected by a string, hang vertically over a perfectly smooth pulley; and, after the system has been in motion for 3 seconds, a weight $\frac{1}{2}P$ is added to P: find the velocity at the end of the fifth second.

1422. Two weights of 5 oz. and 3 oz. respectively, are connected by a string which passes over a fixed pulley: find the velocity with which they will move, and the tension of the string.

1423. A body A, weighing 16 oz., in running down a plane 96·6 feet long, inclined to the horizon at an angle of 30°, draws another body B, weighing 6 oz., vertically up by a string which passes over a smooth pulley at the top of the plane. Find the time in which it will run from the top to the bottom of the plane.

1424. A railway train weighing 20 tons starts from rest down an incline of 1 in 200, and runs unimpeded for 5 minutes; find the constant pressure which, acting in the opposite direction to the motion, would then bring it to rest in 3 minutes.

1425. A sphere of 4 oz. is projected up a plane 4 feet long, inclined at an angle of 30°, at the same instant that another sphere of 10 oz. is allowed to roll from the top; with what velocity must the former be projected so that, after collision, the latter may rebound to the top of the plane; the coefficient of elacticity being ·75?

1426. Two bodies, weighing 10 and 5 ounces respectively, are connected by a string passing over a pulley at the top of a double inclined plane; find the tension of the string when the inclinations of the planes are 50° and 45° respectively.

1427. A sack of corn weighing 2 cwt. is lowered from a height of 50 feet by a rope, which, passing over a fixed pulley, has a bundle of empty sacks weighing $1\frac{1}{2}$ cwt. attached to the other end, and these are drawn up by the descent of the full sack; how long will the sack take to descend, and with what velocity will it strike the ground?

1428. Two weights hang freely over a single fixed pulley; prove that, if friction and the inertia of the pulley be neglected, the force of gravity at different places on the earth will be proportional to the spaces descended from rest in the same time by the heavier body.

1429. A weight of 16 ounces draws another of 10 ounces down a plane inclined at an angle of 45° to the horizon; find the vertical velocity of the latter at the end of three seconds, and the tension of the string.

1430. A weight of 5 oz. hanging freely, pulls a weight of 20 ounces along a perfectly smooth horizontal table, by means of a string to which both are attached; how long will the latter take to reach the edge of the table, from which it was originally 10 feet distant, and at what horizontal distance from the edge of the table will it strike the ground, supposing the length of the string and the height of the table to be each 10 feet?

1431. The velocity acquired by a body in falling from rest, down a plane, is 25; find the velocity it would acquire if projected from the top, down the plane, with a velocity of 25.

1432. A railway train weighing 100 tons, moving at the rate of 40 miles per hour, comes to an incline of 1 in 50; what additional pull, in tons, will be required from the engine to keep the speed at the same amount? and how far would it run up if the engine were to shut off the steam at the base of the incline?

1433. The "ways" along which a ship was launched were found to have an inclination of 1 in 20: find the velocity with which she was propelled along the surface of the water, supposing the time she took to slide off the ways to be 10 seconds.

1434. What velocity will a railway train acquire by its descent of an incline of 1 in 70, one mile in length?

1435. Find the velocity acquired by a railway train running down an incline of 1 in 100, one mile long.

1436. Find the line of quickest descent from a given point to a given plane.

1437. Find the length of the line of quickest descent from a given straight line inclined at an angle a to the vertical, to a point in the same vertical plane at a perpendicular distance p from the given line.

1438. If a be the shortest horizontal distance of a given point from a given plane whose inclination is i, show that the length of the line of quickest descent from the point to the plane is

$$2a \,.\, \sin\frac{i}{2}.$$

1439. Find the plane of quickest descent from a given plane to a given point.

1440. Find the straight line of quickest descent from a given point to a given circle.

1441. Find the line of quickest descent from a given circle to a given circle within the former.

1442. A balloon with its car, &c. weighs 750 lbs., and is capable of just lifting 1200 lbs.; find the height to which it will rise with the first-named weight in twenty seconds, supposing the resistance of the air to be equivalent to a constant downward pressure of 50 lbs. Find also the time in which a body, released from the car at that point, will reach the earth.

1443. Investigate from first principles the expression

$$\sin(2e - i) = \frac{gr}{V^2}\cos^2 i + \sin i,$$

where e is the elevation which, with the velocity V, will give the range r upon a plane inclined at an angle i to the horizon.

1444. A body projected obliquely, and acted on by gravity, strikes a plane inclined to the horizon at an angle β; find the range on this plane, and the time of flight, the angle and velocity of projection being respectively a and V.

1445. If a body be projected at an angle of 60°, and strike a plane inclined at an angle of 30°; show that the range in feet will be sixteen times the square of the time of flight in seconds, if $g = 32$.

1446. Find the elevation and velocity with which a body must be projected, in vacuo, in order that it may pass through two points distant h and h' from the point of projection, and k and k' above its level; find also the time of flight to the latter point.

1447. If any number of bodies be projected from the same point, at the same time, with the same velocity in the same vertical plane, but at various angles, they will all, at any subsequent instant, be in the circumference of the same circle.

1448. What would be the range, *in vacuo*, of a 32-lb. shot fired at an elevation of 10°, with a 6-lb. charge, upon a plane having an upward slope of 4°, supposing the initial velocity to be expressed by

$$V = 1600 . \sqrt{\frac{3C}{W}},$$

where C is the weight of the charge, and W that of the shot?

1449. Investigate an expression for the determination of e, the elevation of a gun, which will cause a shot fired with a velocity v to strike a plane, inclined to the horizon at an angle i, at a distance r from the gun.

1450. Find the range of a shell, fired at an angle of 45° with a velocity of 500 feet, upon a horizontal plane, and the length of fuse requisite to cause it to explode upon reaching the ground, *supposing* the fuse to burn at the rate of 1 inch in six seconds.

1451. Show that if a body be projected from the bottom of an inclined plane and strike the plane perpendicularly, the time of flight will be, in the usual notation, $\frac{V.\cos(\alpha - i)}{g.\sin i}$.

1452. Find the range of a shot, fired at an elevation of 30° with a velocity of 400 feet per second, upon a descending plane of 15°.

1453. Prove that the maximum range upon a plane inclined to the horizon is obtained when the body is projected in the line of direction which bisects the angle between the vertical and the plane; and that equal differences of elevation above and below this line will give corresponding equal ranges.

1454. A sphere is rolled with a velocity of 20 up a smooth board 8 feet in length, inclined at an angle of 30°: find the horizontal distance from the foot of the board at which it will strike the ground.

1455. A shell is to be fired from the top of a cliff 300 feet high with a velocity of 600 feet per second, to strike a ship at anchor 600 yards from the base of the cliff; what must be the elevation of the gun and the length of the fuse, supposing it to burn at the rate of ·167 inch per second?

1456. Let *ABC* be an isosceles triangle, resting with its base *AB* upon a horizontal plane, and let its surface be inclined to the horizon at an angle of 60°; it is required to find the velocity with which a ball must be projected in the direction *AC*, so as to roll off the surface of the triangle at the point of bisection of *BC*.

1457. A particle is projected at an angle α, and strikes a plane inclined at an angle i; find the time at which it is a feet distant from the plane.

1458. A perfectly elastic particle is projected with a velocity v, at an elevation α, directly up a plane inclined at an angle i to the horizon. Find the greatest perpendicular distance of the particle from the plane; show that the velocities parallel to the plane, at the successive points of impact, are in arithmetical progression; and find the number of bounds the particle will make before it commences its descent down the plane.

1459. If x and y be the rectangular co-ordinates of a point referred to horizontal and vertical axes, whose origin is the point of projection of a body, show that

$$y = x \tan e - \frac{gx^2}{2V^2 . \cos^2 e};$$

where e is the inclination of the line of projection to the horizon and V the velocity of projection.

1460. Two particles are projected with the same velocity so as to have the same range on the same horizontal plane; compare their times of flight.

1461. If the wind were blowing at the rate of 40 miles an hour, what would be the equation to the line of motion of the balloon in Question 1442? And what would be its position and the direction of its motion at the end of ten seconds from the time of starting?

1462. Find the length which a fuse must be cut in order that it may burst a shell when it strikes the ground at a range of 1200 yards upon a horizontal plane, if the gun be fired at an elevation of 4°; the fuse burning at the rate of a fifth of an inch per second.

1463. What is the charge of powder which will give a range of 1650 yards on a horizontal plane to a 13-inch shell fired at an elevation of 45°? And what would be its range with that charge on a plane inclined to the horizon at an angle of 2° 40′?

1464. Find the range of a shot fired at an elevation of 30° with a velocity of 161 feet per second, upon a plane which rises 1 in 15.

1465. If a triangle ABC have its base AB horizontal, and a body be projected from A at an elevation e, so as to pass through

C and *B*; show that $\tan e$ is equal to twice the area of the triangle divided by the rectangle of the segments into which the perpendicular from *C* divides the base.

1466. A perfectly elastic body is projected from a point *A* at the base of a plane inclined to the horizon, and strikes the plane perpendicularly: what is the distance from *A* at which it will again strike the plane?

1467. Two elastic spheres discharged at the same instant from points in a horizontal plane, describe the same parabola in contrary directions; after impact one of them retraces its path and the other falls vertically; find their elasticity of recoil and the ratio of their masses.

1468. A sphere whose elasticity is $\frac{1}{3}$, is propelled, with a velocity 10, down a plane inclined at an angle of 60° to the horizon, and at a distance of 15 feet from its starting-point strikes a hard and immoveable obstacle, which presents a horizontal surface to its impact; find the point where the sphere will again strike the plane.

1469. Two equal inclined planes are placed back to back, and a body projected up one of them flies over the top and strikes the ground just at the foot of the other; find the velocity of projection, the inclination of each plane being 30°, and their common altitude *h* feet.

1470. A perfectly elastic sphere is let fall from a height of 16·1 feet above the point where it strikes a plane inclined 45° to the horizon; find the position of the body at the end of 2 seconds.

1471. A body is projected with a velocity of 800 at an elevation of 30° upon a plane inclined 15° to the horizon, and strikes the plane at an angle of 40°; find the range.

1472. If an elastic body be projected on a horizontal plane, show that the successive ranges will be in a geometric progression, having the common ratio *e*.

1473. The "sighting" of a rifle fixes the angle between the direction of the object aimed at and the line of fire: show whether, if aim be taken at the top of a vertical wall with the sight adapted to the horizontal distance of its base, the bullet will pass above or below the top of the wall; and find the error in the oblique range.

1474. Three points, B, C, and D, in the same horizontal line, are equidistant from one another, and from the horizontal line AH, vertically below them; prove that if any line CA be drawn to the horizontal line, the angle BAH is less than $2CAH$, and that if a body be projected from A at an angle $a=2CAH$ so as to pass through B ascending, it will also pass through D; except in the case when A is vertically below B.

1475. A shell is projected with a velocity of 805 feet per second; find its greatest range upon a plane inclined upwards at an angle of 30°.

1476. A perfectly elastic ball is suspended from the centre of the ceiling of a room h feet high, and $2a$ feet wide, by a string; find the length l of the string, so that the ball, being struck horizontally by an equal and perfectly elastic ball projected from the middle of the side of the floor, may just touch the ceiling.

1477. A shot is fired with a velocity of 400 at an elevation of 30°, and is observed to strike an object at the end of 4 seconds; find the inclination of the line which joins the object and the gun.

1478. A pendulum oscillating in 2 seconds at the sea-level, has one inch cut from its length, and is then carried up a mountain 17600 feet high; find the time of oscillation in that position, the radius of the earth being 3960 miles.

1479. A seconds'-pendulum is carried to the top of a mountain 3000 feet high; assuming that the force of gravity varies inversely as the square of the distance from the earth's centre, and that the radius of the earth is 4000 miles, find the number of oscillations lost in a day.

1480. If two clocks, having pendulums of different lengths, gain or lose to the same amount, show that the corrections of their pendulums will be nearly in the ratio of their lengths.

1481. Find the velocity with which the earth must revolve in order that bodies at the surface, in latitude 60°, may lose all their weight.

1482. Show that if the vertical radius of a quadrant of a circle be divided into n segments, which are to one another as the odd numbers 1, 3, 5, 7, &c., and horizontals be drawn to meet the arc, the points of intersection will be such that a body falling down the arc from rest at the extremity of the horizontal radius, will have, at those successive points, velocities which are as the natural numbers 1, 2, 3, 4, &c.

1483. A pendulum which makes n oscillations in a certain interval, at the surface of the earth, makes $n-3$ oscillations at the bottom of one mine, and $n-5$ at the bottom of another, in the same absolute interval as before; compare the depths of the mines.

1484. A heavy particle is suspended from the centre of the ceiling of a room 15 feet wide by an inextensible string 6 feet long, and being drawn aside, in a plane perpendicular to the side of the room, until it touches the ceiling, is allowed to swing; find the point at which it will strike the opposite wall if the string be cut when the particle has made half an oscillation.

1485. A simple seconds' pendulum is found to make 10560 oscillations at the level of the sea, and 10558 in the same time at the summit of a mountain; find the height of the mountain, the radius of the earth being 7960 miles: find also how far a body let fall at the top of the mountain would fall in 3 seconds.

1486. The number of vibrations made in 3 hours by a pendulum at the surface of the sea was 20000; and the number made in the same time, by the same pendulum, at a height of 10560 feet was 19990: find the radius of the earth.

1487. If L be the length of the seconds' pendulum at the level of the sea, and l the length of the seconds' pendulum at a height h, show that the radius of the earth is

$$h \cdot \frac{\sqrt{l}}{\sqrt{L}-\sqrt{l}}.$$

1488. Find the change in the daily rate of a clock having a brass pendulum beating seconds, due to a rise of temperature of 20°; the expansion of brass being ·00001 of its linear dimensions for each degree.

1489. Find the length of the simple pendulum which oscillates half-seconds; the force of gravity being expressed by 32·2.

1490. Find the daily rate of a clock the pendulum of which is 38·64 inches long, supposing each vibration to mark one second upon the dial plate.

1491. Find the length of the seconds' pendulum when the force of gravity is 32·2; and determine the daily rate of a clock having such a pendulum made of brass, due to an increase of temperature of 10 degrees, supposing brass to expand ·00001 of its length for each degree.

1492. A body is suspended freely by a fine thread from the ceiling of a railway carriage, and it is observed to attain a deviation of 40′ from the vertical; find the radius of curvature of the line of rails at that point, supposing the train to be moving at the rate of 15 miles an hour.

1493. A horseman is galloping in a horizontal circle of 20 feet radius at the rate of 12 miles an hour; find the natural inclination of the plane which passes through the axes of the bodies of the man and horse, to the vertical.

1494. How much must the velocity of the earth's rotation be increased in order that bodies at its equator may lose all their weight; the radius being assumed as 4000 miles?

1495. A curve on a railway has a radius of a quarter of a mile; find the difference in level of the lines of rails in order that the pressure on the wheels may be equal, when a train runs round the curve at the rate of 20 miles an hour; the "gauge" or distance between the rails being 4 feet.

1496. A weight W is suspended from the ceiling of a railway carriage by a string a feet long; find the tension of the string and its inclination to the vertical when the carriage is running round a curve of 1 mile radius at the rate of m miles an hour; find also the number of small oscillations which it would then make in one minute.

1497. A 24-lb. and a 12-lb. shot are successively whirled round on a horizontal table with a given length of string of the same strength attached to them, the velocity in each case being increased until the string breaks; find the ratio of these breaking velocities.

1498. A boy whirls round a stone in a sling 3 feet long at the rate of six turns in one second, and the stone is discharged at an angle of 30° to the horizontal plane; find the distance at which the stone will fall and the strain upon the sling, supposing the velocity to be doubled by the action of the boy's arm at the instant of discharge.

1499. Find the centrifugal force at the surface of the earth in latitude 60°, estimated in the direction of the radius of the earth at that point, supposing the earth to be a sphere whose radius is 4000 miles.

1500. A particle suspended in the car of a balloon revolves as a conical pendulum in a plane 21 inches below its point of support, and it is observed to make 7 revolutions in 10 seconds. Find how high the balloon will have risen above the earth in 2 minutes, supposing the force of gravity and the acceleration of the balloon to have remained constant.

1501. A body being suspended by a string 2 feet long, and made to revolve as a conical pendulum, it is found that the tension of the string is twice the weight of the body; compare the time of revolution with the time of oscillation of the body as a simple pendulum.

1502. At what angle is the arm of a conical pendulum inclined when it revolves in the time of its oscillation as a simple pendulum?

HYDROSTATICS.

1503. The volumes of three fluids and their respective specific gravities being given; find the specific gravity of their mixture, supposing a loss of one-tenth in volume.

1504. A cylindrical tube 1 foot long and 1 square inch in section is found to weigh w lbs. when empty, and W lbs. when filled with a certain fluid; find the specific gravity of the fluid, water being the standard.

1505. Explain the experimental process by which the equal transmission of fluid pressure in all directions is proved; and define the meaning of the term "pressure at a point."

1506. The volumes of two fluids are as n to m, and their specific gravities are ρ and ρ'; when mixed together, they lose one pth of their volume: find the specific gravity of the mixture.

1507. A coin, known to be composed of platinum and silver, is found to be of exactly the same size and weight as a sovereign; find the relative weights of the two metals in it, the specific gravities of platinum, silver, and gold being 21, 10·5, and 17·5 respectively.

1508. The specific gravity of silver being 10·5 and of copper 8·9; find the relative weights of the two which must be mixed in order to form a compound which shall weigh one-ninth more in air than in water.

1509. A cubic foot of teak weighs 100 times as much as a cubic inch of lead; compare their specific gravities.

1510. A certain mass of metal weighs 30 oz. in one fluid, and 35 oz. in another; what will be its weight when immersed in a mixture of equal volumes of the two fluids, if it weigh 40 oz. in air?

1511. In the hydrostatic bellows, when the weight is constant, and water is poured into the pipe, find the rise in the surface of the water in terms of the volume poured in and of the sectional areas of the pipe and of the bellows.

1512. The radii of two spheres are 2 inches and 3 inches, and their weights are 8 lbs. and 10 lbs. respectively; find the ratio of their specific gravities.

1513. The specific gravity of copper being 8·8 and of tin 7·3, find the weights of each of these metals in a mass of gun-metal weighing 500 lbs., its specific gravity being $8\cdot\dot{6}$.

1514. A rectangle is immersed vertically to a depth a below the surface of a fluid, and has two of its sides horizontal; find the position of the horizontal line which divides the surface into portions the pressures upon which are equal.

1515. A fluid A has a specific gravity 1·25, and another fluid B has a specific gravity ·85; 5 fluid ounces of A are mixed with 7 fluid ounces of B, and 3 fluid ounces of water are added: find how many grains a mass of lead weighing 745 grains, will weigh when immersed in the mixture; the specific gravity of lead being 11·4.

1516. A piece of gun-metal composed of copper and tin, is found to weigh W ounces in air, and w ounces in water; show that the ratio of the weights of copper and tin is expressed by the fraction

$$\frac{c}{t} \times \frac{W-(W-w)t}{c(W-w)-W},$$

where c is the specific gravity of the copper and t that of the tin.

1517. A vertical cylinder contains a quantity of water of which the depth is equal to twice the diameter of its circular base, and a right cone of a density equal to five times the density of the fluid, having its base exactly fitting the cylinder, rests (vertex downwards) upon the fluid; find the total pressure upon the curved surface of the cylinder in terms of the weight of the water, supposing the axis of the cone to be equal to the diameter of its base.

1518. Find the ratio of the pressures upon the upper and lower halves of a regular hexagon immersed vertically in two fluids, the one of double the density of the other, one of the sides of the hexagon being in the surface of the upper fluid, and its centre of gravity in the surface of the lower.

1519. A vessel in the form of an inverted frustum of a cone is filled with water: compare the pressure upon the bottom or smaller end with the weight of the water, the frustum being 12 inches deep, and the radii of its ends 4 inches and 1 inch.

1520. A watch-chain which weighs 200 grs. in air, weighs only 184·7 grs. in water: find the ratio of the volumes of brass and gold in it; the specific gravity of brass being 7·8, and of gold 19·3.

1521. A hollow equilateral triangular prism, the sides of which are squares, is placed on end and half-filled with water; it is then placed horizontally upon one of its sides. Compare the pressures on the end in the two cases.

1522. If a hollow right cone, standing with its base downwards upon a horizontal plane, be completely filled with water, show that the weight of the cone must be equal to twice the weight of the water in order to prevent the cone from rising; and thence deduce the amount of the whole *normal* pressure upon the curved surface, and the consequent position of the centre of gravity of that surface.

1523. A cylinder is completely filled with water; compare the pressure sustained by its curved surface when the axis is vertical and when it is horizontal.

1524. Compare the pressures upon the upper and lower portions of a circle immersed vertically in a fluid, the circle being divided by a horizontal diameter, and the centre of gravity of a semicircle being at a distance equal to $\frac{4}{3\pi}$ of the radius from the centre; and find the depth to which the circle must be immersed in order that the pressure upon the lower half may be the double of that upon the upper.

1525. Compare the pressures upon the three upper and upon the three lower faces of a cube suspended by one of its angles in a homogeneous fluid, at a depth equal to the side of the cube.

1526. Compare the pressure on the base and on the three sides of a regular tetrahedron filled with water, and having its base horizontal.

1527. A cylinder is divided by a plane which cuts its axis perpendicularly at a distance from the lower end equal to one-fourth of its length; find the depth to which it must be vertically immersed in a fluid, that the total fluid pressures upon the whole surfaces into which it is divided may be equal.

1528. A hollow thin square prism, filled with water, stands upon one of its rectangular faces, and one of its ends is loose, though water-tight; find the magnitude and point of application of a single force which would hold that end in its position.

1529. Supposing both ends of the prism in the last question to be fixed, and the prism to be filled with equal volumes of two fluids of densities ρ and 3ρ which do not mix; find the pressure produced upon that portion of each end in contact with the latter fluid.

1530. An isosceles triangular lamina floats vertically in a fluid with one of its equal sides parallel to the surface of the fluid, and with its centre of gravity in that surface, the vertex of the triangle being supported by a string: find the ratio of the densities of the fluid and lamina, and the tension of the string.

1531. A solid body is floating between two fluids of specific gravities s and s', and the part immersed in the denser fluid is observed to be the same as if it were floating in a mixture of equal volumes of the two fluids; find the specific gravity of the solid.

1532. A hollow water-tight 8-inch cube has one of its vertical sides loose, and capable of revolving about its upper edge; find the pressure at the lower edge necessary to keep the side closed when the cube is filled with water.

1533. An isosceles triangle is immersed in a fluid so that its vertex is at the surface and its base horizontal; find the locus of the centres of pressure of the straight lines drawn in the triangle perpendicular to its base.

1534. A rectangular sluice gate $ABCD$ has its two sides AD, BC and the axis EF upon which it turns, horizontal; find the height to which the water must rise above EF before the gate turns, when $AB = 7$ feet, and $AE = 4$ feet.

1535. A uniform cylinder of length $2b$ inches, and density ρ, floats in a fluid, with its axis vertical and half immersed; another fluid is then poured upon the top of the former (with which it does not mix) to the depth a inches, which causes the cylinder to rise b inches in the lower fluid. Find the density of the upper fluid.

1536. A cylinder rounded off at one end in the form of a hemisphere, is found to float in water with one-half of the radius of the hemisphere immersed, and the equilibrium is found to be neutral; find the density of the body.

1537. A heavy cone, having its base 6 inches in radius, is suspended in water by a point in the circumference of its base so as to be completely immersed; find the magnitude and direction of the resultant pressure upon its curved surface, and compare the former with the total pressure upon the same surface, the point of suspension being 2 feet below the surface of the water, and the height of the cone 12 inches.

1538. A hollow cube having a weight suspended at one of its angles, floats with the three nearest angles at the surface of the water; find the additional weight which will just bring the highest angle down to the surface, the edges of the cube being two feet long.

1539. Find the thickness of an iron spherical shell 20 inches in diameter which will just float in water, the specific gravities of iron, water, and air being 7·5, 1, and ·00125 respectively.

1540. A ship in dock is observed to have risen 3 inches out of the water, owing to the discharge of 50 tons of her cargo; find the area of her section at the water-line.

1541. A body in the form of an equilateral cone and a hemisphere on the same base, floats in a fluid; find the position and character of the equilibrium when the density of the fluid is double of that of the body.

1542. An empty hollow sphere of 4 inches radius, and weighing 3 lbs., is fastened to the bottom of a vessel filled with water by a fine string; find the tension of the string when the sphere is wholly immersed in the water, and also when the water is replaced by the same bulk of oil, the specific gravity of which is ·92.

1543. A thin hollow cone 8 inches in diameter and 12 inches deep has a sphere of lead 3 inches in diameter placed within it; find the depth to which it will sink in a fluid the specific gravity of which is 1·135, that of lead being 11·35.

1544. Four cylindrical pontoons 20 feet long, having hemispherical ends of 2 feet radius, are lashed together and a platform is laid upon them, and it is found that they are then half immersed; find the additional weight they will carry.

1545. Four cylindrical barrels 4 feet long by 2 feet 6 inches diameter are lashed together to form a raft, and it is found that, when floating in the water, one-third of their diameters is immersed; find the additional weight which the raft will support when the barrels are completely immersed.

1546. A cylindrical pontoon, having hemispherical ends, is 25 feet long and 2 feet in diameter, and is immersed to half its depth; find the additional weight it will bear before it is wholly immersed: find also the total pressure upon its surface in the latter case.

1547. A diving-bell is sunk until the depth from surface to surface is 11 fathoms; find the volume of air which must be pumped in, in order to fill the bell, the temperature of the air in the bell being t^0 lower than that of the external air.

1548. A cylindrical diving-bell, 9 feet high, is to be sunk to the bed of a river 40 feet deep; find the height to which the water will rise within it.

1549. The water in a conical diving-bell has risen one-third of its height inside the bell, where the thermometer stands at 60°, the atmospheric temperature being 40°. Find the depth from surface to surface.

1550. A balloon filled with gas at a temperature of 60° and pressure of 30 inches, ascends to a height where the thermometer indicates 35° and the barometer 25 inches; find the change of volume of the gas, and the height to which the balloon has risen.

1551. Calculate the height of a point on Shooter's Hill, above the wharf in the Arsenal, from the following observations:—

	Barometer.	Temp. of Mer.	Temp. of Air.
Arsenal	29·475	37	34
Shooter's Hill	29·022	35	32

using the formula

$$H \text{ (in feet)} = 60000 \log \frac{h'}{h},$$

in which the values of h and h' are supposed to be observed at temperature 32°.

1552. A sphere, *sp. gr.* 1·25, is placed in a reservoir of water 20 feet deep; find the time in which it would reach the bottom, neglecting the resistance of the water.

1553. An india-rubber hollow sphere, which contained air of three times the external density, was suspended within a diving-bell, which was then sunk to a depth x, when the sphere was found to be one-tenth less in circumference than when the bell was at the surface. Find x, on the supposition that the force of compression by the india-rubber remains practically unaltered.

1554. Describe the "Air-Pump;" and determine the increase of temperature required to restore the elastic force of the air in the receiver after two strokes of the piston, supposing the temperature of the air to be 50°, and the capacity of the receiver to be ten times that of the pump.

1555. Describe the action of the air-pump; find the density of the air in the receiver after 5 strokes of each piston, the capacity of the receiver being to that of each barrel as 9 : 1; and calculate the increase of temperature from 40° Fahrenheit which, under a constant pressure, would have produced the same amount of rarefaction in the air.

1556. Describe the common suction-pump; and determine the force necessary to move the handle downwards, when the length of the pipe from the piston to the surface of the water is 15 feet, the diameter of the piston 6 inches, the length of the handle 5 feet, that of the arm at right angles to it and connected with the piston-rod, 1 foot, and the play of the handle 60°.

DIFFERENTIAL CALCULUS.

1557. Find the limiting value of $\frac{7x+2}{3x-5}$, when x approaches infinity.

1558. Define the "differential coefficient" of a function; and find the differential coefficient of

$$u=\log\frac{1+x\sqrt{2}+x^2}{1-x\sqrt{2}+x^2}+2\tan^{-1}\frac{x\sqrt{2}}{1-x^2}.$$

1559. Show that $d.xyz = xydz + xzdy + yzdx$, and apply this form to the differentiation of the function

$$u = x^3 . e^x . \tan x.$$

1560. Prove that $d.a^x = a^x . \log_e a dx$.

1561. Investigate the following formulæ:

$$(a) \quad d.(x^n) = nx^{n-1} . dx;$$

$$(b) \quad d(xy) = x . dy + y . dx;$$

$$(c) \quad d.\left(\frac{x}{y}\right) = \frac{y . dx - x . dy}{y^2};$$

$$(d) \quad d . \tan x = \sec^2 x . dx.$$

Differentiate the following functions:

1562. $u = \sqrt{\dfrac{1+x}{1-x}}$. 1563. $u = \dfrac{x^2 - 2}{3} \sqrt{(1 + x^2)}$.

1564. $u = \dfrac{\sqrt{(x^2 + a^2)} + \sqrt{(x^2 + b^2)}}{\sqrt{(x^2 + a^2)} - \sqrt{(x^2 + b^2)}}$.

1565. $u = \dfrac{1}{x + \sqrt{(1 - x^2)}}$. 1566. $u = \dfrac{(a^2 + x^2)^{\frac{3}{2}}}{(a^2 - x^2)^{\frac{1}{2}}}$.

1567. $\dfrac{x - \sqrt{(x + 1)}}{x + \sqrt{(x + 1)}}$. 1568. $u = \dfrac{x^2 - x + 1}{x^2 + x - 1}$.

1569. $u = \dfrac{x^n}{(a + x)^n}$. 1570. $y = \dfrac{x}{a^2 (a^2 + x^2)^{\frac{1}{2}}}$.

1571. $y = \left(\dfrac{a + bx^2}{5b^2} - \dfrac{a}{3b^2}\right) . (a + bx^2)^{\frac{3}{2}}$.

1572. $u = \log \{\sqrt{(1 + x^2)} + \sqrt{(1 - x^2)}\}$.

1573. $u = x^3 - 3 \log (1 + x^3)^{\frac{1}{3}}$. 1574. $u = \log \dfrac{x + \sqrt{(x^2 - a^2)}}{\sqrt{(x^2 - a^2)}}$.

1575. $u = \dfrac{x . \log x}{1 - x} + \log (1 - x)$.

1576. $\log (x . e^{\cos x})$ and $\tan^{-1} x \sqrt{\dfrac{b}{a}}$.

1577. $u = x^n . e^{\sin x}$. 1578. $u = \tan^{-1} \dfrac{2x}{1 - x^2}$.

1579. $u = \sin (nx) . \sin^n x$. 1580. $u = \cos^{-1} \dfrac{x^{2n} - 1}{x^{2n} + 1}$.

1581. $u = e^x . \log \sin x$. 1582. $u = \dfrac{\cos x}{2 \sin^2 x} - \dfrac{1}{2} \log \tan \dfrac{x}{2}$.

1583. $u = \dfrac{2}{\sqrt{(a^2 - b^2)}} . \tan^{-1} \dfrac{\tan \dfrac{x}{2}}{\sqrt{\left(\dfrac{a - b}{a + b}\right)}}$.

1584. $u = \tan^{-1} x + \tan^{-1} x^3$. 1585. $u = \sqrt{\{\sin (x + a) . \sin (x - a)\}}$.

1586. $u = e^x . \log \sin x$. 1587. $u = \sin x . \sin^{-1} x$.

1588. $u = \log . \tan e^{\sqrt{x}}$. 1589. $u = \dfrac{1}{\sqrt{3}} \tan^{-1} \dfrac{2x - 1}{\sqrt{3}}$.

1590. $y = x . \sqrt{(a^2 - x^2)} + a^2 . \sin^{-1} \dfrac{x}{a} + \log \dfrac{x^2 - 2ax + a^2}{a^2 - x^2} - 3e^{\sqrt[3]{x}}$.

1591. $y = \dfrac{1}{6} \log \dfrac{(x - 1)^2}{x^2 + x + 1} - \dfrac{1}{\sqrt{3}} . \tan^{-1} \dfrac{2x + 1}{\sqrt{3}}$.

1592. $y = \log \dfrac{x}{(1 + x)^{\frac{1}{2}} (1 + x^2)^{\frac{1}{4}}} - \dfrac{1}{2} \tan^{-1} x$.

1593. Investigate Maclaurin's Theorem; and, by means of it, expand $\sin^{-1} x$ in series arranged according to the ascending powers of x:

1594. $u = \log . \cos x$. 1595. Vers x.

1596. $u = \cos^{-1} x$. 1597. $e^x . \cos (x \sqrt{3})$.

1598. $e^x . \tan x$. 1599. $u = \tan^{-1} x$.

1600. If $u = f(x + y)$, show that the first differential coefficient will be the same whether y be considered constant and x variable or *vice versâ;* and show how this theorem may be employed in the investigation of Taylor's Theorem.

Expand by Taylor's Theorem in series arranged according to the ascending powers of h:

1601. $\sin^{-1}(x+h)$. 1602. $\cos^{-1}(x+h)$.
1603. $\log(x+h)^n$. 1604. $\tan(x+h)$.
1605. $\tan^{-1}(x+h)$.

1606. Show, by Taylor's Theorem, that if $u=f(x)$ have a maximum or a minimum value, depending upon the variable x; then, for that value, $\frac{du}{dx}=0$; and that if, for that value, $\frac{d^2u}{dx^2}$ do not also vanish, it must be negative in the former case and positive in the latter.

1607. Define the meaning of "maxima and minima values" of the function of a variable; and determine the form of a thin hollow cone which, with a given capacity, shall be constructed with the least expenditure of material.

1608. Explain how the maxima and minima values of a function of one variable are determined; and find the shortest line to a parabola from a given point in its axis. Explain the result when the distance of the point from the vertex is less than half the latus rectum.

1609. A boatman, 3 miles out at sea, wishes to reach a point on the beach 5 miles from the nearest point of the coast; he can pull at the rate of 4 miles an hour, but he can walk at the rate of 5 miles an hour: find the point at which he must land.

1610. Find the greatest cone B which can be described within a given cone A; the vertex of B being the centre of the base of A.

1611. Inscribe the greatest rectangle in a given parabolic area terminated by a double ordinate perpendicular to the axis.

1612. Find the dimensions of a cylinder of a given volume V, such that its surface shall be the least possible.

1613. Find the greatest right cone which can be cut from a given sphere.

1614. A steamer whose course is due west and speed 10 knots is sighted by another steamer going at 8 knots; what course must the latter steer, so as to cross the track of the former at the least possible distance from her?

1615. If from a circular piece of paper a sector be cut out, and the straight edges of the remaining sector be joined, a cone will be formed; find the arc of the first sector, so that the cone may have the greatest volume possible.

1616. Let *ABCD* be a parallelogram of which *AC* is a diameter; it is required to find in *AC* a point *F*, such that *BF* being drawn and produced to meet *CD* in *E*, the sum of the triangles *AFB* and *EFC* may be a minimum.

1617. Within the angle *ACB* made by two straight lines *CA*, *CB*, a point *P* being given; it is required to draw a straight line through *P*, so that the sum of the segments of the other lines intercepted between it and the angular point *C* may be the least possible.

1618. Let *D* be a point in the diameter *AB* of a semicircle *APB*; it is required to find the position of the point *P*, so that the sum of the distances *AP* and *PD* may be a maximum.

1619. A square piece of sheet-lead is to have its edges turned up perpendicularly, so that the vessel thus formed may contain the greatest possible quantity of fluid. Find the depth of the vessel.

1620. A weight *P* draws another *Q* up by a string passing vertically over a pulley; find *Q* so that the momentum acquired by *Q* in a given time may be a maximum.

1621. Find the subtangent at the point, $x = a$, of the curve $y^3 = x^2 . (2a - x)$; and also its asymptote.

1622. Find the equation to the normal in the "Witch,"

$$xy^2 = a^2 (a - x).$$

1623. Show that if $y = fx$ be the equation to a curve having a rectilinear asymptote, the intercept of the asymptote on the axis of x is the limit towards which $x - y \frac{dx}{dy}$ approaches as x approaches infinity. And find the asymptote of the curve $y^3 = x^3 + ax^2$.

1624. Find the subtangent and the radius of curvature of an ellipse, at the point whose abscissa, measured from the centre, is one-fourth of the major axis of the curve.

1625. Find the radius of curvature of the parabola at the vertex and at the extremity of the latus rectum.

1626. Find the radius of curvature of a cycloid at the extremities of the axis and at the points corresponding to $x = a$, the equation of the curve being

$$x = a \,.\, \text{vers}^{-1} \frac{y}{a} - (2ay - y^2)^{\frac{1}{2}}.$$

1627. Find the radius of curvature at the point where $x = \frac{a}{2}$ in the curve $xy = a^2$, and show that the equation to the evolute is

$$(\alpha + \beta)^{\frac{2}{3}} - (\alpha - \beta)^{\frac{2}{3}} = (4a)^{\frac{2}{3}}.$$

1628. Find the radius of curvature at the vertex of the curve $y^2 = a^2 (x^2 - b^2)$, and if it have asymptotes determine them.

1629. Trace the curve $y^3 = a^3 - x^3$; finding its asymptotes and points of inflexion.

1630. Trace the curve $y^2 - x^2 = 4ax$, and investigate its properties with regard to asymptotes and points of inflexion. Find also its radius of curvature at the point $x = 0$.

1631. Trace the curve $y(a^2+x^2)=ax^2$, and determine its properties with respect to asymptotes and points of contrary flexure.

1632. Trace the curve $y=x-x^3$; finding its points of inflexion and asymptotes, if any. Find also its radius of curvature at the extremity of its greatest ordinate.

INTEGRAL CALCULUS.

Find the values of the integrals:

1633. $\int \frac{2x^3dx}{(a+3x^4)^{\frac{3}{2}}}$.

1634. $\int \frac{r-x}{(2rx-x^2)^{\frac{1}{2}}}\,dx$.

1635. $\int \frac{dx}{a+bx+cx^2}$.

1636. $\int \frac{dx}{x(1-2x+3x^2)^{\frac{1}{2}}}$.

1637. $\int \frac{3x+1}{x^3+2x^2+x}\,dx$.

1638. $\frac{30dx}{x^3-x^2-6x}$.

1639. $\frac{3x^3dx}{\sqrt{(a^4-x^4)^3}}$.

1640. $\int \frac{x^2dx}{(1+x)^{\frac{3}{2}}}$.

1641. $\int \frac{dx}{x^2-1}$.

1642. $\int \frac{x^3dx}{\sqrt{(x^2-a^2)}}$.

1643. $\frac{x^5dx}{(1+x^2)^3}$.

1644. $\int \frac{dx}{(a+bx)^3}$.

1645. $\int b^2.(a^2-x^2)^{\frac{1}{2}}.\,dx$.

1646. $\int \left\{\frac{1}{x}-\frac{1}{\sqrt{(x^2+1)}}\right\}dx$.

1647. $\int \frac{x^3dx}{(a^2-x^2)^{\frac{1}{2}}}$.

1648. $\int 5x^2(3-2x^3)^{\frac{2}{3}}.\,dx$.

1649. $du=\frac{(a-x)^{\frac{1}{2}}}{(a+x)^{\frac{1}{2}}}.\,dx$.

1650. $\int mnbx^{n-1}.(a+bx^n)^{m-1}.\,dx$.

1651. $\int \frac{x.\,dx}{(x-2)^2}$.

1652. $\int \frac{x+1}{x^2+x-6}\,dx$.

1653. $\int \frac{x^3 dx}{(a+bx)^{\frac{1}{2}}}$.

1654. $\int (2ax - x^2)^{\frac{1}{2}} . dx$.

1655. $\int \frac{dx}{x^2 (a^2 + x^2)^{\frac{1}{3}}}$.

1656. $\frac{(2x+3)\, dx}{x^3 - x^2 + 2x}$.

1657. $\int \frac{rydy}{(2ry - y^2)^{\frac{1}{2}}}$.

1658. $\int \frac{dx}{(a + bx + cx^2)^{\frac{1}{2}}}$.

1659. $\int \frac{x^3 dx}{(2ax - x^2)^{\frac{1}{2}}}$.

1660. $\int \frac{dx}{\sqrt{(2ax + x^2)^5}}$.

1661. $\int \frac{\log x}{(1-x)^2}\, dx$.

1662. $\int x^5 \log x . dx$.

1663. $\int x^n \log x . dx$.

1664. $\int \frac{dx}{x . (\log x)^n}$.

1665. $\int x^2 e^x dx$.

1666. $du = \frac{dx}{\epsilon^x - \epsilon^{-x}}$.

1667. $\int x . e^{\sqrt{x}} . dx$.

1668. $\int \frac{\sin x}{\cos^3 x} dx$.

1669. $\int \frac{\sin^3 x dx}{\cos^2 x}$.

1670. $\int \tan^7 x . dx$.

1671. $\int x \sin^{-1} x . dx$.

1672. $\int \frac{x^2 . \sin^{-1} x}{(1-x^2)^{\frac{1}{2}}}\, dx$.

1673. $\int \sin^4 x . \cos^3 x . dx$.

1674. $\cos^2 x . dx$.

1675. $\frac{dx}{\cos^4 x}$.

1676. $\int \frac{dx}{\cos^3 x}$.

1677. $\int \frac{adx}{\sin^3 x}$.

1678. $\int \tan^4 x . dx$.

1679. Find the value of $\int_0^{a\sqrt{\frac{a}{b}}} \frac{x^2 . dx}{\sqrt{(a^3 - bx^2)}}$.

1680. The equation to the cycloid (origin at the vertex) being

$$y = a \operatorname{Vers}^{-1}\frac{x}{a} + (2ax - x^2)^{\frac{1}{2}},$$

show that the length of a cycloidal arc measured from the origin, is twice that of the chord of the corresponding arc of its generating circle.

1681. Find the area of the portion of a parabola cut off by the latus-rectum; and the volume of a conic frustum generated by the revolution of a rectangular trapezoid round its perpendicular side.

1682. Find the area of the curve $y^2 = a^2 . (x^2 - b^2)$ between the abscissas a and b.

1683. Find the area of the curve $xy^2 = 2a - x$ between the limits $x = 0$ and $x = a$.

1684. Find the area of the curve

$$\frac{x^2}{a^2} = \frac{y - a}{y}$$

from $x = 0$ to $x = \frac{a}{2}$.

1685. Find the area of the curve $y^3 = x^3 . (x - a)^2$ between the limits $x = 0$ and $x = a$.

1686. Find the area of the "Witch of Agnesi" bounded by the curve whose equation is $xy^2 = 4r^2(2r - x)$, and by a straight line perpendicular to its axis and passing through the centre of the generating circle.

1687. Find the area of the curve $y = x - x^3$ intercepted between the axes.

1688. Find the area of the curve

$$y . (4a^2 + x^2) = 2(x + a)a^2$$

between the limits $x = 0$ and $x = 2a$.

1689. Find the area of the curve expressed by the equation

$$xy^2 = (a - x)^2,$$

between the limits $x = 0$ and $x = a$.

1690. Find the volume of the solid generated by the revolution of the curve

$$xy = (a + x)(b^2 - x^2)^{\frac{1}{2}}$$

round the axis of y.

1691. An ellipse revolves round a tangent at the extremity of the major axis; find the volume of the ring generated by the area of the semi-ellipse furthest from the tangent.

1692. Find the volume of the solid formed by the revolution of the curve $xy^2 = (a - x)^2$ round the axis of y, between the limits $x = 0$ and $x = a$.

1693. Let DC and CA be the semi-axes, minor and major, of an ellipse, and from any point E in the arc DA, draw EF parallel to AC and meeting DC in F; and let $FC = h$: it is required to find the area DFE, and the volume of a conoidal bullet generated by that area about FE.

1694. Find the volume of the solid generated by the revolution of the curve

$$y^2(a^2 + x^2) + a^2x = a^4$$

about the axis of x; from $x = 0$ to $x = a$.

1695. Find the volume and curved surface of a paraboloid between the limits $x = a$ and $x = b$, the equation to the generating curve being $y^2 = 4mx$.

1696. Find the centre of gravity of a quadrant of a circle.

1697. Find the centre of gravity of a circular sector of which the arc is $2a$; and thence deduce that of a semicircular area.

1698. Find the centre of gravity of a segment of a circle in terms of the radius r of the circle, the semi-arc a of the segment, and the radius p of the base of the segment: and show what this becomes in the case of the semicircle.

1699. Find the centre of gravity of a semi-parabola of which the abscissa is a and the ordinate b.

1700. Find the centre of gravity of a circular arc a, in terms of the arc, its chord, and the radius of the circle.

1701. Find the centre of gravity of the solid generated by the revolution of the figure formed by two straight lines VA, AB at right angles to one another, and the parabolic arc VB about VA; when V is the vertex of the parabola of which the axis is parallel to AB.

1702. Find the centre of gravity of a material line, the density of which varies directly as the distance from one of its ends.

1703. Let the density of the sections of a right cone parallel to its base vary inversely as their distances from the vertex; find the centre of gravity.

1704. Find the centre of gravity of a cone, the density of every point of which varies inversely as the nth power of its distance from the plane of the base.

1705. Find the centre of gravity of the solid formed by the revolution of the area of the curve

$$y(a^2-x^2)^{\frac{1}{2}}=(a-x)^2$$

about the axis of x; between the limits $x=0$, and $x=a$.

1706. At a point D, in an ellipse, the ordinate DH is equal to the abscissa HC, C being the centre. Find the centre of gravity of the segment cut off by the double ordinate DHE.

1707. Let the density of a triangle vary as the nth power of the distance of any point in it from a straight line drawn through the vertex parallel to its base; find its centre of gravity.

1708. Let the density of a quadrant of a circle of uniform thickness vary as the nth power of the distance of any point in it from the centre of the circle; find its centre of gravity.

MOMENTS OF INERTIA.

Find the moment of inertia of

1709. A uniform rod about an axis through its centre of gravity and perpendicular to its length.

1710. A circle about an axis passing through its centre perpendicular to its plane.

1711. An equilateral triangle about one of its perpendiculars.

1712. A circular arc about the diameter which bisects the arc.

1713. A circular arc about an axis passing through its vertex and perpendicular to its plane.

1714. The circumference of a circle about any tangent.

1715. A circular area about any diameter.

1716. A circular ring about an axis perpendicular to its plane passing through its centre.

1717. A cylinder about its axis.

1718. A sphere about any diameter.

1719. A right cone about its axis.

1720. A spherical shell about a diameter.

1721. A hollow cylinder about its axis.

1722. A spherical lamina about a diameter.

1723. A cylindrical lamina about its axis.

1724. A parallelogram about an axis perpendicular to its plane and passing through the intersection of its diagonals.

1725. A parabolic area about an axis perpendicular to its plane and passing through its vertex.

1726. A cube about its diagonal.

1727. A cube about the diagonal of one of its faces.

1728. A cube about one of its edges.

1729. A cone about its slant side.

1730. A spheroid about its axis of generation.

1731. An ellipsoid about one of its axes.

CENTRE OF OSCILLATION.

Find the time of a small oscillation of

1732. An equilateral triangle about an axis perpendicular to its plane, through one of its angles.

1733. A cube about one of its edges.

1734. A sphere about an axis touching its surface.

1735. A right cone about an axis touching the circumference of its base.

1736. Show that an arc of a circle will oscillate about an axis through its middle point perpendicular to its plane in the same time as if its mass were collected at the opposite extremity of the diameter of the complete circle.

1737. Show that a cylinder of 8 inches radius will oscillate about an axis on its surface parallel to its geometrical axis, in the same time as if its mass were collected at a point one foot distant from the axis.

1738. Show that a hemispherical surface will oscillate about a diameter of its base in the same time as a simple pendulum, the length of which is two-thirds of the diameter.

MOTION IN A RESISTING MEDIUM.

1739. Find the time in which a body of given weight falling from rest through the air will acquire a velocity v; assuming that the resistance varies as the square of the velocity.

1740. Find the space through which the body will have fallen when it has acquired the velocity v.

1741. If a particle be projected in a medium the resistance of which varies as the velocity, find the space described in the time t, supposing no other forces to act.

1742. A body is projected vertically upwards with a velocity V; find the height to which it will rise, the resistance of the air varying as the square of the velocity.

1743. If in 1741 the resistance vary as the square of the velocity, find the space described in the time t by a body whose weight is w.

1744. A body is projected with a velocity V obliquely into the air at a small angle of elevation a; show that, if the vertical resistance of the air be neglected, the range on a horizontal plane is

$$\frac{w}{kg} \cdot \log\left\{\frac{kV^2}{w}\sin 2a + 1\right\},$$

where w is the weight of the body, and k the resistance due to a unit of velocity.

EXAMINATION PAPERS.

A.

I. GEOMETRY.

1. *Deduction.* If two sides of a triangle be bisected, show (*from the First Book of Euclid*) that the line joining the points of bisection is parallel to the third side, and equal to half that side; and thence show that if all the sides of a quadrilateral figure be bisected, and the adjacent points of bisection be joined, the figure so formed will be a parallelogram equal to half the given quadrilateral.

2. In every triangle the square of the side opposite any of the acute angles is less than the squares of the sides containing that angle, by, &c.

3. If two straight lines within a circle cut one another, the rectangle contained by the segments, &c.

4. Equal triangles which have one angle of the one equal to one angle of the other, have their sides about the equal angles reciprocally proportional; and triangles which have one angle of the one equal to one angle of the other, and their sides about the equal angles reciprocally proportional, are equal.

5. The circumferences of circles are to one another as their diameters.

6. If two straight lines meeting one another be parallel to two other straight lines which meet one another, but are not in the same plane with the first two; the plane which passes through them is parallel to the plane passing through the other.

COORDINATE GEOMETRY.

7. Investigate the relation between a and a', so that the lines $y = ax + \beta$, $y = a'x + \beta'$, may be perpendicular to one another. The line $y = ax + b$ passes through the point $(1, -2)$, and is perpendicular to the line $5y - 10x + 12 = 0$; find the values of a and b.

8. Construct the circle denoted by the equation

$$x^2 + y^2 - 6x + 10y - 15 = 0;$$

and find the position of that diameter of it which passes through the origin of coordinates.

A.

II. ARITHMETIC AND ALGEBRA.

1. Define the terms "fraction," "power," "root," "index," "logarithm" and "modulus." Explain also the reasoning by which it is shown that $a-(b-c)=a-b+c$.

2. A person owes £800 bearing interest at 5 per cent. per annum. At the end of each year he pays £120 for interest and in part payment of the principal. Find the amount of his debt at the end of the second year.

3. Reduce $\dfrac{(a+b)^2-(c+d)^2}{(a+c)^2-(b+d)^2}$ to its most simple form; and find the value of $\dfrac{a^2+b^2-c^2+2ab}{a^2-b^2-c^2+2bc}-\dfrac{a+b+c}{a-b+c}$.

4. Solve the following equations: $5x+6=\frac{1}{5}x+\frac{1}{6}$ ... (1);

$$\sqrt{(x+8)}-\sqrt{(x+5)}=\sqrt{x} \;\ldots\ldots\ldots\ldots\ldots\ldots\; (2);$$

$$(x-y)(a^2x^2+b^2y^2)=a^2(x^3-y^3),\; xy=c^2 \;\ldots\ldots\ldots\; (3).$$

5. *A* and *B* agree to pay their expenses for a certain time in the proportion of the numbers 4 and 7. At the end of this period it was found that *A* had paid the sum of £102, and *B* £73. What has the one to pay and the other to receive in order to settle the account?

6. The equation $x^3-x^2-33x-63=0$ has two equal roots: find them by means of the *derived equation;* find also the third root.

7. Define a geometrical progression, and show that if each term be subtracted from the preceding, the successive differences constitute also a geometrical progression. Sum the latter series to n terms, when the first term of the original series is 2 and ratio $\frac{1}{3}$.

8. Investigate the formula for the number of shot in a square pile; and show that if the number in a square pile be to the number in a triangular one of the same number of courses as p to q, then the number of courses in each is $\dfrac{2p-q}{2q-p}$.

A.

III. PLANE TRIGONOMETRY.

1. Explain the law of the "algebraic sign" in each of the four quadrants of the circle, in reference to the "tangent" of an arc or angle; and prove the relations

$$\tan(90^\circ + A) = -\cot A, \text{ and } \tan(180^\circ - A) = -\tan A.$$

2. If sine $a = \frac{1}{4}$, what are the surd expressions for the cosine, secant, tangent and cotangent of a?

3. The side a and its opposite angle A, of a right-angled triangle ABC (right-angled at C), are 152 feet and 18° 25′ 16″; find the other parts of the triangle.

4. Prove the following relations:—

$$\sin(A + B) = \sin A \cos B + \sin B \cos A;$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}; \quad \cos^2 A - \sin^2 B = \cos(A + B)\cos(A - B).$$

5. If a, b, c be the sides, and A, B, C the opposite angles of a plane triangle, and also $s = \frac{1}{2}(a + b + c)$; show that

$$\tan\frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.$$

6. Given $a = 126$ feet, $b = 132$ feet, and $c = 140$ feet; find A, B and C.

7. An object is seen from two ships A and B in a river, 480 feet asunder. At A the elevation of the object above the level of the river is found to be 28° 20′, and the angle subtended by it and the ship B, 48° 25′. Also at B the angle subtended by the object and the ship A is found to be 20° 15′. Find the height of the object above the level of the river.

8. Prove that if the sides a and b of a plane triangle ABC include an angle of 60°,

$$\cos(60^\circ - B) = \frac{a + b}{2c}.$$

A.

IV. SPHERICAL TRIGONOMETRY.

1. State *Napier's Rules* for the solution of right-angled spherical triangles; and exemplify them: 1st, when a side is taken as middle part; 2nd, when the complement of an angle is taken as middle part; 3rd, when the complement of the hypothenuse is taken as middle part.

2. If a, b, c are the sides opposite to the angles A, B, C of a spherical triangle, show that

$$\cos a = \cos b \cos c + \sin b \sin c \cos A.$$

3. Show that, if $s = \frac{1}{2}(a + b + c)$,

$$\cos \frac{1}{2} A = \sqrt{\frac{\sin s \sin (s - a)}{\sin b \sin c}}.$$

4. The three sides of a spherical triangle are 50° 37′, 83° 19′, and 40° 12′; find its three angles.

ASTRONOMY.

5. Define the following terms:—"pole" and "equator" of the heavens; "meridian"; "ecliptic"; "the obliquity of the ecliptic"; "the declination and right-ascension"; "latitude and longitude of a heavenly body;" "the latitude and longitude of a place on the earth."

6. Point out, by means of a figure, how the time of rising of the sun, and its azimuth when rising on a given day and at a given place, are determined.

7. What angle or arc in the heavens is the measure of the latitude of a place on the earth? What other angle or arc is it equal to? and how is the latitude of a place determined by observations on a circumpolar star?

8. Point out, by means of a figure, how the latitude and longitude of a star are determined from its observed right-ascension and declination.

A.

V. STATICS.

1. A straight lever is inclined at an angle of 60° to the horizon, and a weight of 360 lbs. hung freely at the distance of 2 inches from the fulcrum is supported by a power acting at an angle of 60° with the lever, at the distance of 2 feet on the other side of the fulcrum: find the power.

2. The arms of a lever are in the ratio of 2 to 1, and are at right angles to each other. A weight of 20 lbs. is suspended freely from the shorter arm, and 5 lbs. from the longer: find the angle which the longer arm makes with the horizon when the lever is in equilibrium.

3. A point is kept at rest by three forces represented by 5 lbs., 6 lbs., 7 lbs.: determine the angles which the directions of these forces make with each other.

4. A weight of 400 lbs. is supported on two props, 4 feet and 8 feet long respectively, resting at their other ends, 10 feet apart, on a horizontal plane: find the pressure on each prop, and the horizontal thrust of each on the plane.

5. Determine the weight that will be supported by a power of 40 lbs. applied to a system of pulleys of the third kind, where each string is attached to the weight, in which there are three *moveable* pulleys, each weighing 5 lbs.; giving the whole investigation, on the principle of the tension of the string applied to this particular case.

6. A power of 10 lbs. supports a weight of 19 lbs. on a plane inclined 30° to the horizon: find the angle which the direction of the power makes with the plane.

7. At what distance from each other must the threads of a screw be cut, that a power of 28 lbs., acting at the extremity of an arm 25 inches long, may press by means of the screw with the weight of 5 tons?

A.

VI. DYNAMICS.

1. A weight of 84 oz. is connected with another of 77 oz. by a string hanging over a fixed pulley: how far will the heavier descend, and what velocity will it acquire in 5 seconds?

2. A body is projected with a velocity of 40 feet per second down a plane inclined to the horizon at an angle of 30°: in what time will it describe 200 feet on the plane, and what will be its velocity at the end of that time?

3. Show that the times of descent down all chords drawn through either extremity of a vertical diameter of a circle are equal, and that the velocities acquired at the lowest point of these chords are proportional to their lengths.

4. Find the straight line of quickest descent: (1) From the circumference of a given circle to a given point within it: (2) From a given point without a circle to the circumference of the circle.

5. Deduce an expression for the time of flight of a projectile, in terms of the range, the angle of elevation of the projectile and the angle of inclination of the range plane; and thence determine the length of fuse for a range of 1200 yards on a plane rising at an angle of 5° 30′, the shell being fired at an elevation of 15°, and the fuse burning at the rate of an inch in three seconds.

6. $A = 3$ oz., $B = 7$ oz., $C = 5$ oz., are three perfectly elastic balls: after A has impinged directly with a velocity of 10 feet per second upon B, at rest, C impinges directly upon B, in the direction opposite to A's first motion, with a velocity of 12 feet per second: find the ultimate velocities of A, B, C, considering the original velocity of A as positive.

7. Find the amount by which a seconds pendulum must be shortened in order that it may keep true time at an elevation of 15000 feet.

A.

VII. HYDROSTATICS.

1. State the principle on which the pressure of a fluid upon any plane surface is determined: and the altitude of a triangular prism and each side of its base being one foot, find the pressures on its sides and ends when filled with water: (1) when the prism is placed upright on one of its ends; (2) when a rectangular face is horizontal with the opposite dihedral angle *upwards;* a cubic foot of water weighing 1000 oz.

2. The specific gravity of gold being 19·25, and of copper 8·9, what are the weights of copper and gold respectively in a compound of these metals which weighs 800 grains in air, and 750 grains in water?

3. A pontoon in the form of a right hexagonal prism floats with one of the rectangular faces horizontal; its length is 8 feet; each side of the hexagonal ends is 1 ft. 6 in.; and its weight with the portion of bridge which it supports is 1461·4 lbs.: to what depth will it be immersed in the water, a cubic foot of which weighs 62·5 lbs.? Find also the additional weight which it bears when sunk to the depth of 2 feet.

4. Describe the *Mercurial Barometer;* state on what the height of the column of mercury in it depends; what it is a measure of; and why it is different at different altitudes above the earth's surface.

5. Describe the *Condenser:* R being the volume of the receiver, and b that of the barrel of the condenser: find the density of the air in the receiver after n strokes of the piston, the density of the external air being taken as the unit.

6. A Diving-bell in the form of a cone, the diameter of the base of which is 8 feet, and the axis 10 feet, is let down into the sea until the water rises 5 feet within it: find the depth to which it was let down, and the density of the contained air.

A.

VIII. DIFFERENTIAL CALCULUS.

1. Find $\frac{du}{dx}$ in the following functions:

$u = (a^2 - x^2)(a^2 + x^2)^{\frac{3}{4}}$, $u = \log_e\{\sqrt{(a^2 + x^2)} - \sqrt{(a^2 - x^2)}\}$,
$u = e^x \sin x$.

2. If $u = \tan x$, find $\frac{d^2u}{dx^2}$; $u = x \log x$, find $\frac{d^3u}{dx^3}$;
$u = xe^x$, find $\frac{d^2u}{dx^3}$.

3. Of all cylinders inscribed in a given hemisphere whose radius is a, find that which has the greatest convex surface.

4. Find the subtangent to a curve whose equation is

$$(y - b)\sqrt{(a^2 - x^2)} = a^2.$$

INTEGRAL CALCULUS.

5. Find the following integrals:

$$\int \frac{a^3x^3\,dx}{(a^4 - x^4)^{\frac{3}{2}}}, \quad \int \frac{dx}{4 - x^2}, \quad \int \frac{x^7\,dx}{\sqrt{(1 - x^4)}}.$$

6. Find the area of a curve whose equation is

$$y^2(a^2 - x^2)^3 = a^6 x^2, \text{ from } x = 0 \text{ to } x.$$

7. Find the volume of a segment of an ellipsoid generated by the revolution of an ellipse about its major axis; and show that the volume of the whole ellipsoid is two-thirds of that of its circumscribing cylinder.

8. Show that the distance of the centre of gravity of a parabola from its vertex is $\frac{3}{5}x$.

9. By the property of Guldin, find the volume of a solid generated by the revolution of a parabola about a double ordinate $= 2b$, the corresponding abscissa or height of the parabola being a.

B.

I. GEOMETRY.

1. Give definitions of the following:—1. "a diameter of a circle;" 2. "a segment of a circle;" 3. "a square;" 4. "parallel straight lines;" 5. "the rectangle contained by two straight lines;" 6. "an angle in a segment of a circle;" 7. "a rectilineal figure described about a circle;" 8. "duplicate ratio," and "triplicate ratio;" 9. "reciprocal figures;" 10. "a dihedral angle;" 11. "a solid or polyhedral angle."

2. Parallelograms upon the same base, and between the same parallels, are &c.

3. If a straight line be divided into any two parts, the square of the whole line is equal to &c.

4. The opposite sides of a quadrilateral described about a circle are together equal to the other two opposite sides.

5. If two triangles have one angle of the one equal to one angle of the other, and the sides about the equal angles proportionals, the triangles shall be equiangular, and &c.

6. If two planes which cut one another be each of them perpendicular to a third plane, their common section shall be perpendicular to the same plane.

CO-ORDINATE GEOMETRY.

7. The line $y = ax + b$ passes through the points $(5, -7)$ and $(-3, 2)$; find the values of a and b. Construct also this line.

8. Determine the radius of the circle which passes through the points $(0, 0)$, $(1, 2)$ and $(1, -2)$; and show also that $(\frac{1}{2}, \frac{3}{2})$ is a point in the same circle.

9. Investigate the equation in general of the conic sections, and show that such equation represents *three* distinct curves.

B.

II. ARITHMETIC AND ALGEBRA.

1. If in quick marching, 110 steps are performed every minute, and each step is $30\frac{1}{2}$ inches in length, what is the rate, in miles, of marching per hour?

2. Find the value of the expression:

$$\frac{1}{x(x-a)(x-b)}+\frac{1}{a(a-x)(a-b)}+\frac{1}{b(b-x)(b-a)}.$$

3. Solve the following equations:

$$a(x-y)=b(x+y),\ x^2-y^2=c^2,$$

$$\sqrt{(3+\sqrt{x})}+\sqrt{(4-\sqrt{x})}-\sqrt{(7+2\sqrt{x})}=0.$$

4. Two railway trains are dispatched from a station, along two different lines, the one starting an hour before the other. As the rate of motion of the later train is 5 miles an hour more than that of the other, it arrives at a distance of 100 miles from the station at the same hour of the day in which the earlier one completes an equal distance. Find their rates of motion.

5. Find the least root of the equation

$$x^2-13{\cdot}572x+24{\cdot}7148=0$$

by the method of continuous approximation, and also by the common method.

6. Investigate a formula for the sum of n terms of an arithmetical series, a being the first term, and d the common difference of the terms. Sum also to 25 terms, the series $18+\frac{33}{2}+15+$ &c.

7. Find an expression for the number of shot in a rectangular pile of n courses; $m+1$ being the number of shot in the top row. Determine also the number of balls in the length and breadth of the lowest course of an *incomplete* rectangular pile of 25 courses; the number of balls in the length and breadth of the top course being, respectively, 24 and 18 balls.

8. Develope a^x in a series of ascending powers of x; and thence deduce the base of the Napierian system of logarithms.

B.

III. TRIGONOMETRY AND MENSURATION.

1. Trace the changes of algebraic sign of the trigonometrical functions *sine* and *cosine* of an arc, for the first four quadrants of the circle; and explain why sec A and sec$(180+A)$, which *coincide*, should be one *positive* and the other *negative*. Prove also that

$$\sec A = \frac{1}{\cos A}, \quad \tan(180-A) = -\tan A.$$

2. Prove the relations:

$$\cos(A+B) = \cos A \cos B - \sin A \sin B,$$
$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A,$$
$$2\sin A \cos B = \sin(A+B) + \sin(A-B).$$

3. Given two sides of a triangle, 200 feet and 250 feet, and their included angle $52^\circ\ 23'$: find the third side and the remaining angles.

4. An object B on the summit of a hill, and two stations C and D are in the same vertical plane: at C the angle of elevation of B above the horizontal plane is $50^\circ\ 18'\ 20''$; at D the angle of elevation of B is $30^\circ\ 16'\ 12''$; the distance $CD = 124$ yards; and the line DC rises from D towards C at an angle of $4^\circ\ 12'$: find the distance of B from D, and its vertical height above that station.

5. Deduce an expression for the area of a triangle in terms of two of its sides a, b and their included angle C.

6. The sides of a plane triangle are 6, $6+\sqrt{(2)}$ and $6-\sqrt{2}$; find its area.

7. The area of a regular polygon of n sides, whose side $=a$, is

$$n \cdot \frac{a^2}{4} \cdot \cot\frac{180}{n}.$$

8. The convex surface of a cylinder inscribed in a sphere is one-third the surface of the sphere: find the radius and altitude of the cylinder in terms of r, the radius of the sphere; and show generally, that if the convex surface of the cylinder is one n^{th} of that of the sphere, the radius of the cylinder, is $\dfrac{\sqrt{(n+2)} \pm \sqrt{(n-2)}}{2\sqrt{n}} \cdot r$.

B.

IV. SPHERICAL TRIGONOMETRY.

1. Define the following:—"a great circle of the sphere;" "a small circle;" "the poles of a circle;" "a spherical angle;" "a spherical triangle;" "the polar triangle."

2. Show that the sides of the polar triangle are the supplements of the angles of the primary triangle; and the angles of the polar triangle are the supplements of the sides of the primary triangle.

3. If a, b, c are the sides opposite to the angles A, B, C of a spherical triangle, show that

$$\cos c = \cos a \cos b + \sin a \sin b \cos C.$$

4. Show that, if $s = \frac{1}{2}(a + b + c)$, $\tan \frac{1}{2} A$

$$= \sqrt{\frac{\sin (s - b) \sin (s - c)}{\sin s \sin (s - a)}};$$

and thence that, $\tan \frac{1}{2}(A + B) = \frac{\cos \frac{1}{2}(a - b)}{\cos \frac{1}{2}(a + b)} \cot \frac{1}{2} C$.

5. The base of a right-angled spherical triangle is $57^\circ 19'$, and the adjacent acute angle $31^\circ 53'$: find the remaining parts of the triangle by Napier's Rules.

ASTRONOMY.

6. Define the following terms:—"Zenith;" "Meridian;" "Prime vertical;" "Azimuth;" "Hour circles;" "Right Ascension and Declination;" "Latitude and Longitude of a heavenly body;" "Latitude and Longitude of a place on the earth."

7. Point out by means of a figure how the time from apparent noon is determined from the observed altitude of the sun on a given day, knowing the latitude of the place of observation.

8. Find the latitude of a place in terms of h, the observed time of the sun being on the prime vertical, and δ the north polar *distance of* the sun.

B.

V. STATICS.

1. Two forces represented by 5 lbs. and 7 lbs. act at a point, in directions making an angle of 50° with each other: what weight will represent their resultant, and what angle will its direction make with that of the greater force?

2. Suppose that in the common gyn, the windlass or axle, round which a rope passing over a pulley fixed at the top of the gyn is coiled, is 10 inches; and the lower block in the system of pulleys (of the second kind, where the same string passes round all the sheaves) contains three sheaves, the rope being fixed to the upper block: what weight will a man support on the hook of the lower block, when applying a force of 100 lbs. on the end of a handspike, 7 feet long, placed in the windlass?

3. If in a system consisting of any number of particles a *point* be taken, and if each particle be multiplied by the square of its distance from the *point*, show that the sum of these products will be the least when the *point* is the centre of gravity of the system.

4. A beam, 8 feet long and weighing 100 lbs., is suspended from a hook, by two cords, 6 feet and 10 feet long respectively, attached to its ends: find the angle which the beam makes with the vertical when in the position of equilibrium, and also the tension of each cord.

5. AB is a beam, which with a weight W, suspended from B, is to be supported against a vertical wall VL, by a chain CD fixed to the wall: given the weight of the beam = 100 lbs., the weight $W = 300$ lbs., the length of the beam $AB = 8$ feet, the angle BAL, which the beam makes with the vertical, $= 30°$; find the distances AC, AD of the points of attachment C, D, of the chain to the wall and to the beam, from the point A, where the beam rests against the wall, so that, CD being perpendicular to AB, the whole may be in equilibrium; and determine the tension of the chain CD, and the pressure against the wall VL.

B.

VI. DYNAMICS.

1. $A = 2$ and $B = 3$ are two perfectly elastic bodies: A, moving with a velocity of 20 feet per second, impinges directly on B moving in the opposite direction with a velocity of 12 feet per second: find their directions and velocities after impact.

2. With what velocity must a body be projected vertically upwards from the base of the Monument to reach the top, 210 feet above, in 3 seconds? and what will be its velocity when it reaches the top?

3. P and Q are two equal weights, each 16 oz., connected by a string passing over a fixed pulley: what weight must be added to P that it may descend through 1 foot in 2 seconds, supposing no inertia in the string or pulley, and that $g = 32$ feet?

4. Find geometrically, the straight line of quickest descent,

(1) From a given straight line to a given point below it;

(2) From the circumference of a given circle to a given point without it, and below the highest point of the circle.

5. A body weighing 3 lbs. revolves as a conical pendulum and makes three revolutions in seven seconds; find the tension of the string, 10 feet long, by which it is held.

6. Given the velocity of projection v, the angle of elevation e; investigate the expression for the range on a plane passing through the point of projection and making a given angle i with the horizon.

7. Investigate the expression for the time of flight in terms of e, i and r the range.

8. If 1 inch of fuze burn 4·9 seconds, find the length that a fuze must be cut, in order that a shell may explode on reaching the ground at a range of 800 yards, on a horizontal plane, when fired at an angle of elevation of 45°.

9. With what velocity must a shell be fired, at an angle of elevation of 30°, to strike an object at the distance of 900 yards on a plane which rises at an angle of 5°?

B.

VII. HYDROSTATICS.

1. A hexagonal prism being filled with water is placed with one of its rectangular faces horizontal; find the pressure on each end, and also on each of the rectangular faces, each side of the hexagonal ends being 1 foot, and the length of the prism 2 feet.

2. The specific gravities of platinum, gold and silver being respectively 21, 17·5 and 10·5, and the value of an ounce of each 30*s*., 80*s*. and 5*s*. respectively, it is required to find the value of a coin composed of platinum and silver which is equal both in weight and in magnitude to a sovereign.

3. A cylindrical pontoon with hemispherical ends, 3 feet in diameter and 20 feet long in the cylindrical part, when floating has a fourth of the diameter immersed; what is the weight of the pontoon; and what additional weight does it bear when only a fourth of the diameter is above the water, a cubic foot of water weighing 62½ lbs.?

4. A diving-bell in the form of a paraboloid, the diameter of its base being 9 feet and its height 12 feet, is let down into the sea until the water rises 4 feet within it; find the depth to which it was let down, and the density of the air within it, the pressure at the surface of the sea being equal to 33 feet of sea-water.

5. Describe the air-pump, and show the degree of rarefaction of the air in the receiver after n turns of the handle, the content of each barrel being an m^{th} part of the content of the receiver.

6. A cylinder 10 feet high being filled with water, it is required to find where a small orifice must be made in its side so that the water issuing from it may strike the horizontal plane on which the cylinder stands at the distance of 8 feet from the base.

7. An oaken sphere is let fall from the surface of the sea; in what time will it strike the bottom at the depth of 20 fathoms, neglecting the resistance of the water, the specific gravity of the oak being 1·17, that of the sea-water 1·03, and $g = 32$?

B.

VIII. DIFFERENTIAL CALCULUS.

1. Find the differentials of the following functions of x:

$$u=(a-x)\sqrt{(a+x)}, \qquad u=\frac{x}{\sqrt{(a^2+x^2)}},$$

$$u=\log_e\frac{x+a}{x-a}, \qquad u=\sin^3x\cos^2x.$$

2. If u is a function of x, and u_1 represents u when, in it, x becomes $x+h$, what is the value of u_1 (Taylor's Theorem)?

3. u being a function of x, show that when u is either a maximum or a minimum, $\frac{du}{dx}=0$; that it is a maximum when $\frac{d^2u}{dx^2}$ is negative; and a minimum when $\frac{d^2u}{dx^2}$ is positive.

4. Of all cylinders which can be inscribed in a given cone whose altitude is a and the radius of whose base is b, find

(1) That which has the greatest convex surface;

(2) That whose whole surface is the greatest.

4. Find the subtangent to a curve whose equation is

$$x^2y^2=a^4-x^4.$$

6. Find the expression for the radius of curvature in the parabola and in the ellipse; and find its value at the vertex in the parabola, and at the extremities of the axes in the ellipse.

B.

INTEGRAL CALCULUS.

1. Find the following Integrals:

$$\int \frac{x^3\,dx}{(a^4-x^4)^{\frac{5}{6}}}, \quad \int \frac{(2x-5)\,dx}{x^2-5x+6}, \quad \int \frac{x^5\,dx}{\sqrt{(a^3+x^3)}}, \quad \int \frac{x^4\,dx}{\sqrt{(1-x^2)}}.$$

2. Find the area of the curve whose equation is $y\,(a-x) = x\,(a+x)$, from $x=0$ to $x=\frac{1}{2}a$.

3. Find the length of the arc of a cycloid whose equation is

$$y=\sqrt{(2ax-x^2)}+a\,\text{vers}^{-1}\frac{x}{a},$$

from $x=0$ to $x=a$, and also from $x=a$ to $x=2a$.

4. Find the volume of a solid formed by the revolution of a curve whose equation is $ay^2=(a-x)\,(a^2+x^2)$, about the axis x, from $x=0$ to $x=a$.

5. Give the expression for determining the distance of the centre of gravity in a solid of revolution; and determine the position of the centre of gravity of a spherical segment.

6. State Guldin's properties of the centre of gravity; and from these properties find the volume and the surface of the solid generated by the revolution of a semicircle, diameter $=2r$, about an axis parallel to its diameter, at the distance c from the centre.

7. Assuming that when a pendulous body vibrates in a cycloid, the force varies directly as the arc from the lowest point, find the time of vibration of such a pendulum whose length is l.

8. The length of the seconds' pendulum in the latitude of London being 39·1386 inches, determine from this the value of g, the number representing the force of gravity.

C.

I. GEOMETRY.

1. Give definitions of the following: "a right angle;" "parallel straight lines;" "an angle in a segment;" "ex æquali;" "a plane perpendicular to a plane;" "a dihedral angle;" "similar solid figures."

2. The difference of any two sides of a triangle is less than the third side.

3. The straight line drawn at right angles to the diameter of a circle from the extremity of it, falls without the circle; and no straight line can be drawn between that straight line and the circumference so as, &c.

4. The sides about the equal angles of equiangular triangles are proportionals; and those which are opposite to the equal angles are, &c.

5. If a straight line be at right angles to a plane, every plane which passes through it shall be at right angles to that plane.

6. Through two given straight lines to draw planes parallel to each other.

CO-ORDINATE GEOMETRY.

7. State clearly what is meant by the "equation of a straight line or curve;" and find the equation of the line which passes through the points $(1, 4)$ and $(-3, 7)$.

8. The equations of the sides of a triangle are

$$y = x - 2, \qquad y = 2x + 3, \qquad \text{and } y = 3x + 1\,;$$

construct this triangle, and find the co-ordinates of its angular points.

9. Investigate the equation of the circle from its geometrical definition; and find the radius of the circle denoted by the equation $x^2 + y^2 - 6x + 4y = 12$.

C.

II. ARITHMETIC AND ALGEBRA.

1. If 30 men can dig a certain trench in 11 days, how many men can dig a trench of the same sectional area, but 4 times as long, in 2 days?

2. Reduce to its simplest form the expression

$$y\sqrt{\frac{x-\sqrt{(x^2-y^2)}}{x+\sqrt{(x^2-y^2)}}};$$

and extract the square root of $23-8\sqrt{7}$ in a binomial surd.

3. Solve the equations: $\sqrt{x}+\sqrt{(2+x)}=\dfrac{4}{\sqrt{(2+x)}}$,

$$ax\sqrt[3]{x}+\frac{bx}{\sqrt[3]{x}}=c, \qquad \begin{cases} \dfrac{1}{x}+\dfrac{1}{y}=a, \\ \dfrac{1}{x}+\dfrac{1}{z}=b, \\ \dfrac{1}{y}+\dfrac{1}{z}=c. \end{cases}$$

4. If 962 men were drawn up in two solid squares, and it were found that one square had 18 ranks more than the other, what would be the strength of each square?

5. Find the position of all the roots of the equation

$$x^4-16x^3+86x^2-168x+69=0,$$

and determine the least root to four places of decimals.

6. The number of shot in a rectangular pile consisting of ten courses is equal to six times the sum of the number of shot in a triangular pile and square pile, each of the same number of courses as the rectangular pile; find the number of shot in the ridge of the rectangular pile.

7. Show that $\log_{10}n=\dfrac{1}{\log_e 10}\cdot\log_e n$; and if $\log_{10}3=\cdot4771213$ and $\log_{10}5=\cdot6989700$, find $\log_{10}6$.

8. The hypothenuse of a right-angled triangle is 10, and the excess of the perpendicular above the base is 2; find the sides of the triangle.

C.

III. TRIGONOMETRY AND MENSURATION.

1. Define the tangent, cotangent, secant, cosecant of an angle; and express each of these functions in terms of the sine. Explain also the double sign in the results; and prefix the proper sign when the angle is between 270° and 360°.

2. The sides a and b of a plane triangle, right-angled at C, are 183 and 197; find the angles and the remaining side.

3. Prove the following formulæ:

$$\frac{\cos B - \cos A}{\cos B + \cos A} = \tan \tfrac{1}{2}(A + B) \tan \tfrac{1}{2}(A - B); \quad \cot A - \tan A = 2 \cot 2A.$$

4. State the three cases for the solution of plane triangles; prove the formula $\dfrac{a+b}{a-b} = \dfrac{\cot \frac{1}{2} C}{\tan \frac{1}{2}(A - B)}$; state to what case this applies, and how.

5. Prove that in a plane triangle right-angled at C,

$$\sin 2A = \frac{2ab}{b^2 + a^2}; \quad \cos 2A = \frac{b^2 - a^2}{b^2 + a^2}.$$

6. A person on a level plain, on which stands a tower AB surmounted by a spire BC, observes that when he is 100 feet distant from the base A of the tower, B is in a line with the top of a hill, and the angle of elevation of B above the horizontal plane is 43°; but when he is 80 feet further from A, C is in a line with the top of the hill, and the elevation of C is 34°; find the height of the spire and of the hill.

7. Investigate a formula for the area of a triangle ABC in terms of the base b and the angles at the base A and C. Determine also the area of the triangle of which the base is 410 feet, and the angles at the base 30° 17′ and 50° 11′.

8. Prove that the areas of an equilateral triangle and hexagon, of equal perimeters, are to one another as 2 to 3.

Find also the absolute values of these areas when the perimeter of each is 15 inches.

C.

IV. STATICS.

1. Define the terms:—"absolutely at rest;" "relatively at rest;" "force;" "equilibrium."

2. Three ropes are fastened at one of their extremities to a ring round a post, and three men *A*, *B*, *C* pull horizontally with forces of 80 lbs., 90 lbs., 100 lbs. at the other extremities of these ropes; find the directions in which *A*, *B*, *C* must pull, that is, the angles which the ropes must make with each other, that the post may be undisturbed.

3. Forces f, f', f'', f''', &c., make angles a, a', a'', a''', &c. with the axis Ax: find the equivalent forces in the directions of the rectangular axes Ax, Ay; find also the resultant of these forces, and the angle which it makes with the axis Ax: and apply this to the case where the forces are represented by 5, 7, 2 and 9 lbs., and the angles which they make with Ax are 0°, 30°, 60°, 90°.

4. Three forces represented by 40 lbs., 60 lbs., 80 lbs. act in a vertical plane upon a point, and their respective directions make angles of 30°, 60°, 120° with the horizon; find the magnitude and direction of a fourth force that shall counterbalance their effect upon the point.

5. A power of 50 lbs. is applied by means of a single moveable pulley to the arm of a screw: what will be the pressure of the screw, the distance between the threads being one-third of an inch, and the length of the arm 12 inches?

6. If the axis of the trunnions of a gun be 2 inches below the axis of the bore, and the centre of gravity of the gun be 5 inches behind the intersection of the perpendicular from the middle of the axis of the trunnion upon the axis of the bore, with this axis; what is the greatest depression that can be given to the gun without its turning over?

C.

V. DYNAMICS.

1. State the three "Laws of Motion."

2. A shot was let fall from the top of a cliff; one second afterwards another shot was projected vertically downwards, from the same point, with a velocity of $35\frac{3}{7}$ feet; and the two were *heard* to strike the sea at the base of the cliff, at the same instant, $6\frac{1}{2}$ seconds after the first was let fall; find the height of the cliff; and, from this, the velocity with which the sound travelled.

3. A weight of 100 oz. is placed on a perfectly smooth plane inclined at an angle of 30° to the horizon, and is attached by a string passing over a pulley at the upper edge of the plane to another weight hanging freely: find what this weight must be that it may descend 27·5 feet in 5 seconds. Find also how far this weight would descend in the same time if the string passed over a pulley at the lower edge of the plane; and find the tension of the string in each case.

4. What is the length of a pendulum which vibrates three times during the time that a body descends 64·4 feet on a plane inclined 30° to the horizon?

5. Determine the equation between v the velocity of projection, e the angle of elevation, and r the range of the shot on a plane which passes through a point at the distance b feet vertically below the point of projection, and which rises at an angle i from the horizon.

6. A shot is to be fired from a battery to clear (1 foot above) a parapet, at the horizontal distance of 600 yards from, and 11 feet above the level of the battery, and then strike a point 60 feet beyond, and 5 feet below the top of the parapet: at what elevation and with what velocity must the shot be fired?

7. $A = 4$ oz., $B = 3$ oz., $C = 5$ oz., are three perfectly elastic spheres; B being at rest, A impinges directly upon it with a velocity of 14 feet; and, immediately after this impact, C impinges directly upon B with a velocity of 12 feet in the direction opposite *to that* in which A had impinged; find the velocities of A, B, C, ***after all the impacts.***

C.

VI. HYDROSTATICS.

1. A prismatic vessel, whose base is an equilateral triangle, and altitude is equal to one side of its base, is filled with fluid: compare the pressure on one of the triangular ends with that on one of the rectangular sides, when it stands upright on a triangular end. Also, when a rectangular side is horizontal, compare the pressure on one end with that on one of the inclined sides, (1) When the horizontal side is upwards; (2) When the horizontal side is downwards.

2. The specific gravities of platinum, gold and silver being respectively 21, 17·5 and 10·5, and the values of an ounce of each *30s.*, *80s.*, and *5s.* respectively, it is required to find the value of a coin composed of platinum and silver which is equal in weight and magnitude to a sovereign.

3. A cylindrical pontoon, 20 feet long and 3 feet in diameter, has a fourth of the diameter immersed when floating; what is the weight of the pontoon, and what additional weight will it just bear, a cubic foot of water weighing $62\frac{1}{2}$ lbs.?

4. Find the altitude of the roof of Severndroog Castle, Shooter's Hill, above the wharf in the Arsenal, from the following observations:—

	Barometer.	Thermometer. Attached.	Thermometer. Detached.
Arsenal	29·475 in	37	34
Severndroog Castle	29·022 in	35	32

5. A diving-bell, in the form of a rectangular prism, and whose height is 8 feet, has descended until its upper surface is 40 feet below the surface of the water: to what height will the water rise within it, and what will be the density of the contained air, that of the external air being 1, the height of the barometer 30 inches, and the density of mercury to that of water 14 to 1 nearly?

6. Describe the common suction-pump and explain its mode of action by means of a figure.

C.

VII. DIFFERENTIAL CALCULUS.

1. u and v being functions of x, show that

$$\frac{d(uv)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx};$$

and thence that, u, s, t being functions of x,

$$\frac{\frac{d(uts)}{dx}}{uts} = \frac{\frac{du}{dx}}{u} + \frac{\frac{dt}{dx}}{t} + \frac{\frac{ds}{dx}}{s}.$$

2. Find the differentials of the following functions of x:

$$u = (2a^2 + 3x^2)(a^2 - x^2)^{\frac{3}{2}}. \qquad u = \frac{x^3}{a^2 - x^2}.$$

$$u = \log_e \frac{a^2 + x^2}{a^2 - x^2}. \qquad u = \log_e \sin x.$$

3. Find the sides of the greatest rectangle that can be inscribed in a given regular hexagon, a side of the rectangle being parallel to a side of the hexagon.

4. Of all cylinders inscribed in a given cone whose altitude is a, and radius of its base b, find,

1st. That which has the greatest convex surface;
2nd. That whose whole surface is a maximum.

5. Find the subtangent to a curve whose equation is $\frac{x^2}{y} = \frac{a^2 - x^2}{y - b}$; and give the values of the subtangent corresponding to $x = \frac{1}{2}a$ and $x = a$.

6. From the variable points C and P, equidistant from a given point A in the same straight line as C and P, perpendiculars, CD, PM, to CP are drawn, of which $CD = AC$, and PM is indefinite. From a given point B, in PC produced, the straight line CDM is drawn cutting PM in M. Find the equation to the locus of the point M, and the expression for the subtangent to the curve.

C.

VIII. INTEGRAL CALCULUS.

1. Find the following integrals:

$$\int \frac{x^{n-1}dx}{(a+bx^n)^{\frac{p}{q}}}, \qquad \int \frac{x\,dx}{3+x-x^2};$$

$$\int \frac{x^2\,dx}{\sqrt{(1-x^2)}}, \qquad \int \frac{x^3\,dx}{\sqrt{(1-x^2)}}.$$

2. Find the area of the curve whose equation is

$a^4y^2=(x^2+y^2)x^4$, from $x=0$ to $x=\frac{1}{2}a$.

3. From the general expression for the volume of a solid of revolution, find an expression,

1st. For the volume of a paraboloid;

2nd. For the volume of a spherical segment; and of the whole sphere.

4. Give the differential equations of rectilinear motion when a body is acted on by any force f.

5. If a meteorolite were to fall to the earth from a height equal to ten times the radius, with what velocity would it strike the earth, the force varying inversely as the square of the distance from the centre, abstracting the resistance of the atmosphere near the surface?

6. Give the expression for determining the distance of the centre of gravity in a solid of revolution; and determine the position of the centre of gravity of a spherical segment.

7. Prove Guldin's properties of the centre of gravity; and, from these properties, find the volume and the surface of the solid generated by the revolution of a semicircle, diameter $=2r$, about an axis parallel to its diameter, at the distance c from the centre.

15—2

D.

I. GEOMETRY.

1. Give Euclid's definitions of the following :—

(1) A rectilineal figure described about a circle; (2) the same ratio, or equal ratios; (3) duplicate ratio and triplicate ratio; (4) similar rectilineal figures.

2. The difference of the angles at the base of any triangle is double the angle contained by a straight line drawn from the vertex perpendicular to the base, and another bisecting the angle at the vertex.

3. In a circle, the angle in a semicircle is a right angle; but the angle in a segment greater than a semicircle is &c.

4. If a straight line be drawn parallel to one of the sides of a triangle, it shall cut the other sides, or these produced, proportionally: and, conversely, if the sides &c.

5. If two straight lines meeting one another be parallel to two others that meet one another, and are not in the same plane with the first two, the first two and the other two shall contain equal angles.

6. *Horizontal Projection.* On a given plane, to draw a straight line to pass through a given point and to have a given inclination to the horizon, not greater than that of the given plane.

CO-ORDINATE GEOMETRY.

7. State clearly what you understand by the terms "co-ordinate axes," "co-ordinates of a point," "positive direction," "negative direction," "equation of a locus," "locus of an equation."

8. Construct the lines represented by the following equations:

$$5y - 6x + 1 = 0, \quad 3y + 8x - 10 = 0, \quad 7y = 6x;$$

and find the angles which they make with the axis of x.

9. Determine the radii of the circles

$$y^2 + x^2 - 6y - 10x - 15 = 0, \text{ and } y^2 + x^2 - 8y - 12x = 12;$$

and show also that the line which joins their centres is equally inclined to the co-ordinate axes.

D.

II. ARITHMETIC AND ALGEBRA.

1. Two persons, *A* and *B*, start from Shooter's Hill and London Bridge, distant 8 miles, at the same time, the former walking at the rate of $3\frac{1}{4}$ miles, and the latter at the rate of $3\frac{1}{2}$ miles, per hour. At what distance from Shooter's Hill will they meet? Find also this distance on the supposition that *A* was detained 20 minutes on the road before he met *B*.

2. Simplify the expression $\dfrac{1-\frac{1}{2}\left\{1-\frac{1}{3}(1-x)\right\}}{1-\frac{1}{3}\left\{1-\frac{1}{2}(1-x)\right\}}$;

and show that $\left(\dfrac{2}{2+\sqrt{2}}\right)^{\frac{1}{2}}=(2-\sqrt{2})^{\frac{1}{2}}$.

3. Solve the following equations: $\sqrt{(5x+10)}=\sqrt{5x}+2$,

$$x=\frac{2}{x+\sqrt{(2-x^2)}}+\frac{2}{x-\sqrt{(2-x^2)}}; \qquad \frac{m}{x}+\frac{n}{y}=a,\ \frac{n}{x}+\frac{m}{y}=b.$$

4. In the front of a detachment from an army were 175 more men than in the depth; and by increasing the front by 50 men, the detachment was drawn up in 20 lines. Find the number of men in the detachment.

5. The first term of an arithmetic series is *a*, the common difference *d*, and the sum of *n* terms *s*; find an expression for *s*. Find also the first term, the common difference, and the sum of *n* terms of the arithmetic series, of which the general form of the n^{th} term is $\frac{1}{5}(2n-1)$.

6. The first term of a geometric series is 5, and the ratio 2: how many terms of this series must be taken, that their sum may be equal to 33 times the sum of half that number of terms?

7. Find the value of $\dfrac{(\cdot005234)^{\frac{2}{7}}\times(\cdot017)^{\frac{1}{2}}}{(\cdot024)^{\frac{1}{3}}}$ by logarithms.

D.

III. TRIGONOMETRY AND MENSURATION.

1. Show by a figure that $\tan A = \dfrac{\sin A}{\cos A}$, and $\text{cotan}\, A = \dfrac{1}{\tan A}$.

2. Shew that $\sin 60 = \dfrac{1}{2}\sqrt{3}$; and thence find $\cos 60°$, $\tan 60°$, $\cot 60°$, $\sec 70°$, $\text{vers}\, 60°$.

3. A and B being any two angles, show that

$$\text{cotan}\,(A \pm B) = \frac{\cot A \cot B \mp 1}{\cot B \pm \cot A}.$$

4. a, b, c being the sides of a triangle, respectively opposite the angles A, B, C, prove that $\dfrac{a+b}{a-b} = \dfrac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)}$; state to what case in the solution of plane triangles this is applicable, and point out how it is applied.

5. The three sides of a triangle are 300, 500, 700; find each of its angles without employing logarithms.

6. Wanting to know the distance between two church towers B and H, standing on a horizontal plane, I measured a base of 880 yards on the same plane, from the extremities A, C of which, I took the following horizontal angles, viz. from the station A, the angles between the other station C and the towers B and H were respectively $91° 13'$ and $67° 17'$; from the station C the angles between the other station A and the towers B and H were respectively $79° 24'$ and $99° 47'$; find the distance between the two churches B and H.

7. The three sides of a triangle are 6, $6+\sqrt{2}$, $6-\sqrt{2}$; find its area.

8. Show that the area of a regular hexagon of which a is the side is $\dfrac{3\sqrt{3}}{2}a^2$: and the distance between two parallel sides of a regular hexagon being 20 yards, find its area and also that of its circumscribing circle.

D.

IV. SPHERICAL TRIGONOMETRY.

1. Define the following terms:—"diametral plane," and "tangent plane, of the sphere;" "side of a spherical triangle;" the "spherical excess;" "quadrantal triangle."

2. State Napier's Rules for the solution of a right-angled spherical triangle; and prove that if a, b, c be the sides of a spherical triangle, right-angled at B,

$$\cos b = \cos a \cos c.$$

3. Given the side $a = 38^\circ 17'$, the angle $A = 50^\circ 16'$, and the angle $B = 90^\circ$, of a spherical triangle ABC, to find the remaining parts by Napier's Rules.

4. Show that in any spherical triangle, if a, b, c be the sides, A, B, C their opposite angles, and $s = \frac{1}{2}(a + b + c)$,

$$\sin^2 \frac{1}{2} A = \frac{\sin(s-b)\sin(s-c)}{\sin b \sin c}, \quad \cos^2 \frac{1}{2} A = \frac{\sin s \sin(s-a)}{\sin b \sin c}.$$

5. The two sides a and b of a spherical triangle ABC are 108° and $99^\circ 21'$, and the angle $A = 120^\circ$; find the angle B. Give also the equations in a logarithmic form, for the determination of the remaining parts of the triangle.

ASTRONOMY.

6. Define the following terms: "The Poles and Equator of the Earth," "The Poles and Equator of the Heavens," "The Declination, and Right Ascension of a heavenly body," "The Latitude and Longitude of a place on the Earth."

7. Show by a figure that the Latitude of a place on the earth is equal to the altitude of the pole above the horizon.

8. State how the Latitude of a place is determined by observations of a Circumpolar star.

9. State the principles upon which the determination of the longitude of a place depends.

D.

V. STATICS.

1. Give definitions of the following: 1. the "resultant" of two or more forces; 2. the "components" of a force; 3. the "centre of gravity;" 4. the "lever;" 5. the "inclined plane;" 6. the "pulley."

2. A force of 30 lbs. in a direction making an angle of 60° with the axis of x is the resultant of two forces which make angles of 45° and 120° respectively with the same axis; find the magnitudes of these forces.

3. A plane figure, consisting of a square with an equilateral triangle described upon one of its sides, is placed with its plane vertical and an angle of the square resting upon a horizontal plane; find the angle which a side of the square makes with that plane when the figure just balances on the point of support.

4. Spheres of which the weights are 4 oz., 6 oz., 5 oz., 7 oz., are placed with their centres at the angular points A, B, C, D of a trapezoid of which AB and DC are the parallel sides; $AB = 10$ inches, $BC = 4$ inches, $CD = 7$ inches, and $DA = 5$ inches: find the position of their centre of gravity—

1st. When the bodies are supposed to be connected by inflexible rods without weight;

2nd. When the connecting rods are of uniform density, and weigh $\frac{1}{4}$ oz. per inch.

5. If two forces acting upon the arms of a lever keep it at rest, they are to each other inversely as the perpendiculars drawn from the fulcrum upon their directions: prove this.

6. A circular plate of metal, weighing 10 lbs., supported by a hook at the point A in its circumference, has a weight of 50 lbs. suspended from the point B diametrically opposite to A: with what force must the point D at the upper extremity of the diameter DE at right angles to AB be pressed vertically upwards, that the diameter AB shall incline downwards at an angle of 30° to the horizon?

D.

VI. Dynamics.

1. Define the terms "inertia;" "velocity;" "momentum;" "accelerating force."

2. A ship's apparent course is N.W. by W., 8 knots an hour, and the tide sets her W.S.W. at the rate of 3 knots an hour: what is her true course? and what her rate of progress in that course?

3. A shot is fired from the ground vertically upwards with a velocity of 322 feet per second; and 6 seconds after, another shot is fired, with a velocity of 1288 feet per second, vertically upwards from the same spot: at what distance from the ground will the second shot pass the first?

4. A weight of 120 oz. is placed on a smooth horizontal plane, and is attached by a string to a weight of 41 oz. hanging vertically over the edge of the plane; find how far this weight will descend in 5 seconds, the velocity it will acquire in that time, and the tension of the string,

1st. When the friction is neglected;

2nd. When the friction is equal to one-eighth of the weight on the plane.

5. From what height must the ram of a pile-driver, weighing 16 cwt., descend upon the head of a pile, that it may strike it with a momentum equal to that of a 42 lb. shot fired with a velocity 1610 feet per second?

6. A railway train, without locomotive engine, descends, from rest, a mile or 1760 yards along an inclined plane which falls 1 foot in 400 feet, and then ascends an inclined plane of 1 foot in 600 feet: how far will it ascend along the latter plane, supposing no motion to be lost by friction or in the transfer from one plane to the other?

7. With what velocity must a shell be fired, at an angle of elevation of 45°, from a battery on a cliff 400 feet above the level of the sea, to strike a ship at the horizontal distance of 2 miles from the base of the cliff?

D.

VII. DIFFERENTIAL AND INTEGRAL CALCULUS.

1. u and v being functions of x, show that

$$d\frac{u}{v}=\frac{vdu-udv}{v^2} \text{ or } \frac{d\,.\,\frac{u}{v}}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}.$$

2. u being a function of x, find $\frac{du}{dx}$, when

$$u=\frac{(1-x^2)^3}{1+x^2},\qquad u=\frac{1+x}{\surd(1-x^2)},\qquad u=\log_e\frac{x-\surd(1-x)^2}{x+\surd(1-x^2)},$$

$$u=3\sin^{-1}x-(2x^3+3x)\surd(1-x^2).$$

3. Of all cylinders which can be inscribed in a given cone whose altitude is a and the radius of whose base is b, find

1st. That which has the greatest convex surface;
2nd. That whose whole surface is the greatest.

4. Find the subtangent to a curve whose equation is

$$x^2y^2=a^4-x^4.$$

5. Find the following integrals:

$$\int\frac{x^{n-1}dx}{(a+bx^n)^{\frac{p}{q}}},\qquad \int\frac{xdx}{3+x-x^2},\qquad \int\frac{x^2dx}{\surd(1-x^2)},\qquad \int\frac{x^3dx}{\surd(1-x^2)}.$$

6. Show, by means of Taylor's Theorem, that the area of a curve is $\int ydx$.

7. Find the area of the curve whose equation is

$$a^4y^2=(x^2+y^2)\,x^4,\text{ from } x=0 \text{ to } x=\tfrac{1}{2}a.$$

8. From the general expression for the volume of a solid of revolution, find an expression,

1st. For the volume of a paraboloid;
2nd. For the volume of a spherical segment, and of the whole sphere.

E.

I. GEOMETRY.

1. If a straight line falls upon two parallel straight lines, it makes, &c.

2. Deduction. Show that the square of a straight line drawn from the vertical angle of any triangle to the middle of the base, together with the square of half the base, is equal to half the sum of the squares of the other two sides of the triangle.

3. If from any point without a circle two straight lines be drawn, one of which cuts the circle, and the other touches it; the rectangle contained, &c.

Prove the case in which the line that cuts the circle does not pass through the centre.

4. Triangles of the same altitude are to one another as their bases.

5. Give the definitions of the following:

(1) A straight line at right angles to a plane; (2) the inclination of a plane to a plane; (3) a solid angle; (4) similar solid figures; (5) a pyramid; (6) a parallelepiped.

6. If two planes which cut one another be each of them perpendicular to a third plane, their common section shall be perpendicular to the same plane.

V. CO-ORDINATE GEOMETRY.

7. Show clearly by means of a figure, that if b be constant and a admit of different values, the equation $y = ax + b$, represents a series of straight lines all passing through a given point, but that if a be constant and b admit of different values, it denotes a series of lines all parallel.

8. Construct the circles $x^2 + y^2 - 10x + 2y - 95 = 0$, and

$$x^2 + y^2 + 4x - 2y - 4 = 0,$$

and thence find the distance between their centres.

E.

II. ARITHMETIC AND ALGEBRA.

1. Define the terms "fraction," "decimal," "power," "index," "logarithm," and "modulus."

2. A person after paying 5 per cent. on his income had £800 left; determine his income. Find also the amount of income-tax, at sevenpence in the pound, on the sum left.

3. Two sets of men, consisting of 12 men in one set, and 17 in the other, are employed to work in succession in removing goods from a building. They are to receive £42 among them. The first set work 11 days and the second 12 days; find the sum that each man receives. Divide also the same sum amongst them when the first set work $8\frac{1}{2}$ hours each day, and the second $10\frac{1}{2}$ hours, the number of days being the same as before.

4. Solve the following equations:

$$x^2 = 21 + (x^2 - 9)^{\frac{1}{2}}, \qquad \left\{\begin{array}{l} xyz = 105 \\ 35x = 3yz \\ 7xy = 15z \end{array}\right\}.$$

$$\left\{\begin{array}{l} x^2 - xy = 153 \\ x + y = 1 \end{array}\right\},$$

5. The soldiers in the front of a column were to the number of ranks as 9 to 2, and the whole column consisted of 882 men. Find the number of men in front, and also the number of ranks.

6. Form the equation of which the roots are

$$2 + \sqrt{3},\ 2 - \sqrt{3},\ -3 + \sqrt{(-1)},\ -3 - \sqrt{(-1)}.$$

7. Sum to n terms, the series, $s_1 + s_2 + s_3 +$ etc., in which

$s_1 = 1 + 2 + 2^2 + 2^3 + \ldots\ldots$ to n terms,

$s_2 = 1 + 2 + 2^2 + 2^3 + \ldots\ldots$ to $(n - 1)$ terms, etc.

8. Decompose the fraction $\dfrac{4x^3 - 5x^2 - 9x - 14}{x^4 - 4x^3 - 7x^2 + 34x - 24}$ into fractions having denominators of the first degree.

9. Expand $\dfrac{b}{a}(x^2 - a^2)^{\frac{1}{2}}$ to five terms by the Binomial Theorem.

E.

III. TRIGONOMETRY AND MENSURATION.

1. Define the terms "sine," "cosine," "tangent," "cotangent," and "secant," of an angle A; and thence point out the method of connecting these expressions with the sides of a plane triangle, right-angled at C.

2. Prove the formulæ

$$2\cos^2\frac{1}{2}A = 1 + \cos A\,; \quad \frac{\sin A + \sin B}{\sin A - \sin B} = \frac{\tan\frac{1}{2}(A+B)}{\tan\frac{1}{2}(A-B)}.$$

3. The side a of a plane triangle is 17 feet, the angles B and C, 50° 16′ 25″ and 90° respectively; find the other parts of the triangle.

4. a, b, c being the sides opposite to the angles A, B, C of a plane triangle, show that

$$\frac{a}{b} = \frac{\sin A}{\sin B}\,; \quad \frac{a}{c} = \frac{\sin A}{\sin C}\,; \quad \frac{b}{c} = \frac{\sin B}{\sin C}\,;$$

explain also the method of using these expressions in the solution of a plane triangle; and point out the ambiguous case.

5. Show that in any plane triangle, $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$.

6. From a station on the bank of a river a person ascends, at right angles to the bank, 65 yards up a slope inclined at an angle of 16° to the horizon, when he observes the angle of depression of an object on the opposite bank and in the same plane as his first station to be 2° 50′. Find the breadth of the river.

7. The four sides of a trapezium are, $AB = 500$ yards, $BC = 400$ yards, $CD = 700$ yards, $DA = 300$ yards, and the diagonal $BD = 600$ yards; find its area.

8. The base of a stone pyramid is a regular hexagon of which each side is 10 feet, and each of the slant *edges* of the pyramid is 50 feet: find its weight, a cubic foot of the stone weighing 165 lbs.: and find also the number of square feet in its slant surface.

E.

IV. STATICS.

1. Prove that, "If two forces acting at any angles on the arms of any lever, are inversely as the perpendiculars from the fulcrum upon their directions, they will balance each other."

2. A weight of 20 lbs. is suspended to a string fixed at two points A, B, 10 inches apart, in the same horizontal line, the distances of the point of suspension, C, of the weight from the ends of the string being, $AC=6$ and $BC=8$ inches: what is the tension of each of the portions, AC and BC, of the string, and what are the horizontal and vertical forces on the fixed points A and B?

3. In the first system of pulleys, where each pulley hangs by a separate string and the strings are parallel, investigate the relation of P to W, when there are three moveable pulleys, the weight of each of which is A; and find what power will support 2 cwt. by means of such a system in which each pulley weighs 8 lbs.

4. If P_1, P_2, P_3, P_4, are bodies, considered as material points, in the same straight line as a given point A, and G is their centre of gravity, show that

$$AG=\frac{P_1 . AP_1+P_2 . AP_2+P_3 . AP_3+P_4 . AP_4}{P_1+P_2+P_3+P_4}.$$

5. A gun with its carriage, weighing 40 cwt., is supported on a plane, inclined at an angle of 15° to the horizon, by means of a rope fixed to the middle of the axletree, and passing thence, parallel to the plane, to the axle of a capstan at the top of the plane; there are four bars to the capstan, on each of which a man presses at the distance of 8 feet from the axis, the axle being 18 inches in diameter: what is the pressure exerted by each man?

6. Four equal beams, each weighing 10 lbs., are suspended as a funicular polygon, from two points in the same horizontal line; the two upper beams make angles of 50°, and the two lower, angles of 30° with the horizon: find the weight suspended at the *lowest point* when the system is in equilibrium.

E.

V. DYNAMICS.

1. Two steamers are moving in parallel but opposite directions; the first with an uniform velocity of 9 miles an hour, the second with an uniform velocity of 15 miles an hour. A shot is fired from the first directly at the paddle-wheel of the second, at the instant when it is exactly opposite to the gun. Supposing the distance between the vessels to be 1200 yards, and the velocity of the shot, 1200 feet, to continue uniform, how far abaft the paddle-wheel will the shot pass?

2. On an inclined plane on a railway to a slate-quarry, the loaded waggons descending on one line are made to draw up the empty ones on the other, by means of a rope passing round a wheel at the top of the inclined plane. Suppose that the length of the plane is 400 yards, its height 100 feet; the weight of a train of empty waggons 10 tons, and of their load 70 tons; in what time will a train descend the plane; what velocity will it acquire at the bottom; and what will be the tension of the rope during the descent?

3. At what elevation and with what velocity must a shot be fired, that after clearing (1 foot above) a rampart at the horizontal distance of 2000 feet from the point of projection, and 15 feet above its level, it may strike a gun 50 feet beyond the rampart and 8 feet below its top?

4. $A = 20$, $B = 11$, $C = 4\frac{1}{2}$, are three glass spheres of which the elasticity is $\frac{15}{16}$; after A with a velocity of 24 feet per second has impinged directly on B at rest, C impinges directly on B with a velocity of 18 feet in a direction opposite to A's motion: find the final velocities of the bodies.

5. Two clocks A and B are each constructed to beat seconds, but A gains 10 seconds in a mean solar day, and B makes only 6000 beats while A makes 6001: how must their pendulums be altered that the clocks may show true time?

E.

VI. HYDROSTATICS.

1. A sluice-gate in the form of an equilateral triangle, each side of which is 4 feet, is placed with the base downwards, and turns upon an axis parallel to the base, the part below the axis being prevented turning in the direction of the pressure, but the upper part being free: at what height above the base must the axis be placed that the gate may open when the water rises more than 6 feet above the vertex?

2. The specific gravity of gold being 19·25, of platinum 20·00, and of silver 10·5; how much platinum and silver may be mixed with 10 oz. of the gold, that the weight of the compound may be 20 oz., and its specific gravity the same as that of the gold?

3. Supposing a pontoon to be made in the form of an equilateral triangular prism; that its length is 20 feet, each side of the triangular ends is 3 feet, and its weight with the portion of the bridge it supports is 750 lbs.: a cubic foot of water weighing 62·5 lbs., find the depth to which it will sink when an additional weight of 3000 lbs. is placed on it:—

1st. When the bridge rests upon one of the rectangular sides of the prism;

2nd. When the bridge rests upon the ridge formed by two of the sides.

4. Describe Nicholson's Hydrometer, and point out how the specific gravities of solids and of fluids are determined by means of this instrument, giving the formulæ for the specific gravities.

5. State how "Fahrenheit's," the "Centigrade" and "Reaumur's" scales on Thermometers are determined; and F^0, C^0, R^0 indicating any, the same, absolute temperature on these scales, give the relations of F, C, R.

6. Describe the principle and mode of action of the "Bramah *Press.*"

E.

VII. DIFFERENTIAL CALCULUS.

1. Find the differentials of the following functions:

$$u = \frac{1+x^2}{1-x^2}, \qquad u = \frac{x}{\sqrt{(1-x^2)}},$$

$$u = \log_e \{x + \sqrt{(x^2-1)}\}, \quad u = x^n e^x.$$

2. u being a function of x, show that when u is either a maximum or a minimum, $\frac{du}{dx} = 0$; that it is a maximum when $\frac{d^2u}{dx^2}$ is negative, and a minimum when $\frac{d^2u}{dx^2}$ is positive.

3. Find the base and altitude of the greatest cone which can be inscribed in a given sphere whose radius is r.

4. AB is a horizontal line and BV is vertical: find the distance BC, so that CA being drawn to the given point A, the time of a heavy body descending down CA shall be less than the time down any other line drawn from BV to the point A. (To be done analytically.)

5. Find the value of the subtangent and of the subnormal in the witch, whose equation is $xy^2 = a^2(a-x)$.

6. Find the expression for the radius of curvature at any point in a parabola; and determine its value at the vertex of the parabola, and also at the extremity of the focal ordinate.

INTEGRAL CALCULUS.

7. Find the following integrals:

$$\int x^2 \sqrt[3]{(a^3-x^3)^2}\, dx,$$

$$\int \frac{20dx}{x^3-7x+6}, \text{ by resolving the fraction,}$$

$$\int x^3 \sqrt{(1-x^2)}\, dx, \text{ integrating by parts.}$$

E.

VIII. INTEGRAL CALCULUS.

1. Show, by Taylor's Theorem, that in all variable motions, $v = \dfrac{ds}{dt}$.

2. When a body vibrates in a cycloid, the force in the direction of the tangent varies as the arc to be described to the lowest point, and is equal to g when this arc is equal to l, the length of the semi-cycloid or twice the diameter of the generating circle: find the time of vibration in a cycloidal arc when the length of the commencing arc, measured from the lowest point, is a; and point out on what supposition and in what manner this is applicable to the vibrations in circular arcs.

3. By a change of temperature, the length of the pendulum of a clock which had kept mean time, beating seconds, was increased ·0114 inch, the length of the seconds' pendulum being 39·1386 inches: how was the rate of the clock affected?

4. Give the expressions for the distance of the centre of gravity, on the axis x, for a curve line; for the area of a curve; for the volume of a solid of revolution and for its surface; and find the distance of the centre of gravity of a semicircle from its centre.

5. Show that the moment of inertia of a plane surface revolving in its own plane, about the origin of the rectangular co-ordinates, is

$$\mu \int \left(\frac{y^3}{3} + x^2 y \right) dx,$$

where μ is the mass of the unit of area.

6. Find the radius of gyration of a parabola revolving in its own plane, about an axis passing through its vertex.

F.

I. GEOMETRY.

1. If $ABCD$ is a parallelogram, and E a point without it: if EA, EB, EC, ED, BD be drawn, show that

$$\Delta EBC = \Delta ABE + \Delta EBD,$$
$$\Delta EAD = \Delta EDC + \Delta EBD.$$

2. If in a circle, a point be taken in a straight line which is not a diameter, dividing it into two segments, and a straight line be drawn from this point to the centre, the square of this line together with the segments of the former will be equal to the square of the radius of the circle.

3. Similar triangles are to one another in the duplicate ratio of their homologous sides.

4. If two straight lines be at right angles to the same plane, they shall be parallel to one another.

ANALYTICAL GEOMETRY.

5. Find the equation to a straight line passing through a point whose co-ordinates are $x' = 3$, $y' = 4$, and perpendicular to a straight line whose equation is $y = 2x - 5$; and construct these lines to a scale by means of their equations, showing by the figure, that the second line is perpendicular to the first and passes through the given point.

6. Find the co-ordinates to the point of intersection of two straight lines whose equations are

$$y = 3x + 7, \text{ and } y = 5x - 9;$$

and construct these lines to a scale by their equations, showing by the figure that their intersection is in the point determined.

7. Find the equation to the parabola referred to rectangular co-ordinates, of which the origin is the middle point of the perpendicular from the focus on the directrix.

8. Find the equation to the ellipse referred to rectangular co-ordinates, as in the parabola (quest. 7).

18—2

F.

II. ARITHMETIC AND ALGEBRA.

1. Give definitions of the terms: "simple quantity," "compound quantity," "binomial quantity," "index," "root," and "co-efficient."

2. If limestone be composed of 28 parts of lime to 22 parts of carbonic acid, and the process of burning drive off the carbonic acid; to what weight will 1 ton 11 cwt. of limestone be reduced by burning?

3. Explain briefly the methods of solving simultaneous equations of the first degree.

4. Solve the following equations:

$$\left.\begin{array}{r} x + y = axy \\ x^2 + y^2 = b^2x^2y^2 \end{array}\right\}, \qquad 8^{2x} = 5 \, . \, 8^x - 6.$$

5. Find all the roots of the equation

$$x^3 - 2x^2 - 2{\cdot}99x + 5{\cdot}61 = 0.$$

6. The number of shot in a square pile of n courses being $\dfrac{(2n+1)(n+1) \, . \, n}{3 \, . \, 2 \, . \, 1}$, show how the formula for the number of shot in a rectangular pile of n courses, and having $m+1$ shot in the top row, is obtained: and compute the number of shot in an incomplete pile of 20 courses, having 30 shot in the longer, and 18 shot in the shorter side of its upper course.

7. Prove that in the expansion of $(1+z)^n$ the second term is nz whether n be an integer or a fraction, positive or negative.

8. From the equation between a number and its logarithm to the base a, show that $\log_a (N \times N_1) = \log_a N + \log_a N_1$,

$$\log_a \frac{N}{N_1} = \log_a N - \log_a N_1, \qquad \log_a N^m = m \log_a N.$$

9. $\text{Log}_{10} 731$ being $2{\cdot}8639174$, prove that

$$\log_{10} {\cdot}00731 = \bar{3}{\cdot}8639174, \text{ and } \log_{10} 731000 = 5{\cdot}8639174.$$

F.

III. TRIGONOMETRY AND MENSURATION.

1. Define the terms "complement," "supplement," "secant," and "cosecant" of an arc or angle; and prove that

$$\cot a = \frac{\cos a}{\sin a}, \quad \sin (90^\circ + a) = \cos a, \quad \sin 30^\circ = \frac{1}{2}.$$

2. Solve the right-angled triangle ABC, of which B is the right angle, and $a = 173{\cdot}4$ feet, $C = 20^\circ\, 17'\, 12''$.

3. Prove that $\sin A - \sin B = 2 \cos \frac{1}{2} (A + B) \sin \frac{1}{2} (A - B)$.

4. Show that the sides of a plane triangle are to one another as the sines of their opposite angles. Also solve the triangle of which $a = 156$, $b = 127$, and $B = 50^\circ$.

5. Two objects A and B subtend an angle of $50^\circ\, 18'\, 20''$ at a station S. Given the distances $SA = 1784$ yards, $SB = 1692$ yards, to find the horizontal distance between the objects: (1) when A, B and the eye (S) of the observer are in the same horizontal plane; (2) when A and S are in a horizontal plane and B 120 feet above it.

6. In an isosceles triangle in which $a = b$, prove that

$$\cos A = \frac{c}{2a}, \text{ and vers } C = \frac{c^2}{2a^2}.$$

7. Investigate the formula for the area of a plane triangle when two sides and the included angle are given; and thence find the area of the triangle in which

$$b = 124 \text{ feet}, \quad c = 161{\cdot}4 \text{ feet}, \quad \text{and } A = 71^\circ\, 12'.$$

8. Assuming that the area of a circle is equal to half the product of the radius and circumference, it is required to deduce the particular form, (1) in terms of the radius, (2) in terms of the diameter, (3) in terms of the circumference. State also, generally, how the expression for the area enunciated above, is obtained by means of the inscribed and circumscribed polygons.

F.

IV. SPHERICAL TRIGONOMETRY.

1. State Napier's rules for the solution of right-angled spherical triangles, and point out how they are applicable to each case.

2. The angles of a spherical triangle being A, B, C, and the sides opposite to them α, β, γ, show from the equations

$$\sin^2\frac{1}{2}A = \frac{\sin(\sigma-\beta)\sin(\sigma-\gamma)}{\sin\beta\sin\gamma} \text{ and } \cos^2\frac{1}{2}A = \frac{\sin\sigma\sin(\sigma-\alpha)}{\sin\beta\sin\gamma},$$

that $$\sin\frac{1}{2}(A+B) = \frac{\cos\frac{1}{2}(\alpha-\beta)}{\cos\frac{1}{2}\gamma}.\cos\frac{1}{2}C \text{ and } \sin\frac{1}{2}(A-B)$$

$$= \frac{\sin\frac{1}{2}(\alpha-\beta)}{\sin\frac{1}{2}\gamma}\cos\frac{1}{2}C.$$

3. Deduce Napier's first and second analogies, and state to what cases in the solution of spherical triangles they are applicable.

4. State what is understood by the spherical excess; and E'' being the spherical excess in a triangle of which the area in feet is n, and r the radius of the Earth in feet, show that

$$E'' = \frac{180.60.60n}{\pi r^2}.$$

5. The radius of the Earth being 3954 miles nearly, find the spherical excess in a triangle whose sides are respectively 40, 50, 60 miles.

ASTRONOMY.

6. Define the following terms: pole of the heavens, meridian, horizon, prime vertical, right ascension, declination, parallax, sidereal year, tropical year.

7. The latitude of the place of observation being known, show how the time of the Sun's rising and setting on a given day, *and also* his azimuth at that time may be determined.

F.

V. STATICS.

1. Forces f, f', f'', f''', &c. make angles a, a', a'', a''', &c. with the axis Ax; find the equivalent forces in the directions of the rectangular axes Ax, Ay; find also the resultant of these forces, and the angle which it makes with the axis Ax: and apply this to the case where the forces are represented by 5, 7, 2 and 9 lbs., and the angles which they make with Ax are $0^\circ, 30^\circ, 60^\circ, 90^\circ$.

2. A power of 50 lbs. is applied by means of a single moveable pulley to the arm of a screw: what will be the pressure of the screw, the distance between the threads being one-third of an inch, and the length of the arm 12 inches?

3. If the weight of a gun and carriage exclusive of the wheels be 18 cwt., and the centre of gravity of this mass be at the horizontal distance of 10 inches from the axletree; what weight will each of two men support when lifting the trail at a horizontal distance of 8 feet from the axletree?

4. A uniform beam AB, 8 feet long and weighing 100 lbs., is hooked at A to a vertical wall VL, and is supported by a chain fixed to it at C and to the wall at D: $AC = 6$ feet, and each of the angles ACD, CAD is 60°; find the strain on CD, and the direction and amount of the reaction at A when a weight of 1000 lbs. is suspended from B.

5. A heavy six-pounder gun weighing 36 cwt. 3 qrs. is to be drawn up a slope of 30° inclination to the horizon by means of drag-ropes parallel to the slope: the friction being one-ninth of the perpendicular pressure on the slope; what is the least number of men that will be able to move the gun, supposing each man to draw with a power of $1\frac{1}{4}$ cwt.?

6. Two uniform beams, each 20 feet long and weighing 100 lbs., rest against each other in the form of a roof, and are supported on the top of two vertical walls, 30 feet apart, to which they are attached: find the direction and the amount of the reaction at the top of each wall, and the amount of the horizontal force tending to overturn the wall.

F.

VI. DYNAMICS.

1. *A* and *B* are two ivory balls whose elasticity is represented by seven-eighths: *A*'s diameter is 1 inch, and *B*'s is $1\frac{1}{2}$ inch, and *A*, moving with a velocity of 5 feet per second, impinges directly upon *B*, moving in the opposite direction with a velocity of 2 feet per second: in what direction and with what velocities will they move after impact?

2. From what height must the ram of a pile-driver weighing 14 cwt. descend upon the head of a pile, that it may drive it into the earth six inches; supposing the resistance of the pile to be represented by a pressure of 16 tons.

3. A shot is fired vertically upwards with a velocity of 1610 feet; find to what height it would rise if the air caused no resistance to its motion; in what time it would rise through the first mile of its ascent; and in what time it would again reach the ground.

4. Two weights, $A = 125$ oz., $B = 36$ oz., are attached to the two ends of a string; *A* being placed on a horizontal plane, the string passes over a pulley fixed at the edge of the plane so that the string may be parallel to it, and *B* hangs freely; find the space described by *A* or *B* in 4 seconds,—1st, supposing no friction on the plane; 2nd, supposing the friction to be a fifth of the weight *A*.

5. Find geometrically the plane of quickest descent: (1) from a given plane to a given point below the plane; (2) from a given point without a circle to the circumference of the circle.

6. Deduce expressions for finding the angle of elevation at which a shot is to be fired, and its velocity of projection, so that the shot may pass through two given points of which the coordinates are p, q, p_1, q_1.

7. An invariable pendulum which made 86380 vibrations in a day at one station was found to make 86423 at another. Compare the force of gravity at the two stations.

F.

VII. Differential and Integral Calculus.

1. If $v=f(x)$, $z=\phi(x)$ and $u=vz$, show that

$$\frac{du}{dx}=z\frac{dv}{dx}+v\frac{dz}{dx}.$$

2. Find $\frac{du}{dx}$ when

$$u=\frac{\sqrt{(1+x^2)}}{1-x},\quad u=\log_e\frac{\sqrt{(1+x^2)}-1}{x},\quad u=\sin^{-1}\frac{x}{\sqrt{(1+x^2)}}.$$

3. Investigate Maclaurin's Theorem; and expand $\sec x$, in a series terminating at the term which involves the fifth power of x.

4. Find the greatest cylinder that can be inscribed in a given cone.

5. Show that in any curve, the value of the subtangent is

$$-\frac{y}{\frac{dy}{dx}}.$$

6. If R be the area of a curve referred to rectangular co-ordinates x and y, show that $\frac{dR}{dx}=y$.

7. Deduce the differential expression for the radius of curvature of a curve; and find the co-ordinates of the centre of the circle of curvature.

8. Find the value of u in the following differential equations:

$$\frac{du}{dx}=\frac{5ax}{\sqrt{(a^2-x^2)}},\quad \frac{du}{dx}=\frac{bx^4}{(a-x)^3},\quad \frac{du}{dx}=\frac{2x-17}{3x^2-7x-6}.$$

9. Find the volume, from $x=0$ to $x=a$, of a solid generated by the revolution, about the axis x, of a curve whose equation is

$$x^3+(a+x)y^2-a^3=0.$$

G.

I. GEOMETRY.

1. If a side of a triangle be produced, the exterior angle is equal, &c.

2. In an isosceles triangle, if a straight line be drawn from the vertical angle to a point in the base, the square of this line together with the rectangle of the segments of the base is equal to the square of one of the equal sides of the triangle.

3. The diameter is the greatest straight line in a circle; and of all others, that which is nearer to the centre, &c.

4. The sides about the equal angles of equiangular triangles are proportionals; and those, &c.

5. If two parallel planes be cut by another plane, their common sections with it are parallels.

6. The scale of slope of a plane being given, to find the index of a point in the plane when its projection is given.

ANALYTICAL GEOMETRY.

7. The equations of two straight lines are

$$y = ax + b, \text{ and } y = ax + \beta;$$

investigate the relation between a and a in order that the lines may be perpendicular to one another.

Find also the point of intersection, and contained angle, of the lines $5y - 6x + 10 = 0$, $3y + 2x - 12 = 0$.

8. Prove that if p and q be the parts of the axes of x and y, between a line and the origin, its equation will be

$$\frac{x}{p} + \frac{y}{q} = 1.$$

9. A circle passes through the points (0, 0), (0, 2) and (3, 4); find its radius, and the co-ordinates of its centre; and also the lengths of the three perpendiculars drawn from the centre on the *three lines* which join the given points.

G.

II. ARITHMETIC AND ALGEBRA.

1. To how much will amount a rate of 2*s.* 7½*d.* in the pound on an estate rated at £1536. 13*s.* 4*d.*? (*By Vulgar Fractions.*)

2. £6800 is to be divided among *A*, *B* and *C*, so that *A*'s share shall be to *B*'s as 2 to 3, and *B*'s to *C*'s as 3 to 5. (*To be done without Algebra.*)

3. A tank may be filled by one pipe discharging into it, in an hour, and by another in three quarters of an hour; in what time will it be filled if both the pipes together discharge into it?

4. Given $\sqrt{x} + \sqrt{y} = 7$ and $x^2 + y^2 = 641$; find the values of x and y.

5. A regiment being formed in the greatest solid square it would admit of, there were 23 men more than the complete square, and the same regiment being formed two deep in a hollow square, it was found that the number of men in each face of it was 94 more than in each face of the solid square: of how many men did the regiment consist?

6. Form the equation of which the roots are $\sqrt{3} + \sqrt{2}$; $\sqrt{3} - \sqrt{2}$; $-\sqrt{3} + \sqrt{2}$; $-\sqrt{3} - \sqrt{2}$: and show that these are the roots of the equation so formed.

7. Prove that the sum of the geometric series

$$a + ar + ar^2 + ar^3 + \text{\&c. to } n \text{ terms is } a\,\frac{r^n - 1}{r - 1};$$

and sum the series $5 + \sqrt[3]{25} + \sqrt[3]{5} + 1$ to 12 terms.

8. By means of the table of logarithms find the value of the fraction $\dfrac{(\cdot 5123786)^3 \sqrt{(\cdot 0031742)}}{(\cdot 0031742)^2 \sqrt[3]{(\cdot 5123786)}}$.

9. The perimeter of a right-angled triangle is 10 yards and its area is 30 square feet; find its sides.

G.

III. TRIGONOMETRY AND MENSURATION.

1. Define the terms "sine," "cosine," "tangent," and "cotangent" of an arc; and show by a figure that

$$\tan a = \frac{\sin a}{\cos a}, \quad \sec a = \frac{1}{\cos a}, \quad \tan(\pi - a) = -\tan a.$$

2. In any right-angled triangle, show that the sine of either acute angle is equal to the ratio of the opposite side to the hypothenuse, and that the tangent of the same is equal to the ratio of the opposite side to the base.

3. Prove the following formula:

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}, \quad \sec 2A = \frac{1 + \tan^2 A}{1 - \tan^2 A}.$$

4. Given $a = 10$, $b = 13$, $c = 15$, to find the angles A, B, C.

5. The horizontal distance between two vertical objects is 121 yards, and the angle which the straight line joining the tops of the objects makes with the horizon, is $15^\circ 25'$; find the distance between the tops.

6. From the bottom of a wall 50 feet high, the angle of elevation of the top of a vertical column in the same horizontal plane was $40^\circ 20' 16''$, and at the top the angle of elevation of the same was $20^\circ 16'$; find the height of the column.

7. Prove that the area of a plane triangle is equal to $\sqrt{\{s(s-a)(s-b)(s-c)\}}$, in which a, b, c are the sides opposite to the angles A, B, C, and $s = \frac{1}{2}(a + b + c)$. Find also the area of the triangle of which the sides are 17, 22, and 31 yards.

8. A log of timber is 16 feet long, 20 inches broad, and 10 inches thick. If 3 solid feet be cut from one end of it, what length will be left?

9. A cone, a hemisphere, and a cylinder have equal bases and altitudes; show that the formulæ for their solid contents are as the numbers 1, 2 and 3. Find also the content of a cone, hemisphere and cylinder, when the radius of each base is 2 feet, and the altitudes of the cone and cylinder are each 17 inches.

G.

IV. STATICS.

1. Define the terms "force," "reaction," "composition of forces," "resolution of forces," the "resultant of forces," the "components of a force."

2. A force represented by 10 lbs. is in equilibrium at a point with two other forces, of which the directions make angles of 120° and 135°, respectively, with that of the first force; find the weights which represent these forces.

3. A rifle bullet is of the form of a cylinder, of which the radius is a and length b, terminated at one end by a cone of which the radius is a and axis c; find the centre of gravity of the bullet.

4. In the second system of pulleys, where each pulley hangs by a separate string and the strings are parallel, investigate the relation of P to W, when there are three moveable pulleys, the weight of each of which is A; and find what power will support 2 cwt. by means of such a system in which each pulley weighs 8 lbs.

5. The distance between the threads of the elevating screw, in question 5, being two-thirds of an inch, and the radius of the arm by which it is turned 5 inches, what power must be applied at the arm to elevate the breech, supposing no friction on the screw?

6. A beam, 8 feet long and weighing 100 lbs., is suspended from a hook, by two cords, 6 feet and 10 feet long respectively, attached to its ends: find the angle which the beam makes with the vertical when in the position of equilibrium, and also the tension of each cord.

7. BC is a vertical wall, and FEA is a uniform beam resting against it and on a prop at E, at the distance of 4 feet from A, so as to make an angle of 30° with the horizon: find the length of the beam AF that it may be in equilibrium in this position.

G.

V. DYNAMICS.

1. $A=2$ and $B=3$ are two ivory spheres of which the elasticity is represented by seven-eighths; A, moving with a velocity of 20 feet per second, impinges directly on B, moving in the opposite direction with a velocity of 12 feet per second: find their directions and velocities after impact.

2. A body being let fall from the top of a cliff 300 feet high, it is required to find with what velocity a body must be projected vertically upwards from the base of the cliff, that it may meet the falling body half way in its descent.

3. A weight of 84 oz. is connected with another of 77 oz. by a string hanging over a fixed pulley: how far will the heavier descend, and what velocity will it acquire in 5 seconds,

1st. Supposing no inertia in the pulley;

2nd. When the inertia of the pulley is represented by 10 oz.?

4. A body is projected with a velocity of 20 feet per second down a plane inclined to the horizon at an angle of 30°: in what time will it describe 100 feet on the plane, and what will be its velocity at the end of that time, supposing $g=32$ feet?

5. Find, geometrically, the straight line of quickest descent, (1) From the circumference of a given circle to a given point without the circle; (2) From a given point within a circle to the circumference of the circle.

6. The ratio of the lengths of two pendulums is 25 to 16, and the difference of their times of vibration is two-fifths of a second: find their lengths.

7. Deduce an expression for the time of flight of a projectile, in terms of the range, the angle of elevation of the projectile and the angle of inclination of the range plane; and thence determine the length of fuze for a range of 1200 yards on a plane rising at an angle of 5° 30′, the shell being fired at an elevation of 15°, and the *fuze* burning at the rate of an inch in 3 seconds.

G.

VI. HYDROSTATICS.

1. A vessel in the form of a pyramid, having each face an equilateral triangle, the side of which is 1 foot, is filled with water, and placed with a face horizontal; find the pressure on each face, (1) when the vertex of the pyramid is upwards, (2) when the vertex is downwards.

2. A rectangular sluice-gate, 3 feet high, turns upon a horizontal axis parallel to the base, the part below the axis being prevented turning in the direction of the pressure, but the upper part being free to turn: at what height above the base must the axis be placed that the gate may open when the water rises more than 7 feet above the top of the gate?

3. The specific gravity of gold being 19·5 and of silver 10·5, what are the weights of silver and gold respectively in a compound of these metals which weighs 900 grains in air, and 840 in water?

4. A cylindrical pontoon with hemispherical ends, 23 feet long and 3 feet in diameter, when floating has a fourth of the diameter below the surface of the water; what is the weight of the pontoon, and what additional weight will it just bear, a cubic foot of water weighing 62·5 lbs.?

5. Describe the Mercurial Barometer; state on what the height of the column of mercury in it depends; what it is a measure of, and why it is different at different altitudes above the Earth's surface.

6. Describe the common suction-pump; point out its principle and mode of action; and explain why by means of it water cannot be raised above a certain height, stating that height.

7. A diving-bell in the form of the frustum of a cone, the diameter of the base being 8 feet, of the top 6 feet, and the height 6 feet (inside measures), is let down into the sea until the water rises 3 feet within it; find the depth to which it descended, and the density of the contained air, the pressure of the atmosphere at the surface of the sea being equal to 33 feet of sea-water.

G.

VII. DIFFERENTIAL CALCULUS.

1. u being any function of x, and u' the value of u when x becomes $x+h$, show that

$$u' = u + \frac{du}{dx} h + \frac{d^2u}{dx^2} \cdot \frac{h^2}{1.2} + \frac{d^3u}{dx^3} \cdot \frac{h^3}{1.2.3} + \&c.$$

2. u being a function of x, find $\frac{du}{dx}$ when,

$$(1)\ u = \frac{(a^2+x^2)}{\sqrt{(a^2-x^2)}}; \qquad (2)\ u = \log_e \frac{x+\sqrt{(x^2-a^2)}}{\sqrt{(x^2-a^2)}};$$

$$(3)\ u = e^x \log_e \sin x.$$

3. Find the radius of the base and the altitude of the least cone that will circumscribe a given sphere.

4. A is a given weight attached to a string hanging over a fixed pulley, and by means of which another weight x is to be raised, by being attached to the other end of the string: it is required to find x in terms of A, so that the momentum generated in x in a given time may be a maximum.

5. Find the value of the subtangent, and of the subnormal in the ellipse, and show that the subtangent is the same, for the same abscissa, as in the circle described on the major axis.

INTEGRAL CALCULUS.

6. Find the following integrals:

$$\int \frac{3x^3dx}{\sqrt{(a^4-x^4)^3}}, \quad \int \frac{(x+1)\,dx}{x^2+x-6},$$

$$\int \frac{x^3dx}{\sqrt{(a^2-x^2)}}, \quad \int \frac{xdx}{\sqrt{(2ax-x^2)}}.$$

7. Investigate the differential expression for the area included by a curve and two rectangular co-ordinates; and point out how this is applied to finding such areas.

H.

I. GEOMETRY.

1. Give Euclid's definitions of the following:

(1) Parallel straight lines; (2) the rectangle contained by two straight lines; (3) the distance of a straight line from the centre of a circle; (4) an angle in a segment of a circle; (5) similar rectilineal figures.

2. If from the vertical angle of a triangle a straight line be drawn to the middle of the base, twice the square of this line together with twice the square of half the base will be equal to the sum of the squares of the other two sides of the triangle.

3. About a given circle to describe a triangle equiangular to a given triangle.

4. Equal parallelograms which have one angle of the one equal to one angle of the other have their sides about the equal angles reciprocally proportional: and parallelograms that have one angle of the one equal to one angle of the other and their sides about the equal angles reciprocally proportional are equal to one another.

CO-ORDINATE GEOMETRY.

5. Find the distance between two points of which the co-ordinates are $x_{,}=7$, $y_{,}=-3$, $x_{,,}=-5$, $y_{,,}=2$; find the equation to the straight line passing through these points, and the angle which it makes with the axis x; and construct the figure to a scale.

6. The co-ordinates to the centre of a circle being $a=5, \beta=3$; find its radius that it may pass through a point of which the co-ordinates are $x=2$, $y=7$; and find the distance from the origin at which it cuts the axis x. Construct the figure to a scale.

7. A parabola being a plane curve in which the distance of any point from a given line is equal to its distance from a fixed point, find the equation to the parabola when the given line and the perpendicular upon it from the fixed point are the rectangular axes to which the curve is referred.

H.

II. ARITHMETIC AND ALGEBRA.

1. If a man can perform a journey of $258\frac{3}{4}$ miles in $6\frac{3}{4}$ days, walking $11\frac{1}{2}$ hours each day, how many hours a day must he walk, at the same rate, to perform a journey of $130\frac{2}{3}$ miles in $3\frac{11}{12}$ days?

2. There are fifteen 24-pounder guns in one fort, and twelve 32-pounder guns in another fort: all the 24-pounders with one 32-pounder are worth £1969; and all the 32-pounders with one 24-pounder are worth the same sum: find the value of a gun of each sort.

3. Given $x(y+z)=20$, $y(x+z)=35$ and $z(x+y)=27$; find x, y, z.

4. Given $\sqrt{\frac{5x}{x+y}}+\sqrt{\frac{x+y}{5x}}=\frac{3}{2}\sqrt{2}$ and $xy-(x+y)=1$; find x and y.

5. Determine all the values of x and y which satisfy the equations $x^4+y^4=612+x^2+y^2$ and $x^2+y^2=129-x^2y^2$.

6. Determine the roots of the equation $x^3+x^2-10x+8=0$, and by means of them resolve the fraction $\frac{x^2-19x+8}{x^3+x^2-10x+8}$ into the sum of three fractions having denominators of the first degree.

7. Find an expression for the sum of n terms of the series

$$1+2r+3r^2+4r^3+\&c.$$

8. The number of shot in the longer side of the base of a complete rectangular pile exceeds the number in the shorter side by 10, and after a certain number of courses had been removed there were 25 shot in the longer side of the upper course: how many shot had been removed?

9. How many signals may be made by means of a red, a yellow, a green, a blue, and a white flag, hoisting them singly, two at a time, three at a time, four at a time, and five at a time, the flags when combined being hoisted over one another in every possible order?

H.

III. TRIGONOMETRY AND MENSURATION.

1. Define the terms (1) complement of an angle, complement of an arc; (2) supplement of an angle, supplement of an arc; (3) sine; (4) cosine; (5) tangent; and show that the sine of an arc is equal to the sine of its supplement; and the tangent of an arc is equal to minus the tangent of its supplement.

2. Prove that $\cos(A+B) = \cos A \cos B - \sin A \, . \, \sin B$; and then show that $\cos 2A = 2\cos^2 A - 1$, or $= 1 - 2\sin^2 A$.

3. The hypothenuse of a right-angled triangle is 175, and one of its sides is 57; solve the triangle.

4. Having given $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$, show that

$$\sin \tfrac{1}{2} A = \frac{2}{bc} \, . \, \{s(s-a)(s-b)(s-c)\}^{\frac{1}{2}}, \text{ } s \text{ being equal to } \frac{a+b+c}{2}.$$

5. The two sides of a triangle being 20 and 30, and their included angle 60°, solve the triangle.

6. The angles of elevation of the top of a tower from two stations in the same vertical plane with it are 60° and 30°, and the distance between the stations is 120 feet: find the height of the top of the tower above each station, (1) When both stations are in the same horizontal plane; (2) When the station nearest to the tower is 10 feet above the level of the more remote.

7. Investigate the formulæ for the areas and perimeters of regular polygons of n sides, described in, and about a circle of given radius r; and apply them to find the perimeters and areas of the regular polygons of 900 sides described in, and about the circle of which the radius is 100 feet.

8. If the circumference of the dial of a clock be 15 feet, what will be the area of the sector passed over by the minute-hand in 12 minutes?

9. The curved surface of a hemisphere being 157·08 feet, find the area and circumference of its base.

H.

IV. SPHERICAL TRIGONOMETRY AND ASTRONOMY.

1. Show that in a right-angled spherical triangle, the tangent of the angle at the base is equal to the tangent of the perpendicular divided by the sine of the base. And give Napier's rules for the solution of right-angled spherical triangles.

2. State Napier's four analogies, and point out the cases in spherical triangles to the solution of which they are applicable.

3. Show that in any spherical triangle

$$\cos A = \frac{\cos \alpha - \cos \beta \cos \gamma}{\sin \beta \sin \gamma}.$$

4. The three sides of a spherical triangle are $50^{\circ}\ 37'$, $83^{\circ}\ 19'$, and $40^{\circ}\ 12'$; find its three angles.

5. If n = number of square feet in the area of a spherical triangle whose sides are small compared with the radius of the sphere, and D = number of feet in a degree on the surface of the sphere, show that the number of seconds in the spherical excess is $20\pi . \frac{n}{D^2}$; and point out how this may be applied to determine the sum of the errors of the observed angles in such a triangle.

6. Define the following terms: equator of the heavens; ecliptic; tropics; oblique ascension; zenith; azimuth; parallax; sidereal day; solar day; equation of time.

7. The right ascension and declination of a heavenly body being determined by observation, show how its latitude and longitude may be computed.

8. Show how, by observing the Sun's altitude on a given day, at a given hour, the latitude of a place on the Earth is determined; and also the Sun's azimuth at the time of observation.

9. An eclipse of one of Jupiter's satellites, which occurred at $19^{h}\ 52^{m}\ 19^{s}\cdot 3$, Greenwich time, was observed at $11^{h}\ 17^{m}\ 25^{s}\cdot 8$: *what is the* longitude of the place of observation?

H.

V. STATICS.

1. Define the terms: "force," "composition of forces," "equilibrium," "centre of gravity," "fulcrum," "moment of a force about a point."

2. If the men on one drag-rope of a gun pull with a force of 400 lbs., those on the other with a force of only 300 lbs., and the directions of the drag-ropes make an angle of 30° with each other; with what force will the gun be urged forward, and what angles will the direction in which it is urged make with the drag-ropes?

Find also the force that would be gained by the forces on the ropes being equally distributed, and the ropes being parallel.

3. Two weights of 100 lbs. each are suspended by a string passing over two fixed smooth points A, B: find by how much the pressure on the upper point exceeds that on the lower when the line AB makes an angle of 45° with the vertical.

4. Find the magnitude and position of the resultant of two parallel forces acting on a rigid body.

5. The diameters of the ends of a frustum of a cone being a and b, and the height of the frustum h; show that the distance of its centre of gravity from the end of which the diameter $= a$ is

$$\frac{h}{4} \cdot \frac{a^2 + 2ab + 3b^2}{a^2 + 2ab + b^2}.$$

6. A system of pulleys of the second kind, in which each pulley hangs by a separate string, and consisting of four moveable pulleys, has the last pulley hooked to the arm of a screw at the distance of 10 inches from its axis: the distance between the threads of the screw being half an inch, and a power of 30 lbs. being applied at the free end of the string passing round the first pulley, so that the strings being parallel, the force on the arm acts at right angles to it: it is required to find the pressure produced on the head of the screw.

H.

VI. DYNAMICS.

1. A body being let fall from the top of a cliff 322 feet high, it is required to find with what velocity a body must be projected vertically upwards from the base of the cliff, that it may meet the falling body half way in its descent.

2. A weight of 84 oz. is connected with another of 77 oz. by a string hanging over a fixed pulley: how far will the heavier descend, and what velocity will it acquire in 5 seconds, (1) Supposing no inertia in the pulley; (2) When the inertia of the pulley is represented by $10\frac{1}{16}$ oz.?

3. Investigate an expression for the velocity with which a shot must be fired at a given angle of elevation e, that it may strike an object at a given distance r from the point of projection, on a plane passing through that point and making a given angle i with the horizon.

4. With what velocity and at what elevation must a shot be fired, that, clearing (1 foot above) a parapet 14 feet above the point of projection and at the horizontal distance of 1800 feet, it may dismount a gun 9 feet below the parapet, and at the horizontal distance of 150 feet beyond it?

5. A stone being let fall from the top of a cliff, a pendulum $84\frac{1}{4}$ inches in length was observed to make 6 vibrations during the descent of the stone to the sea: determine the cliff's height, the length of the seconds' pendulum being $39\frac{1}{7}$ inches nearly.

6. A particle suspended by a string 3 feet long is struck horizontally so as to produce in it a velocity of 12 feet per second; find the arc which the particle will have described when the string begins to slacken, and the highest point to which the particle will rise.

H.

VII. DIFFERENTIAL CALCULUS.

1. Show that $\frac{d.\sin x}{dx} = \cos x$, and $\frac{d.\cos x}{dx} = -\sin x$.

2. u being a function of x, find $\frac{du}{dx}$, when

$$u = \frac{x^2}{(1-x^3)^{\frac{2}{3}}}; \qquad u = \log_\epsilon \sqrt{\{\sqrt{(1+x^2)} - \sqrt{(1-x^2)}\}};$$

$$u = \frac{x^2}{\sqrt{(x^2+1)}+x}; \qquad u = \epsilon^x \sin x . \log_\epsilon x.$$

3. Find the value of x which renders $\frac{x-1}{(x+1)^2}$, a maximum or a minimum.

4. Find the altitude and diameter of the base of the greatest cone that can be inscribed in a given sphere.

5. Given the base of an inclined plane $= b$, to find its height so that the time of a body falling down the plane may be a minimum.

6. ADB is a semicircle of which the diameter $AB = 2a$; the chord DB is bisected in E and AE joined, intersecting DP, perpendicular to AB, in M: show that the equation to the locus of the point M is $y^2(2a+x)^2 = x^2(2a-x)$, where $x = AP$, and $y = PM$; and from this equation determine the value of the subtangent and of the subnormal to a point in the curve which is the locus of the point M.

7. The equation of the cycloid being

$$y = \sqrt{(2ax - x^2)} + a \text{ vers}^{-1} \frac{x}{a},$$

show that the length of a cycloidal arc is twice the corresponding chord of the generating circle.

8. Find the area, from $x = 0$ to x, of a curve whose equation is

$$\frac{x^2}{a^2} = \frac{y-a}{y}.$$

H.

VIII. INTEGRAL CALCULUS.

1. Find the following integrals:

$$\int \frac{ax^{n-1}dx}{(b-ex^n)^{\frac{1}{2}}}, \qquad \int \frac{dx}{1-x^2}, \qquad \int \frac{x^2dx}{\sqrt{(1-x^2)}}.$$

2. Find the volume of the solid generated by the revolution of the curve whose equation is $y^2(a^2+x^2)+a^3x=a^4$, about the axis x, between the values $x=0$ and $x=a$.

3. Find the distance of the centre of gravity of the segment of a sphere, from the extremity of the diameter; and state what this distance is in a hemisphere.

Find also the distance of the centre of gravity of the segment of a spherical shell of very small thickness, and deduce the distance for the hemispherical shell, including the plane circular base.

4. Investigate the 1st of Guldin's properties of the centre of gravity.

5. Show by Taylor's theorem, that in all variable motions

$$f=\frac{dv}{dt}.$$

6. A body attracted to a centre by a force varying inversely in the sesquiplicate ratio of the distance, begins to fall at the distance a; what will be its velocity at the distance $\frac{1}{2}a$, supposing the force at that distance to be equal to g, the force of gravity?

7. Show that, in a system of material points, the Moment of Inertia with respect to any given axis is equal to the moment about an axis parallel to this and passing through the centre of gravity, plus the moment of the whole system collected in its centre of gravity, about the given axis.

8. The Moment of Inertia of a circle, radius r, about an axis passing through its centre and perpendicular to its plane being $\frac{\pi r^4}{2}$, find the Moment of Inertia of a sphere, radius a, about its diameter.

J.

I. GEOMETRY.

1. If a straight line touch a circle, and from the point of contact a straight line be drawn cutting the circle, the angles which this line makes, &c.

2. The sides about the equal angles of equiangular triangles are proportionals; and those which are opposite to the equal angles are, &c.

3. If two straight lines be cut by parallel planes, they shall be cut in the same ratio.

4. The triangles formed by joining the corresponding extremities of three equal and parallel straight lines, not all in the same plane, are equal, and their planes are parallel.

5. The traces of two planes being given, find the projections of their intersection, when the traces of the given planes intersect in two points.

6. Draw a plane that shall pass through a given point and through a given straight line.

CO-ORDINATE GEOMETRY.

7. Determine the equation to a straight line which is drawn through a point whose co-ordinates are $x^1 = 5$, $y^1 = -2$, perpendicular to the straight line whose equation is

$$y = 2x + 5,$$

and construct the line by scale.

8. The co-ordinates to the centre of a circle referred to rectangular axes are α and β: find the equation to the circle. Also determine the position and magnitude of the circle which is the locus of the equation

$$y^2 + x^2 - 4y + 6x - 12 = 0.$$

9. Give the definition of a "conic section," and state when the conic section is a "parabola," an "ellipse," or an "hyperbola."

J.

II. ARITHMETIC AND ALGEBRA.

1. What sum of money lent at £3. 4*s.* per cent. per annum, simple interest, will amount to £10000 in 7½ years?

2. A father left $\frac{2}{5}$ of his property to his eldest son, $\frac{5}{9}$ of the remainder to his second, and the rest to his third: the difference between the shares of the second and third was £1800: what was the share of each?

3. Given the sum of three numbers equal to 13; the sum of the products of every two equal to 47; and the squares of the numbers in arithmetic progression: find the numbers.

4. When the price, per cwt., of tin to that of copper was as 3 : 2, 11 cwt. of gun-metal cost £64. 8*s.*, but when the prices of tin and copper were reversed, 11 cwt. of gun-metal cost £89. 12*s.*: what was the price of copper in the first case, and what the weight of copper and tin in the metal?

5. Solve the following equations:

$$(1)\quad \frac{1}{x-\sqrt{(2-x^2)}} - \frac{1}{x+\sqrt{(2-x^2)}} = 1.$$

$$(2)\quad v^2z+vz^2=84 \text{ and } (v^2-z^2)(v-z)=7.$$

6. The equation $x^3-11x^2+7x+147=0$ has two equal roots: find them by means of the derived equation, and the third root by depressing the equation.

7. Decompose the fraction $\frac{x^2+5x+1}{x^3-2x^2-x+2}$ into three fractions having denominators of the first degree.

8. Investigate an expression for the number of shot in a square pile of n courses, that is, an expression for the sum of n terms of the series $1^2+2^2+3^2\ldots\ldots n^2$.

9. By means of a table of logarithms find the value of the fraction

$$\frac{(3\cdot14159)^2\sqrt[3]{(\cdot08357)}}{(\cdot4753)^3\sqrt{(5\cdot37462)}}.$$

J.

III. TRIGONOMETRY AND MENSURATION.

1. A and B being any two angles, show that

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A . \tan B},$$

and
$$\operatorname{cotan}(A \pm B) = \frac{\operatorname{cotan} A \operatorname{cotan} B \mp 1}{\operatorname{cotan} B \pm \operatorname{cotan} A}.$$

2. From $\sin 30^\circ = \frac{1}{2}$ and $\sin 45^\circ = \frac{1}{2}\sqrt{2}$, show that

$$\tan 15^\circ = 2 - \sqrt{3}, \text{ and } \tan 75^\circ = 2 + \sqrt{3}.$$

3. Show that, in any triangle, the sides are to each other as the sines of their opposite angles; and apply this to finding the remaining side and angles of a triangle of which two sides are 315·753, 238·825, and the angle opposite the greater of these two sides is 97° 13′.

4. From the top of a hill I observed two church spires exactly in the same direction on the horizontal plane below the hill: the angle of depression of the base of the nearer spire was observed 15° 23′, and of the further spire 4° 12′: the top of the hill being known to be 315 feet above the horizontal plane on which the churches stand, it is required to find, from these observations, the distance between the two spires.

5. Two sides of a court-yard in the form of a parallelogram are 40 feet and 70 feet, and its shorter diagonal is 50 feet; what will it cost paving at *3s. 9d.* a square yard?

6. The side of an equilateral triangle, of a square and of a regular hexagon are in arithmetic progression; the sum of this progression is 36 feet; and the perimeters of the three figures are equal; find their areas.

7. The diameter of the horizontal bottom of a circular reservoir is 200 feet; the sides of the reservoir being inclined all round at an angle of 45°, find the number of cubic feet of water contained in it when the depth of the water is 10 feet.

J.

IV. STATICS.

1. Define the terms (1) "force," (2) "gravity," (3) "weight," (4) "resultant," and (5) state what is meant by the "parallelogram of forces."

2. Assuming the parallelogram of forces, show that if f and f_1 represent two forces acting at a point, θ the angle which their directions make with each other, and r their resultant,

$$r^2 = f^2 + f_1^2 + 2ff_1 \cos\theta:$$

find the resultant of two pressures acting at a point in directions making an angle of 60° with each other, and find also the angle which the resultant makes with the direction of the pressure 7 lbs.

3. The extremities of a string 14 inches long are fixed to two points 10 inches apart, in the same horizontal line, and a weight of 8 lbs. is suspended,

1st, from a knot in the string 6 inches from one extremity;

2nd, from a smooth ring which slides freely along the string:

find in both cases the tensions of each of the parts into which the string is divided, and state clearly the mechanical principles involved in each step of the investigation.

4. Define the centre of gravity of a body, or system of bodies; and show that if m, m_1, m_2, m_3 are bodies in a straight line, and a, a_1, a_2, a_3 are their distances from a fixed point in that line, the distance of their centre of gravity from the same point is

$$\frac{am + a_1m_1 + a_2m_2 + a_3m_3}{m + m_1 + m_2 + m_3}.$$

5. If a and b be the two parallel sides of a trapezoid, and h the line which bisects these sides, show that the centre of gravity of the trapezoid will be in this line, and its distance from a along it will be $\frac{h}{3}\cdot\frac{a+2b}{a+b}$.

J.

V. DYNAMICS.

1. A body projected vertically upwards from the bottom of a tower with a velocity of 60 feet per second reaches the top in 2 seconds: what is the height of the tower? and how much above the top does the body rise?

2. A weight of 3 oz. draws a weight of 12 oz. down a plane inclined 30° to the horizon, by means of a string passing over a pulley at the bottom of the plane: find the vertical descent of each of the weights in 3 seconds, the friction and inertia of the pulley being 1 oz., and $g = 32$ feet.

3. Investigate the expression for the velocity of a shot which, being fired at an angle of elevation e, shall strike a mark at the distance r, on a plane passing through the point of projection and making an angle i with the horizon.

4. Show how the value of e is determined from the equation

$$r = \frac{2v^2}{g} \cdot \frac{\cos e \sin (e - i)}{\cos^2 i}$$

in terms of r, v, i; and prove, that the range on a given plane is a maximum when the direction of projection bisects the angle between the plane and the vertical.

5. At what elevation must a shot be fired with a velocity of 400 feet that it may range 2500 yards on a plane which descends at an angle of 30°?

6. The length of the seconds' pendulum being 39·139 inches, find (1) the value of g the force of gravity, (2) the length of the pendulum which vibrates 80 times in a minute.

7. A clock intended to beat seconds gains 7·4 seconds a day: how must its pendulum be altered that it may go right?

J.

VI. HYDROSTATICS.

1. A cylindrical vessel, of which the diameter is 6 inches, and the length of the axis is 1 foot, being filled with water is closed at both ends: find the pressure on the concave surface, and also on one end of the cylinder,

(1) When the axis is vertical;

(2) When the axis is horizontal; and compare the whole pressures in the two cases.

2. Define the "centre of pressure;" and deduce the differential expression for the distance of the centre of pressure of a plane surface immersed in a homogeneous fluid from the surface of the fluid.

3. A sluice-gate in the form of an isosceles triangle, of which the altitude is 4 feet and base 5 feet, the vertical angle being downwards, turns upon a horizontal axis parallel to the base, the part below the axis being prevented turning in the direction of the pressure, but the upper part being free to turn: at what distance from the top of the gate must the axis be placed, that the gate may open when the water rises more than 8 feet above its top?

4. Some silver being alloyed with 100 grains of copper of the specific gravity 8·79, the alloy was found to weigh 1150 grains in air and 1034·5 grains in water: what was the specific gravity of the silver employed?

5. The whole length of a cylindrical pontoon with hemispherical ends is 23 feet; its diameter is 3 feet; and its weight, with the portion of the bridge it supports, is 1660: what additional weight does it bear when the axis of the cylinder is on the level of the surface of the water?

6. Describe Nicholson's hydrometer, and point out the manner in which it is applied for the determination of the specific *gravities* both of solids and of fluids.

J.

VII. DIFFERENTIAL CALCULUS.

1. u being a function of x, find $\frac{du}{dx}$, when

(1) $u=\frac{x^2}{(1+x)^3}$; (2) $u=(a^2+x^2)^{\frac{3}{2}}\sqrt{(a^2-x^2)}$;

(3) $u=\log_e\frac{\sqrt{(x^2+a^2)}+x}{\sqrt{(x^2+a^2)}-x}$; (4) $u=e^x\sin x$.

2. Show that when u, a function of x, is a maximum,

$$\frac{du}{dx}=0, \text{ and } \frac{d^2u}{dx^2} \text{ is negative.}$$

3. Find the value of x which renders $\frac{1+x}{1+x^2}$ a maximum or a minimum.

4. Find the altitude and the radius of the base of the least cone that can circumscribe a given sphere whose radius is r.

5. Find the subtangent to a point in the curve whose equation is $ay^2=(a+x)^2(2a-x)$.

INTEGRAL CALCULUS.

6. Find the following integrals:

$$\int\frac{7xdx}{3(a^2+x^2)^{\frac{3}{2}}},\quad \int\frac{(3x+2)dx}{x^2+x-2},\quad \int\frac{x^2dx}{\sqrt{(a^2-x^2)}},\quad \int\frac{dx}{(a^2-x^2)^{\frac{3}{2}}}.$$

7. Show that the area of a curve between the curve and the co-ordinates x, y, is $\int_x^0 ydx$.

8. Find the volume, from $x=a$ to $x=2a$, of the solid generated by the revolution about the axis x, of the curve whose equation is $xy^2=(a+x)^2(2a-x)$.

K.

I. GEOMETRY.

1. In a circle, the angle in a semicircle is &c.

2. The rectangle contained by the diagonals of a quadrilateral inscribed in a circle is equal to both the rectangles contained by its opposite sides.

3. If, in a circle, an equilateral polygon be inscribed, and an equilateral polygon of the same number of sides be described about the circle, the perimeter of the inscribed polygon will be to the perimeter of the circumscribed, as the perpendicular from the centre on a side of the inscribed polygon, is to the radius of the circle.

4. If two straight lines meeting one another be parallel to two other straight lines which meet one another, but are not in the same plane with the first two; the plane which passes through these is parallel to the plane passing through the others.

5. *Horizontal Projections.* A plane being given by three points a_7, b_{23}, c_{19}, not in a straight line, determine its scale of slope.

CO-ORDINATE GEOMETRY.

6. Find the equation to a straight line which, passing through a point of which the co-ordinates are $x_{,}=-5$, $y_{,}=3$, makes an angle of 45° with the axes; find the distances from the origin at which this line cuts the axes; and construct the figure to a scale.

7. Find the co-ordinates to the points in which a straight line whose equation is $y = 3x - 5$, intersects a circle described from the origin as its centre, with a radius equal to 4.

8. An ellipse being a plane curve in which the sum of the distances of every point from two fixed points is equal to a given line, find the equation to the ellipse referred to rectangular co-ordinates of which the origin is the middle of the straight line joining the fixed points; the distance between these points being given $=2c$, and the sum of the distances of a point in the curve from them $=2a$.

K.

II. ARITHMETIC AND ALGEBRA.

1. Define the terms, "fraction," "decimal," "root," "index," "power," and "logarithm."

2. Reduce the following to equivalent fractions with rational denominators $\frac{2}{\sqrt{5}-\sqrt{2}}$, $\frac{2\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$, and $\frac{20}{4\sqrt{5}+3\sqrt{2}}$.

3. A ship whose value was £2000, was wrecked; $\frac{3}{10}$ of it belonged to A, $\frac{2}{3}$ to B, and the rest to C. An insurance office pays them £1500 of their loss. How much of this must each receive?

4. Find the values of x and y in the following equations:

$$\left\{\begin{array}{l}\frac{a}{x}+\frac{b}{y}=m \\ \frac{b}{x}+\frac{a}{y}=n\end{array}\right\}; \quad \frac{3x}{x+1}+\frac{2x-5}{3x-1}=3\frac{3}{40}; \quad \left\{\begin{array}{l}x^3-y^3=a \\ x-y=b\end{array}\right\}.$$

5. Find the negative root of the equation

$$x^4-24x^3+144x^2-60x-5=0,$$

to 7 places of decimals, and determine the first figure of the greatest root.

6. If s be the sum of n terms of a series in geometrical progression, a the first term, l the last, and r the common ratio, show that $l=ar^{n-1}$, and $s=a\frac{r^n-1}{r-1}$.

Find also the limiting value of s, when r is less than unity; the series being convergent and extended without limit.

7. A, who travels three times as fast as B, sets off from a certain place eight days after B in order to overtake him: in how many days will he effect this?

8. The number of balls in the shorter side of the base of a complete rectangular pile is 21; how many must there be in the other side that the pile may contain 4697 balls?

9. The first two terms of the expansion of $(1+x)^n$ being $1+nx$, whether n be a whole number or a fraction, positive or negative, determine the coefficients of the succeeding terms of the expansion.

K.

III. TRIGONOMETRY AND MENSURATION.

1. Given $\sin\phi = \cos 2\phi$; find $\sin\phi$, and thence ϕ.

2. A and B being any two angles, show that

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B,$$
$$\text{and } \cos(A \pm B) = \cos A \cos B \mp \sin A \sin B.$$

3. A and B being any two angles, show that

$$\frac{\sin A + \sin B}{\sin A - \sin B} = \frac{\tan \frac{1}{2}(A + B)}{\tan \frac{1}{2}(A - B)}.$$

4. a, b, c being the sides of a triangle ABC respectively opposite the angles A, B, C, prove that $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$; and apply this to finding each of the angles of a triangle of which the sides are 200, 300, 400 feet.

5. A base of 300 yards was measured along one of the banks of a river, and the angles between this base and the lines from each of its extremities to a conspicuous object on the opposite bank were observed to be $54^\circ\ 15'$ and $49^\circ\ 37'$: find the breadth of the river, that is, the perpendicular distance of the object on the opposite bank from the measured base.

6. M, N are two inaccessible objects whose distances from each other is required. A base AB is so measured that each of the angles MAB, NBA is a right angle; the angle MAN is one-third of a right angle, and the angle MBN is half a right angle: determine from these data the distance MN in terms of the base $AB = b$.

7. Show that the area of a triangle is $\frac{1}{2}bc \sin A$; and apply this to finding the area of a triangle of which the sides are $4 + \sqrt{3}$ and $4 - \sqrt{3}$, and their included angle is $32^\circ\ 12'$. Find also the third side of this triangle.

K.

IV. SPHERICAL TRIGONOMETRY.

1. A, B, C being the angles of a spherical triangle A, B, C, show that the area of the triangle is $\pi r^2 . \frac{A+B+C-180^\circ}{180^\circ}$.

2. State "Napier's Rules" for the solution of right-angled spherical triangles; and exemplify them: (1) when a side is taken as middle part; (2) when the complement of an angle is taken as middle part; (3) when the complement of the hypothenuse is taken as middle part.

3. α, β, γ being the sides opposite to the angles A, B, C of a spherical triangle, and assuming the equation

$$\cos\beta\cos\gamma + \sin\beta\sin\gamma\cos A - \cos\alpha = 0, \ldots\ldots\ldots (1).$$

show that $\tan^2\frac{1}{2}A = \frac{\sin(\sigma-\beta)\sin(\sigma-\gamma)}{\sin\sigma(\sin\sigma-\alpha)}$, where $\sigma = \frac{1}{2}(\alpha+\beta+\gamma)$.

4. Assuming the equation (1), show that

$$\cos\alpha = \frac{\cos A + \cos B\cos C}{\sin B\sin C}.$$

5. Define the following terms: "pole" and "equator of the heavens," "meridian," "ecliptic," "the obliquity of the ecliptic," "the declination and right-ascension," "latitude and longitude of a heavenly body," the "latitude and longitude of a place on the earth."

6. Point out, by means of a figure, how the time of rising of the sun, and its azimuth when rising on a given day and at a given place, are determined.

7. What angle or arc in the heavens is the measure of the Latitude of a place on the earth? What other angle or arc is it equal to? and how is the latitude of a place determined by observations on a circumpolar star?

8. Point out, by means of a figure, how the latitude and longitude of a star are determined from its observed right-ascension and declination.

K.

V. STATICS.

1. A gun, weighing 12 cwt. and suspended by a rope, is held out from the vertical line passing through the point of suspension by means of a rope attached to the gun; supposing that the suspending rope makes an angle of 15° with the vertical, and that the second rope makes an angle of 30° with the horizon; what will be the tension of this second rope or the force exerted on it? and what will be the pressure on the point of suspension, or the tension of the suspending rope?

2. The directions of three forces, 50 lbs., 30 lbs., 70 lbs., make respectively angles of 30°, 45°, 60° with the horizon: find their resultant and the angle which its direction makes with the horizon.

3. A beam of uniform density rests between two planes, one of which is vertical, and the other is inclined to the horizon at an angle of 30°: find the inclination of the beam to the horizon when in the position of equilibrium.

4. Two rulers, each 30 inches long and weighing 15 ounces, are placed lengthwise over each other, and to the under one a weight of 45 ounces is hung at one end $4\frac{1}{2}$ inches from an edge on which it rests: in what direction and how far must the upper ruler be slid upon the lower that the whole may be in equilibrium?

5. A field-gun, which with its carriage weighs 24 cwt., is supported on a plane inclined to the horizon at an angle of 30°, by means of ropes going round on the outside of the wheels, and each hooked to a block of pulleys of the first kind, in which the same rope goes round all the sheaves, and having two sheaves in the lower block, the other block being fixed at the top of the plane so that the ropes are parallel to the plane; what power must be applied at the free end of each of the ropes that the gun may just be supported independently of friction?

K.

VI. DYNAMICS.

1. A body is projected vertically upwards with a velocity of 161 feet per second, and one second after, another body is projected vertically upwards with the same velocity: where will the two bodies meet?

2. A weight of 4 lbs. hanging vertically, draws a weight of 12 lbs. along a horizontal table: supposing that a weight of 1 lb. is just sufficient to overcome the friction, how far will the weight of 4 lbs. descend in 5 seconds, g being taken equal to 32 feet?

3. Prove that the times of descent down all chords drawn from either extremity of a vertical diameter of a circle are equal.

4. Find geometrically the plane of quickest descent from a given point without a given circle, to the circumference of the circle.

5. Deduce the equation which subsists between v the velocity of projection of a shot, e the angle of elevation, r the range, and i the inclination of the range-plane to the horizon.

6. Find the velocity with which a shell must be fired at an angle of elevation of 12°, to strike a mark at the distance of 750 yards on a horizontal plane.

7. There are three ivory balls A, B, C whose elasticity is $\frac{7}{8}$, $A = 8$ oz., $B = 7$ oz., $C = 5$ oz.; A impinges directly, with a velocity of 12 feet per second, on B, at rest, and, immediately after, C impinges directly on B with a velocity of 4 feet per second in a direction opposite to that of A's motion: find the final velocities of A, B and C.

8. The length of the seconds' pendulum being 39·139 inches, how much will a clock with a seconds' pendulum movement lose in a day if its pendulum be a metre or 39·37 inches in length?

K.

VII. DIFFERENTIAL CALCULUS.

1. v and t being functions of x, shew that:

$$(1)\quad \text{when } u = vt,\quad \frac{du}{dx} = v\frac{dt}{dx} + t\frac{dv}{dx}.$$

$$(2)\quad \text{when } u = \frac{v}{t},\quad \frac{du}{dx} = \frac{1}{t}\cdot\frac{dv}{dx} - \frac{v}{t^2}\cdot\frac{dt}{dx}.$$

2. u being a function of x, find $\dfrac{du}{dx}$ when

$$u = \frac{\sqrt{(a^2+x^2)} + \sqrt{(a^2-x^2)}}{\sqrt{(a^2+x^2)} - \sqrt{(a^2-x^2)}},\qquad u = \log_e \sqrt{\left\{\frac{a+x}{a-x}\cdot\frac{b+x}{b-x}\right\}}$$

$$u = \frac{e^x-1}{e^x+1},\qquad u = \cos^{-1}(4x^3 - 3x).$$

3. Find the greatest rectangular building that can be erected on a plot of ground in the form of a segment of a circle, the height of the segment being 30 feet, its base or chord 120 feet, and consequently the radius of the circle 75 feet.

4. A weight W is to be raised from the horizontal plane HB to the point C, up an inclined plane, by means of a weight P attached to a string passing over a pulley at C, and hanging vertically on the other side: given the height $BC = a$, to find the inclination of the plane AC, that W may be drawn up it in the least time possible.

5. Find the subtangent and subnormal in the conchoid of Nicomedes, the axis x being that passing through the generating point.

INTEGRAL CALCULUS.

6. Find the following integrals:

$$\int\frac{5x^2dx}{\sqrt[3]{(a^3-x^3)^2}};\quad \int\frac{8xdx}{x^2-6x-7};\quad \int\frac{dx}{x^2(a^2-x^2)^{\frac{3}{2}}};\quad \int\sqrt{(2ax-x^2)}\,dx.$$

7. Find the area, from $x=0$ to $x=\frac{1}{2}a$, of the curve whose equation is $x\sqrt{(a^2+y^2)} = ay$.

K.

VIII. INTEGRAL CALCULUS.

1. Show that the area included between the co-ordinates x, y to a point in a curve and the curve is $\int y dx$.

2. Find the length of the curve from $x=0$ to $x=\frac{1}{2}a$ in a curve whose equation is $\frac{y}{\sqrt{2}}=\frac{1}{2}a \operatorname{vers}^{-1}\frac{2x}{a}-\sqrt{(ax-x^2)}$.

3. Find the volume from $x=a$, to $x=0$, of the solid described by the revolution, about the axis x, of a curve whose equation is

$$y\sqrt{(a^2-x^2)}=(a-x)^2.$$

4. Find the distance of the centre of gravity of a parabola whose altitude is a and base $2b$, from its vertex.

5. Find the distance of the centre of gravity of a cone, whose altitude is a and radius of its base b, from its base; and also of the frustum of this cone, in terms of b the radius of its base, β the radius of its upper section, and h its height.

6. By means of the property of Guldin find the volume of the solid generated by the revolution of a parabola whose altitude is a and base $2b$, 1st, about its base; 2nd, about a tangent to the parabola at its vertex; and give the ratio of the two solids thus generated.

7. A body attracted to a centre by a force varying directly as the distance from the centre, begins to fall at the distance a from the centre; find the velocity of the body when arrived at the distance $\frac{1}{2}a$, and the time of falling to that distance, the force at the distance m being equal to g: and show that with this law of force the time of falling to the centre is independent of the distance from which the body begins to fall.

8. If r is the distance of particles in a body M from a fixed axis about which the body revolves, and I the moment of inertia of the body about that axis, show that $I=\int r^2\frac{dM}{dr}dr$.

L.

I. GEOMETRY.

1. If four straight lines be proportionals, the rectangle contained by the extremes will be &c. And if &c.

2. If the sides of a pentagon, no two sides of which are parallel, be produced till they meet, the sum of all the angles at these points of intersection will be equal to two right angles.

3. If two circles touch each other externally or internally, any straight line drawn through the point of contact will cut off similar segments.

4. If a straight line stand at right angles to each of two straight lines at their point of intersection, it shall also be at right angles to the plane which passes through them, that is, to the plane in which they are.

CO-ORDINATE GEOMETRY.

5. Find the equation to a straight line passing through a point whose co-ordinates are $x' = 5$, $y' = -3$, and parallel to a straight line whose equation is $y = 3x + 1$.

6. The co-ordinates to two points are $x_{,} = 9$, $y_{,} = 4$, $x_{,,} = 5$, $y_{,,} = 7$: find the length of the line which joins these points, and also the length of the perpendicular let fall upon this line from the origin of the co-ordinates.

7. Find the co-ordinates to the intersections of a straight line whose equation is $y = 3x - 4$, with a parabola whose equation is $y^2 = 16$.

8. Show that in the parabola, the locus of the middle points of any number of parallel chords is a straight line parallel to the axis of the parabola.

9. Show that the locus of a point whose distances from two fixed points are together always equal to a given line, is an ellipse of which the major axis is that line.

L.

II. ARITHMETIC AND ALGEBRA.

1. £9240 is to be divided among *A*, *B*, and *C*, so that *A*'s share shall be to *B*'s as 3 to 5, and *B*'s share to *C*'s as 7 to 2: find their shares, without employing Algebra.

2. A person having purchased $\frac{9}{16}$ of a copper mine which was valued at £20,000, afterwards sold $\frac{2}{3}$ of his share for £12,000: how much per cent. did he gain on the purchase-money of the part sold?

3. Extract the cube root of ·098 to 10 places by Horner's method, contracting the work for the final figures.

4. Solve the following equations:

(1) $x^2 + 2\sqrt{(x^2 - ax)} = 2\{ax + \sqrt{(ax)}\}$.

(2) $x + y = axy$, and $x^2 + y^2 = b^2x^2y^2$.

(3) $7^{2x} = 2 + 7^x$, applying logarithms.

5. Form an equation of which the roots are

$$1 + \sqrt{3},\ 1 - \sqrt{3},\ 3 + \sqrt{(-1)},\ 3 - \sqrt{(-1)}.$$

6. Find the least positive root of the equation

$$x^4 - 10x^3 + 33x^2 - 48x + 20 = 0,$$

by Horner's method, and state the character of the other roots.

7. The number of shot in a square pile of n courses being $\frac{(2n+1).(n+1).n}{3.2.1}$, show how the formula for the number of shot in a rectangular pile of n courses, and having $m + 1$ shot in the top row, is obtained; and compute the number of shot in an incomplete pile of 20 courses, having 47 shot in the longer and 23 shot in the shorter side of the upper course.

8. Given the sum of two sides of a triangle $= s$, the perpendicular from the vertical angle on the base $= a$, and the diameter of the circumscribing circle $= d$: find all the sides of the triangle.

9. If a^x is expanded in a series $1 + Ax + A_2x^2 + A_3x^3 +$ &c., show how the co-efficients A_2, A_3, A_4, &c. are determined in terms of A, and state what A represents.

L.

III. TRIGONOMETRY AND MENSURATION.

1. A and B being any angles, prove that

$$\sin A + \sin B = 2 \sin \tfrac{1}{2}(A+B) \cos \tfrac{1}{2}(A-B),$$
$$\sin A - \sin B = 2 \cos \tfrac{1}{2}(A+B) \sin \tfrac{1}{2}(A-B).$$

2. Prove that, in any plane triangle, the sides are to each other as the sines of their opposite angles; and state to the solution of which case in plane triangles this applies, and in what manner.

3. a, b, c being the sides opposite to the angles A, B, C of a plane triangle, prove that $\frac{a+b}{a-b} = \frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)}$; and state to the solution of which case in plane triangles this applies, and in what manner.

4. If $s = \frac{1}{2}(a+b+c)$, prove that $\sin^2 \frac{1}{2} A = \frac{(s-b)(s-c)}{bc}$; and point out how this is applied to the determination of the three angles of a triangle, when the three sides are given.

5. A straight line or base of 500 yards having been measured along the south bank of the Thames near Woolwich, the horizontal angles contained by the base and straight lines drawn from its extremities to a station on the Essex bank of the river were observed to be $69^\circ\ 43'$ and $77^\circ\ 40'$: find the perpendicular breadth of the river at the Essex station.

6. From the deck of a ship, 10 feet above the sea, the angle of elevation of the top of a cliff was observed $21^\circ\ 34'$, and from a mast-head, 65 feet vertically above the former point of observation, the angle of elevation of the top of the cliff was observed $15^\circ\ 31'$: find the height of the top of the cliff above the level of the sea, and the distance of the ship from its base.

7. The area of a square inscribed in a circle is 200 square feet; find the area of the equilateral triangle inscribed in the same circle.

L.

IV. STATICS.

1. Define the terms: "density of a body," "resultant," "composition of forces," "resolution of forces," "horizontal and vertical components of a force," "moment of a force about a point."

2. Assuming the parallelogram of forces, show that, if three forces P, Q, R, acting at a point, are in equilibrium, $P : Q$ :: sine of the angle formed by the directions of Q and R : sine of the angle formed by the directions of P and R.

3. If forces f, f_1, f_2, &c., the directions of which make the angles a, a_1, a_2, &c. with the axis x, be resolved in the directions of that axis and of the axis y at right angles to it, show that, if

$$X = f\cos a + f_1 \cos a_1 + f_2 \cos a_2 + \&c.,$$

$$\text{and } Y = f\sin a + f_1 \sin a_1 + f_2 \sin a_2 + \&c.$$

R the resultant of the forces f, f_1, f_2, &c. is equal to $\sqrt{(X^2 + Y^2)}$; and, θ being the angle which R makes with the axis x, $\tan\theta = \frac{Y}{X}$.

4. If P, P', P'', &c. are any parallel forces acting upon a rigid body, at points of which the perpendicular distances from any plane are z, z', z'', &c.; if R be the resultant of these forces; and if Z be the distance of its point of application from that plane; show that, $R = P + P' + P'' + \&c.$; and

$$(P + P' + P'' + \&c.)\, Z = Pz + P'z' + P'z' + \&c.:$$

and point out how this applies to finding the position of the center of gravity of any system of bodies considered as points.

5. A right-angled triangle, of which the sides are 3, 4, 5, is suspended by a string fixed at the right-angle; in what position will it rest, that is, what angle will its hypothenuse then make with the horizon?

6. An uniform beam rests with one end against a vertical wall, and the other on a plane inclined to the horizon at an angle a: find the inclination of the beam when in a position of equilibrium, the coefficient of friction being μ.

L.

V. DYNAMICS.

1. Two bodies A and B, of which the elasticity is represented by e, move in the same direction with velocities a, b (a greater than b); show that, after the impact of A on B,

$$\frac{A+B}{(1+e)B}=\frac{\text{relative velocity}}{\text{velocity lost by } A}; \text{ and } \frac{A+B}{(1+e)A}=\frac{\text{relative velocity}}{\text{velocity gained by } B}.$$

2. A 9 lb. shot is fired vertically upwards with a velocity of 250 feet per second, and at the same instant an 18 lb. shot is let fall from a point 1000 feet vertically above the point of projection: find where the two shot will meet; and supposing the impact of the one upon the other to be direct, and the elasticity of cast iron to be $\frac{1}{2}$, find the time in which the 9 lb. shot will reach the ground, and the velocity with which it will strike, assuming $g=32$.

3. A body $B=10$ lbs., which rests upon a horizontal plane, is connected by a string with another body $A=6$ lbs. which hangs freely; A in its descent drawing B along the plane, it is required to find the time in which A will describe 16 feet, and the velocity it will have acquired, supposing the friction of B on the plane to be $\frac{1}{5}$ of its weight.

4. Show that the times down all chords of a circle, having one of their extremities in a vertical diameter, are equal: and find the straight line of quickest descent from the circumference of a circle to a given point within it.

5. A pendulum 15 inches in length was observed to make 7 vibrations during the time that a heavy body was falling from the top of a cliff to the sea: what was the height of the cliff?

6. A clock which had been regulated to mean time at the equator was found to gain $1^m\ 12^s$ a day: find the increase of the force of gravity at the place of observation, in terms of the force of gravity at the equator.

7. Find an equation connecting v, e, r and i, when the range plane does not pass through the point of projection, but at the distance b vertically below it.

L.

VI. DIFFERENTIAL AND INTEGRAL CALCULUS.

1. v being a function of x, and u a function of v, show that

$$\frac{du}{dx} = \frac{du}{dv} \cdot \frac{dv}{dx}.$$

2. u being a function of x, find $\frac{du}{dx}$ when

$$u = \frac{1+x}{(1-x^2)^2}; \qquad u = \log_e \{\sqrt{(a^2+x^2)} - \sqrt{(a^2-x^2)}\};$$

$$u = \frac{\sqrt{(a^2-x^2)}}{(a^2+x^2)^{\frac{3}{2}}}; \qquad u = \log_e \frac{1-e^x}{1+e^x}.$$

3. Find the values of x which render $x^4 - 12x^3 + 52x^2 - 96x$ a maximum or a minimum.

4. Find the following integrals:

$$\int 10x^2 \cdot \sqrt[3]{(a^3-x^3)^2}dx; \qquad \int x^2 \cdot \sqrt{(a^2+x^2)}\, dx;$$

$$\int \frac{(5x+3)\,dx}{x^2+2x-3}; \qquad \int \frac{dx}{x^2 . \sqrt{(a^2-x^2)}}.$$

5. Find the area of the Cissoid of Diocles, between the values $x = 0$ and $x = \frac{1}{2}a$.

6. Show that if V be the volume of a solid of revolution

$$\frac{dV}{dx} = \pi y^2.$$

7. Give the general expression for the distance of the centre of gravity of a mass from the axis of y; and apply this to finding the distance of the centre of gravity of any segment of a sphere, and also of a hemisphere, from the centre of the sphere.

8. State Guldin's properties of the centre of gravity: and, by this method, find the volume of the solid formed by the revolution of a parabola whose altitude is a and base $2b$; 1st about a tangent at its vertex; 2nd about its base.

M.

I. GEOMETRY.

1. Show that in any triangle, if straight lines be drawn from each of the angles to the middle of the opposite side, four times the sum of the squares of these lines is equal to three times the sum of the squares of the sides of the triangle.

2. *ADB* is a semicircle of which the centre is *C*, and *AEC* is another semicircle on the diameter *AC*; *AT* is a common tangent to the two semicircles at the point *A* : show that if from any point *F*, in the circumference of the first, a straight line *FC* is drawn to *C*, the part *FG*, cut off by the second semicircle, is equal to the perpendicular *FH* on the tangent.

3. Equal triangles, which have an angle in the one equal to an angle in the other, have their sides about the equal angles, &c.: and triangles which have, &c.

4. If two straight lines be cut by parallel planes, they shall be cut in the same ratio.

CO-ORDINATE GEOMETRY.

5. Find the equation to a straight line drawn through a point whose co-ordinates are $x' = 5$, $y' = 7$, and parallel to a straight line whose equation is $y = 3x - 4$; and construct both these lines to a scale by means of their equations, showing by the figure that the second line is parallel to the first and passes through the point required.

6. Find the intersections of a straight line whose equation is $y = 2x + 5$ with a circle whose radius is 5, the centre of the circle being the origin of the co-ordinates.

7. Find the equation to the tangent at a given point of a parabola; and show that the subtangent is equal to twice the *abscissa*.

M.

II. Arithmetic and Algebra.

1. A person purchased an estate at the rate of £84. 7*s.* 6*d.* per acre, and sold it at the rate of £90. 14*s.* 0¾*d.* per acre: how much per cent. did he gain on the purchase-money?

2. Divide £16815 among three persons *A*, *B*, *C*, so that *A*'s share shall be to *B*'s as 3 to 7, and *B*'s to *C*'s as 6 to 5; without using Algebra.

3. Given $\sqrt{x} - \sqrt{y} = 2$ and $(x+y)\sqrt{xy} = 510$; determine all the values of x and y which satisfy these equations.

4. The number of courses in an incomplete square pile is equal to the number of courses wanting to complete the pile, and the number of shot in the incomplete pile is equal to 6 times the number of shot wanting to complete it: find the number of shot in the incomplete pile.

5. Find the greatest root of the equation, $x^3 - 3x^2 - 2x + 1 = 0$, and the integer limits of the other two roots.

6. The sum of three numbers in geometric progression is 21, and the sum of their reciprocals is $\frac{7}{12}$: find the numbers.

7. Decompose the fraction $\frac{30}{x^3 - x^2 - 6x}$ into three fractions having denominators of the first degree.

8. Show that
$$a^x = 1 + Ax + \frac{A^2x^2}{1.2} + \frac{A^3x^3}{1.2.3} + \&c.$$
$$\text{where } A = a - 1 - \frac{1}{2}(a-1)^2 + \frac{1}{3}(a-1)^3 - \&c.$$

9. Show that, if y and $y_{,}$ are any two numbers,
$$\log_a (yy_{,}) = \log_a y + \log_a y_{,}, \text{ and } \log_a \frac{y}{y_{,}} = \log_a y - \log_a y_{,};$$
also, n being any number whole or fractional, $\log_a y^n = n \log_a y$.

M.

III. TRIGONOMETRY AND MENSURATION.

1. Sin 30^0 being $\frac{1}{2}$, determine the values of sin 15^0, cos 15^0.

2. Show than $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \,.\, \tan B}$.

3. Show that $\dfrac{\sin A + \sin B}{\sin A - \sin B} = \dfrac{\tan \frac{1}{2}(A - B)}{\tan \frac{1}{2}(A + B)}$.

4. In a plane triangle, show that

$$\frac{a}{b} = \frac{\sin A}{\sin B}, \quad \frac{a}{c} = \frac{\sin A}{\sin C}, \quad \frac{b}{c} = \frac{\sin B}{\sin C}:$$

state to what case in the solution of plane triangles this is applicable, and point out when the solution is ambiguous and when not.

5. From the top of a tower, 60 feet high, standing near the edge of a cliff, the angle of depression of a ship's hull was observed $15^0\ 19'\ 36''$, and from the base of the tower, the angle of depression of the ship's hull was $13^0\ 5'\ 20''$: find the horizontal distance of the ship from the tower, and the height of the tower's base above the level of the sea.

6. Simultaneous observations at two stations, *A* and *B*, in the same vertical plane as a balloon, gave the angles of elevation of the bottom of the car $13^0\ 59'\ 40''$, and $21^0\ 00'\ 12''$: the station *A* was 6810 feet further from the balloon, in horizontal distance, than *B*, and 300 feet above the level of *B*: find the vertical height of the car above the level of *B* and its horizontal distance from that station.

7. The three sides of a triangle are 6, $6 + \sqrt{2}$, $6 - \sqrt{2}$; find its area.

8. Show that the volume of the frustum of a cone whose altitude is a, and the radii of whose base and upper surface are R and r, is

$$\tfrac{1}{3}\pi a\,(R^2 + Rr + r^2).$$

9. *ABC* is a cone whose altitude $AD = 4$ inches, and diameter of its base $BC = 6$ inches; and *EFGH* is a cylinder inscribed in it. The convex surface of the cylinder is one-fourth *that of* the cone: find the dimensions of the cylinder.

M.

IV. STATICS.

1. Show that, if from any point L, in the direction of the resultant R, of two forces P and Q, perpendiculars, LM, LN, be let fall upon the directions of the forces P and Q, then $P : Q :: LN : LM$.

2. The ends of a thread, 16 inches long, are fixed to pins at the points A and B, 14 inches apart, and at the point C in it, at the distance of 6 inches from A, a force of 30 lbs. is applied by means of another thread attached there, and so that, when the threads are stretched, the angle ACD is a right angle: find the strain upon the pins A and B, that is, the tensions of the strings CA and CB.

3. An equilateral triangle has a square described on one of its sides: find the position of the centre of gravity of the whole figure.

4. Show how the resultant of any number of forces, acting in the same plane, is determined, by resolving the forces in the directions of rectangular axes; and apply this to finding the resultant of three forces, $A = 3$, $B = 4$, $C = 2$, $D = 6$, and the angle which its direction makes with that of the force A, the angles which the directions of the forces B, C, D make with that of A being 30°, 45°, 60° respectively.

5. Suppose that the vertical pressure of a mortar on a prop placed under a particular part of its bed would be 9 cwt.; that a 6-feet handspike is placed under the bed so that this point rests upon the handspike at the distance of 6 inches from its lower end, which rests on the platform; and that the handspike is inclined to the horizon at an angle of 30°: with what force must a man press perpendicularly to the handspike, at its upper extremity, so as just to sustain the vertical pressure of the mortar, the coefficient of friction being ·5.

M.

V. DYNAMICS.

1. Two inelastic bodies, A and B, move in the same direction with uniform velocities a and b (a greater than b), show that after the impact of A on B they will move with a common velocity equal to $\frac{Aa + Bb}{A+B}$.

2. Two ivory spheres, 3 inches and 4 inches in diameter, move in opposite directions with uniform velocities, 25 feet and 10 feet per second; find their motions after impact, the elasticity of ivory being represented by the fraction $\frac{6}{7}$.

3. To what height would a shot rise if fired vertically with a velocity of 1200 feet per second, and in what time would it again reach the ground, abstracting the resistance of the air?

4. A body, A, resting upon a plane inclined at an angle of 30° to the horizon, is connected with an equal body B by means of a string passing over a fixed pulley at the top of the plane, B hanging freely: in what time will A describe 100 feet on the plane?

(1) Supposing no friction on the plane.

(2) Supposing the friction to be one-tenth of the pressure on the plane.

5. Find the straight line of quickest descent from a given point without a given circle to the circle.

6. The length of the pendulum vibrating seconds (in a vacuum) in the latitude of London having been found to be 39·139 inches, determine the value of g which represents the force of gravity.

7. If a clock constructed to beat seconds gains 1 minute in a day, how, and to what extent, must its pendulum be altered?

8. Show how the velocity and angle of elevation may be determined, that a shot may pass through two given points of which the co-ordinates are p, q and p_1, q_1.

M.

VI. DIFFERENTIAL CALCULUS.

1. u being a function of x, find $\dfrac{du}{dx}$ when

$$u = \frac{x(1+x^2)}{1-x}; \qquad u = \log_e \sqrt{\frac{\sqrt{(x^2+a^2)}-x}{\sqrt{(x^2+a^2)}+x}};$$

$$u = \frac{(a^2+x^2)^{\frac{3}{2}}}{\sqrt{(a^2-x^2)}}; \qquad u = e^x \sin^2 x \,.\, \cos x.$$

2. Find the values of x which render $(x^2+10x+11)(7-x)^2$ a maximum or a minimum.

3. Of all cylindrical pontoons terminated by equilateral cones, having the same given surface $= a^2$, to find that which has the greatest volume.

4. Find the value of the subtangent in a curve whose equation is

$$y^3x^2 = b\,(a^2-x^2)^2.$$

INTEGRAL CALCULUS.

5. Find the following integrals:

$$\int \frac{6x^4\,dx}{(a^5+x^5)^{\frac{3}{5}}}; \qquad \int \frac{3dx}{2-5x+2x^2}; \qquad \int x^2\sqrt{(1-x^2)}\,dx.$$

6. Find the area of the curve whose equation is

$$a^2x^2 = y^2(a^2+x^2), \text{ from } x = 0 \text{ to } x = \frac{1}{2}a.$$

7. In what time would a body let fall from a point at a distance from the earth's surface equal to its radius, reach the earth, and with what velocity would it strike; the force of gravity varying inversely as the square of the distance from the earth's centre?

8. Give the differential expression for the distance of the centre of gravity of any body from the axis y; and point out what this becomes for an area, and also for a solid of revolution about the axis x.

9. Find the distance of the centre of gravity of a parabola, and also of a paraboloid, from the vertex.

N.

I. GEOMETRY.

1. Bisect a triangle by a line drawn from a given point in one of its sides.

2. In a circle, the angle in a semicircle is a right angle; the angle in a segment greater than a semicircle is less than a right angle, and the angle in a segment less than a semicircle is greater than a right angle.

3. Triangles, and also parallelograms, having the same altitude, are to one another, &c.

4. If a straight line stand at right angles to each of two straight lines at the point of their intersection, it shall be at right angles to the plane in which they are.

CO-ORDINATE GEOMETRY.

5. Find the equation to a straight line passing through two given points, the rectangular coordinates to which are respectively $x' = 3$, $y' = 5$, $x'' = 7$, $y'' = 2$; and find the distance from the origin at which this line cuts the axes, and also the angle which it makes with the axis of x.

6. The equation to a given straight line being $y = 3x - 4$, find the length of the perpendicular let fall on it from a point whose co-ordinates are $x' = 2$, $y' = 5$; and represent correctly the positions of the given line and the given point.

7. Find the equation to the circle whose radius is r, and the co-ordinates to its centre x', y'; and thence deduce the values of y corresponding to $x = 7$, in the circle whose radius is 5 and co-ordinates to its centre $x' = 9$, $y' = 2$.

8. In the parabola, if a perpendicular on the tangent at any point be drawn from the focus, the tangent at the vertex will pass through the point of intersection.

9. In an ellipse show that the square of the transverse axis is to the square of the conjugate, as the rectangle of the abscissæ is *to the* square of the ordinate,

N.

II. ARITHMETIC AND ALGEBRA.

1. A father left $\frac{2}{5}$ of his property to one son, $\frac{5}{9}$ of the remainder to a second, and the rest to a third: the difference between the shares of the second and third was £1200: what was the share of each? (*Arithmetically.*)

2. Solve the following equations:

(1) $x^2 + 2\sqrt{(x^2 - ax)} = 2\{ax + \sqrt{(ax)}\}$.

(2) $x + y = axy$, and $x^2 + y^2 = b^2x^2y^2$.

(3) $7^{2x} = 2 + 7^x$, applying logarithms.

3. Form an equation of which the roots are $5 + \sqrt{3}$, $5 - \sqrt{3}$, $2 + \sqrt{(-1)}$, $2 - \sqrt{(-1)}$ and -3.

4. Find the least positive root of the equation

$$x^3 - 10x^2 + 33x - 20 = 0,$$

by Horner's method, and state the character of the other roots.

5. The first term of a geometric series is 5 and the ratio 2: how many terms of this series must be taken, that their sum may be equal to 33 times the sum of half that number of terms?

6. Show, by the method of indeterminate coefficients, that the number of shot in a square pile, of which n is the number of courses, is

$$\frac{(2n+1).(n+1).n}{3.2.1}.$$

7. Assuming that the coefficient of the second term of the expansion of $(1+x)^n$ is n, whatever n may be, prove the Binomial Theorem.

8. Given the sum of two sides of a triangle $= s$, the perpendicular from the vertical angle on the base $= a$, and the diameter of the circumscribing circle $= d$: find all the sides of the triangle.

9. In a triangle, given the perpendicular from the vertical angle on the base $= a$, the base $= 2b$, and the difference of the other two sides $= 2d$; to find those sides.

N.

III. TRIGONOMETRY AND MENSURATION.

1. Find the value of cot $(A \pm B)$ in terms of cot A and cot B.

2. Given $\tan\theta = 2\sin\theta$; find θ.

3. In any triangle, a, b, c being the sides opposite to the angles A, B, C, show that

$$\frac{a+b}{a-b} = \frac{\tan\frac{1}{2}(A+B)}{\tan\frac{1}{2}(A-B)},$$

and point out how this is applied to the solution of the case where two sides and the included angle are given.

4. a, b, c being the sides of a triangle opposite to the angles A, B, C, show that

$$\cos B = \frac{a^2+c^2-b^2}{2ac};$$

and deduce from this the value of $\cos\frac{1}{2}B$.

5. If the exterior face of a front of fortification is 360 yards, the face of each bastion 100 yards, the angles which the lines of defence make with the exterior face each 16^0, and the flanks are perpendicular to the lines of defence,—what is the length of the flank and of the curtain?

6. Investigate the formula for the area of a triangle when its three sides are given; and find the area of a triangle of which the three sides are 4, $4+\sqrt{3}$ and $4-\sqrt{3}$.

7. How many square feet are in the floor of a room, which is a regular octagon having each diagonal 16 feet?

8. Deduce an expression for the surface of a square pyramid of which the altitude is a, and a side of the base is b.

9. If the depth of the ditch of a field-work is 10 feet, and its width at the bottom 8 feet less than at the top, what must be the width at the top that the ditch may furnish earth for the work the area of whose vertical section is 143 feet, allowing an increase in area of a tenth in the earth thrown out, when thus applied?

N.

IV. SPHERICAL TRIGONOMETRY.

1. Show that in a spherical triangle *ABC*, right-angled at *B*, α, β, γ being the sides opposite the angles *A*, *B*, *C*,

$$\sin A = \frac{\sin \alpha}{\sin \beta}, \text{ and } \tan A = \frac{\tan \alpha}{\sin \gamma};$$

and state *Napier's Rules* for the solution of right-angled spherical triangles.

2. Show that, in any spherical triangle,

$$\cos A = \frac{\cos \alpha - \cos \beta \cos \gamma}{\sin \beta \sin \gamma}.$$

3. Show that, in any spherical triangle, if $\sigma = \frac{1}{2}(\alpha + \beta + \gamma)$,

$$\sin^2 \frac{1}{2} A = \frac{\sin (\sigma - \gamma) \sin (\sigma - \beta)}{\sin \beta \sin \gamma}; \quad \cos^2 \frac{1}{2} A = \frac{\sin \sigma \sin (\sigma - \alpha)}{\sin \beta \sin \gamma}.$$

ASTRONOMY.

4. Define the following terms: Meridian, Prime Vertical; Right Ascension, Declination, Latitude, Longitude of a *heavenly body;* Equator of the Earth, Latitude and Longitude of a Place on the Earth.

5. Show, by a figure, how the obliquity of the ecliptic is determined from the Sun's meridian altitudes at the summer and winter solstices.

6. Given the latitude of the place and the Sun's declination, show by a figure how the length of the day, and the Sun's azimuth at rising or setting, may be determined.

7. Show how the hour lines are determined in a vertical south dial (in north latitude), and also in a vertical east or west dial.

8. Show how the latitude of a place is determined from two observed altitudes of the Sun on a given day, and the observed interval of time between the observations.

N.

V. STATICS.

1. A force represented by 100 lbs. acting in a direction which makes an angle of 60° with the horizon, is counteracted by two forces making angles of 30° and 120° with the horizon: find the weights which represent these forces.

2. Four equal bodies are placed in the angular points of the four solid angles of a triangular pyramid, show that their centre of gravity coincides with that of the pyramid.

3. A handspike is placed under the breach of a heavy gun, and rests on its carriage to support it for elevating: what force must be applied at the further end of the handspike, supposing its whole length to be 5 feet 6 inches; the distance of the line of support on the carriage cheek from the end on which the gun rests, 9 inches; the distance of the part of the breech of the gun resting on the handspike from the axis of the trunnions 3 feet; the distance of the centre of gravity of the gun from the same axis 6 inches; and the weight of the gun 28 cwt. 2 qrs.?

4. In a system of pulleys in which the strings are parallel, and each string is attached to the weight, there being 4 pulleys in the system: show that, when the power P is in equilibrium with the weight W,

(1) Neglecting the weight of the pulleys, $W = (2^4 - 1) P$,

(2) If A is the weight of each pulley, $W = (2^n - 1)(P + A) - 4A$.

5. A power of 50 lbs. is applied at the extremity of an arm, 15 inches long, to a screw in which the distance between the threads is a third of an inch: what is the pressure exerted at the head of the screw, in the direction of the axis?

6. A 13-inch shell weighing 200 lbs. rests between two planes AB, BC inclined at angles of 30° and 45° to the horizon: find the pressure on each plane.

N.

VI. DYNAMICS.

1. A glass sphere half an inch in diameter, moving with a velocity of 24 feet per second, impinges directly on another sphere of glass 1 inch in diameter and moving in the same direction with a velocity of 6 feet per second: in what directions and with what velocities will the bodies move after impact, the elasticity of glass being $\frac{15}{16}$?

2. A body is projected vertically upwards with a velocity of 483 feet per second, in what time will it rise through 1610 feet?

3. Two bodies *A* and *B*, each weighing 5 lbs. avoirdupois, are connected by a string passing over a fixed pulley: what space will they describe in 10 seconds when an ounce weight is added to *A*, and what velocity will they have at the end of that time?

4. Find geometrically the straight line of quickest descent;

(1) From a given straight line to a given point.

(2) From a given point within a circle to the circumference.

(3) Between the circumferences of two given circles, the one being wholly within the other.

5. Investigate the expression for the range of a projectile on a given plane passing through the point of projection, in terms of v, the velocity of projection; e, the angle of elevation of the projectile; and i, the inclination of the plane of the horizon.

6. In Atwood's machine, a weight of 22 oz. being placed on one side, and 23 oz. on the other, in what time will the heavier descend 64 inches, and what velocity will it have acquired, supposing the inertia of the wheels to be 3 oz. and $g = 32$ feet?

7. A particle is allowed to roll down the exterior of a parabola whose axis is horizontal; find the point where it will quit the curve.

N.

VII. HYDROSTATICS.

1. If W be the weight of a body in air, and w its weight in water, show that its specific gravity $=\frac{W}{W-w}$; and find the specific gravity of a piece of metal which weighs 102·74 grains in air, and 97·29 grains in water.

2. The whole length of a cylindrical pontoon with hemispherical ends is 22 feet; its diameter is 2 feet 8 inches; and its weight with the portion of the bridge it supports is 1550 lbs.: what additional weight does it bear when the axis of the cylinder is on the level of the surface of the water?

3. What weight of fir-wood, specific gravity 0·57, must be attached to a piece of copper, specific gravity 8·79, and weighing 10 oz., that the mass may just float?

4. A prismatic diving-bell, of which the height inside is 8 feet, is sunk in the sea to the depth of 70 feet; find the height to which the water will rise inside the bell, and the density of the included air compared to that at the surface of the sea where the pressure of the atmosphere is 33 feet of sea water.

5. Describe the principle of the barometer, and how this instrument is applied to the determination of the difference in the level of points on the earth's surface.

6. Describe the action of the Fire Engine.

7. Describe the action of the Bramah Press.

N.

VIII. DIFFERENTIAL CALCULUS.

1. Find the differentials of the following functions of x:—

$$u=(a+x)\sqrt{(a-x)}, \qquad u=\frac{x}{\sqrt{(a^2-x^2)}},$$

$$u=\log_e\frac{x-a}{x+a}, \qquad u=\sin x\,.\cos x.$$

2. If u is a function of x and u^1 represents u when in it x becomes $x+h$, what is the value of u^1 (Taylor's Theorem)?

3. From the expression for the range, $r=\frac{2v^2}{g}\,\frac{\cos e\sin(e-i)}{\cos^2 i}$, find the value of e which gives r a maximum on a given plane whose inclination is i, the shot being fired with a given velocity v.

4. Find the value of the subtangent in the ellipse.

INTEGRAL CALCULUS.

5. Find the following integrals:

$$(1)\ \int\frac{x^4dx}{(a^5-x^5)^{\frac{4}{5}}}. \qquad (2)\ \int\frac{dx}{1-3x+2x^2}. \qquad (3)\ \int\frac{x^2dx}{(1-x^2)^{\frac{3}{2}}}.$$

6. Find the area of a curve whose equation is

$$ax=y\sqrt{(a^2-x^2)}, \text{ from } x=0 \text{ to } x=a.$$

7. Find the area of an elliptic segment in terms of a circular segment radius $=a$ (the semi-axis major), and having the same abscissa x.

8. Find the volume of a paraboloid; and show that it is equal to half the circumscribing cylinder.

9. Find the volume of a ring in the form of a double cycloid, and having an elliptical section.

ANSWERS.

ANSWERS.

1. **67.**

1. 317 generations and $2\frac{616}{857}$ years. **2.** 35. **3.** £900.

4. 11 hrs. 59 m. 26·34 s. **5.** 10000. **6.** £98959. 8*s.* 5·17*d.*

7. 18. **8.** 27·5. **9.** 60. **10.** 2000. **11.** 15.

12. £85. 17*s.* $2\frac{5}{8}$*d.* **13.** £119. 11*s.* 5·14*d.*

14. 75 cwt. nitre, 12·5 cwt. charcoal, 12·5 cwt. sulphur.

15. $2\frac{2}{3}$ in. **16.** 98. **17.** 3 hrs. 36 m. **18.** $6\frac{1669}{1701}$ days.

19. 1016. **20.** 9 hrs. 49 m. $5\frac{5}{11}$ s.

21. (1) 3 hrs. 16 m. 21·82 s. **22.** One twenty-fourth.
(2) 3 hrs. 49 m. 5·45 s.
(3) 3 hrs. 32 m. 43·64 s.

23. 24854·85. **24.** Excess of French gun 1 cwt. 47·4 lbs.

25. 8 days 8 hrs. **26.** 4100 days. **27.** 111. **28.** 3·84 fr.

29. 6 hrs. 40 m. **30.** 12, 22; 7 galls. **31.** ·880339 quart.

32. $\frac{152}{299}$. **33.** $\frac{747}{215}$. **34.** ·018374. **35.** ·15.

36. $\frac{1}{7}$. **37.** 22 shillings. **38.** $\frac{3}{4}$. **39.** $\frac{5}{48}$. **40.** $\frac{2}{3}$.

41. $\frac{15}{16}$. **42.** ·858333 crown, $\frac{103}{384}$ Napoleon.

43. £54. 14*s.* $9\frac{1}{2}$*d.* **44.** £100.

45. To *A* 4 cwt. 2 qrs. 18 lbs. $10\frac{2}{3}$ oz. To *B* 13 cwt. 1 qr. 16 lbs. $12\frac{4}{5}$ oz.

46. £16000. **47.** $\frac{112}{153}$. **48.** 1*s.* 4*d.* **49.** £100.

50. £100. **51.** 1418464·5125. **52.** ·163057.

53. 276·824165. **54.** ·049261; 00027004.

55. 2736·577; 63·216. **56.** 9655·651. **57.** 848·3968.

58. Sixth place of decimals. **59.** 27·69957.

60. ·0000000092976. **61.** $\frac{42343}{16650}$. **62.** $\frac{171}{185}$.

63. $1\frac{2}{3}$*d.*; 4*s.* 8*d.*; $1\frac{5}{11}$ inch. **64.** $\cdot06\dot{5}6\dot{8} = \frac{203}{99} \times \frac{9}{281}$.

65. ·00138251. **66.** $\frac{119}{3330} \times \frac{33}{853}$. **67.** 23·11422; 8·114384.

68. ·0021814. **69.** 1·4142135; 1·259921. **70.** 1·79256.

71. 2·571281599. **72.** 3·0365889721. **73.** 1·912931182.

74. 10921227. **75.** 10000. **76.** 12000. **77.** $7\frac{1}{2}$. **78.** £702. 9s.

79. 51 square feet, 11 rectangles of 1 foot × 1 in. and 4 square in.

80. 5 sq. ft. 7 (ft. × in.) 1 sq. in. 6 (in. × line) 8 sq. lines.

81. 30·479.

82. 57 sq. feet, 2 (ft. × in.) 2 sq. in. 3 (in. × line) 11 sq. lines.

83. 18096021930. **84.** 112344424214; 30332.

85. 46530163. **86.** 131102101. **87.** 155047.

88. 245. **89.** ·49825373. **90.** 28.

91. (1) $\frac{135}{2}$; (2) $\frac{9}{2}$; (3) $\frac{21}{2}$; (4) 0; (5) ∞. **92.** 2·707.

93. $\frac{2(b+1)(b+2)(8b+3)}{3}$. **94.** ·158385. **95.** 4·4083.

96. $a^{\frac{10}{20}}$; $b^{\frac{20}{20}}$; $3^{\frac{5}{20}}\,.\,a^{\frac{25}{20}}$; $d^{\frac{70}{20}}$. **97.** $\frac{3x^2-6xy+2y^2}{3\,.\,\sqrt{(2xy)}}$.

98. square both sides. **99.** square both sides.

100. $\frac{12+3\sqrt{6}}{5}$. **101.** $\frac{(x+y+y^2)(x-y)}{y(x^2+y^2)}$. **102.** $3-2\sqrt{2}$.

103. $\frac{-\sqrt{15}}{300}$. **104.** $\frac{2\sqrt{(x^2-y^2)}}{y}$.

105. Erratum in question; $\frac{\{x-\sqrt[3]{(xy^2)}\}^4\,.\,\{x+\sqrt[3]{(xy^2)}\}^2}{x^2+y\sqrt[3]{(x^2y)}-2\sqrt[3]{(x^4y^2)}}$.

Ans. $x^4+xy^2\,.\,\sqrt[3]{(xy^2)}-2x^2y\,.\,\sqrt[3]{(x^2y)}$.

106. $2\,\frac{a^2-b^2}{a^2+b^2}$. **107.** $(x-y)\sqrt{(x^2-y^2)}$. **108.** $\frac{x+y}{x-y}$.

109. $\frac{x^a+1}{x^a-1}$. **110.** $2\,\frac{x-(a^2-b^2)}{x+(a+b)^2}$. **111.** $4x$.

112. $\frac{(2x)^{\frac{1}{2}}}{4y^{\frac{2}{5}}}$. **113.** $2x^2$. **114.** $\frac{1}{x}$. **115.** $\frac{(a+x)^{\frac{1}{4}}}{a^{\frac{2}{5}}x^{\frac{4}{5}}}$.

116. $\frac{a^{\frac{1}{n}}+b^{\frac{1}{n}}}{a-b}$. **117.** $\frac{3x+y}{4x^2+2xy-y^2}$. **118.** $\frac{x+a}{x^2+4ax-5a^2}$.

119. Question should be $\frac{\{\sqrt[3]{(a^2b)}-\sqrt{(ab^3)}\}^3}{1-3a^{-\frac{1}{6}}b^{\frac{7}{6}}+3a^{-\frac{1}{3}}b^{\frac{7}{3}}-a^{-\frac{1}{2}}b^{\frac{7}{2}}}$. Ans. a^2b.

120. $\frac{a+b}{ab}$. **121.** $\frac{x^2-2x+2}{x^2-x-1}$. **122.** $\frac{a-b}{a+b}\surd\{a-b\surd(-1)\}$.

123. $1-\surd(a+x)$. **124.** 0. **125.** $\frac{2x}{y^2}\surd(x^2-y^2)$.

126. $2\frac{x^2}{a}$. **127.** 1. **128.** $\frac{x+y}{x-y}$.

129. $x^{\frac{3m+1}{m}}-x^3y^{\frac{1}{n}}+y^3x^{\frac{1}{m}}-y^{\frac{3n+1}{n}}$. **130.** $\frac{1}{x^4-y^4}$. **131.** $2\cdot\frac{x^2-y^2}{x^2+y^2}$.

132. 2. **133.** $\frac{4\,.\,x^{\frac{4}{3}}\,.\,y^{\frac{4}{3}}}{y^2-x^2}$. **134.** $\frac{1+\surd(1-x^2)}{x}$.

135. $21x^5-101x^4+10x^3+136x^2+12x-27$; $7x^3-\frac{38x^2}{3}-\frac{62x}{3}-42$;

remainder $-490x-279$.

136. $fk+el-cm-bn$. **137.** $\surd(x^{6n}-y^{6n})$. **138.** $(a^3-b^3)^{\frac{p+q}{pq}}$.

139. $4a^4b^4-2a^3b^5-14a^2b^6+7ab^7+12b^8-6a^{-1}b^9$. **140.** 2.

141. $a^6x^{\frac{3}{2}}-3a^{\frac{9}{2}}b^{\frac{1}{2}}xy^{\frac{1}{5}}+3a^3bx^{\frac{1}{2}}y^{\frac{2}{5}}-a^{\frac{3}{2}}b^{\frac{3}{2}}y^{\frac{3}{5}}$.

142. $\frac{2}{3}y^2z-6yz^2-\frac{3}{2}z^3$. **143.** $x^{\frac{1}{2}}-\frac{3}{4}x^{\frac{1}{3}}+1$. **144.** $a-\frac{3}{4}a^{\frac{1}{2}}+1$.

145. $\sqrt{\left\{\frac{c(a+b+c)+ab}{2}\right\}}+\sqrt{\left\{\frac{c(c-a-b)+ab}{2}\right\}}$.

146. $\sqrt[4]{3}\,.\,(\surd 2-1)$; $w^2-x=1$. **147.** $\sqrt[4]{3}\,.\,(\surd 2-1)$.

148. $\frac{1}{2}\cdot\sqrt[4]{20}\,.\,(\surd 3-1)$. **149.** $\surd(55)-1$. **150.** $5-3\surd 2$.

151. $\surd 5+\surd 2$. **152.** $3+2\surd 3$.

153.

$$\begin{array}{r|l}
1 & 1+(a^2-2b^2)\quad-(a^4-b^4)\qquad\qquad-(a^6+2a^4b^2+a^2b^4) \\
+(a^2+b^2) & \quad+(a^2+b^2)\quad+(2a^4+a^2b^2-b^4)+(a^6+2a^4b^2+a^2b^4) \\
\hline
 & 1+(2a^2-b^2)\quad+(a^4+a^2b^2)
\end{array}$$

Quotient $x^4+(2a^2-b^2)x^2+a^4+a^2b^2$.

154. $6(x+y)-20$.

155. $\frac{6a}{xy}-10+\frac{21xy}{a}-\frac{120x^2y^2}{a^2}+\frac{135x^3y^3}{a^3}-\frac{663x^4y^4}{a^4}+$ &c.

156. $\frac{3x}{4}-\frac{15}{4}x^{-1}+\frac{15}{2}x^{-3}+x^{-4}-15x^{-5}-2x^{-6}+$ &c.

157.

$$\begin{array}{r|rrrrrrrrrr}
1 & 1-3 & -31 & +25 & +\ 3 & -15 & -\ 8 & +\ 19 & +\ 3 & +\ 10 & \\
+7 & & +7+28 & -21 & +\ 7 & \pm\ 0 & +14 & -(21) & \pm(0) & -(21) & +(14)+(56) \\
\pm 0 & & \pm\ 0 & \pm\ 0 & \pm\ 0 & \pm\ 0 & \pm\ 0 & \pm\ 0 & \pm(0) & \pm\ (0) & \pm\ (0)\pm\ (0) \\
-3 & & & -\ 3 & -12 & +\ 9 & -\ 3 & \pm\ 0 & -\ 6 & +\ (9) & \pm\ (0)+\ (9) \\
+2 & & & & +\ 2 & +\ 8 & -\ 6 & +\ 2 & \pm 0 & +\ 4 & -\ (6)\pm\ (0) \\
\hline
 & 1+4 & -\ 3 & +\ 1 & \pm\ 0 & +\ 2 & -\ 3 & \pm\ 0 & -\ 3 & +\ 2 & +\ 8+65
\end{array}$$

The coefficients in brackets are not used in determining the final remainder.

Quotient $\frac{x^5}{3}+\frac{4x^4}{3}-x^3+\frac{x^2}{3}+\frac{2}{3}$.

"Final remainder" $-3x^3+21x^2-3x+14$.

Continuation of Quotient $-\frac{1}{x}-\frac{1}{x^3}+\frac{2}{3x^4}+\frac{8}{3x^5}+\frac{65}{3x^6}+\&c.$

158. $1-2x^{-1}+3x^{-2}-5x^{-3}+8x^{-4}-13x^{-5}$;

$$\frac{3x}{2}+\frac{9}{2}+6x^{-1}-\frac{11}{2}x^{-2}-\frac{75}{2}x^{-3}-\&c.$$

159. $x+x^{-1}+x^{-2}+x^{-3}+\frac{x^{-3}}{x-1}$;

$$\frac{7}{2}=3+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{27\times 2}=\frac{189}{54}=\frac{7}{2}.$$

160. $x^5-3x^3+2x^2+6x-1$; $-2x^2+2x+1$. **161.** $3y^3+18y^2-10$.

162. $a^4b^6+3a^5b^5-5a^6b^4+7a^7b^3-9a^8b^2+\frac{a^9b}{2}$; $2a^5-3a^6b^{-1}-4a^7b^{-2}$.

163. $x^5z^2-5x^4yz^3-11x^3y^2z^4-10x^2y^3z^5-40xy^4z^6-55y^5z^7$;
$461xy^7z^6-350y^8z^7$.

164. $a^6b^{-6}+3a^5b^{-5}-5a^4b^{-4}+7a^3b^{-3}-2a^2b^{-2}+4ab^{-1}+6$;
$7a^{-2}b^5-4a^{-3}b^6+5a^{-4}b^7$.

165. $x^5y^{-9}+2x^4y^{-8}-3x^3y^{-7}+6x^2y^{-6}+3xy^{-5}$;
$22x^{-1}y^3+21x^{-2}y^4-6x^{-3}y^3$.

166. $x^{-2}z^2+2x^{-1}y^{-1}z^3+3y^{-2}z^4-4xy^{-3}z^5+5x^2y^{-4}z^6+6x^3y^{-5}z^7-7x^4y^{-6}z^8$;
$-10x^{10}y^{-7}z^7-2x^{11}y^{-8}z^8$.

167. $6x^{-3}y^6+4x^{-2}y^5-2x^{-1}y^4+7y^3+9xy^2-8x^2y$; $-6y^{-3}+4xy^{-4}$.

168. $8a^{-8}b^8-7a^{-7}b^7+6a^{-6}b^6-5a^{-5}b^5+4a^{-4}b^4-3a^{-3}b^3+2a^{-2}b^2-a^{-1}b$.

169. $ax^4y^5+3x^5y^4-2a^{-1}x^6y^3+5a^{-2}x^7y^2+6a^{-3}x^8y-4a^{-4}x^9+a^{-5}x^{10}y^{-1}$;
$5a^{-6}x^6-3a^{-7}x^7y^{-1}+6a^{-8}x^8y^{-2}+a^{-9}x^9y^{-3}$.

170. $1 - 2x^{-1} + 9x^{-2} - 23x^{-3} + \dfrac{71x^{-2} - 60x^{-3} - 23x^{-4}}{x^2 + 2x - 3 - x^{-1}}$

$$-3 = \frac{1+2}{1+2-3-1} = 1-2+9-23+\frac{71-60-23}{1+2-3-1} = -15+12 = -3.$$

Arrange the work thus:

-5	$5-4\pm0\pm0-1$	$1-1\pm0\pm0-1+1$	$\frac{1}{5}-\frac{1}{25}$
	$^*5\pm0+25-30$	$1-\frac{4}{5}\pm0\pm0-\frac{1}{5}$	
	$-4-25+30-1$	$-\frac{1}{5}\pm0\pm0-\frac{4}{5}+1$	
	$-4\pm0-20+24$	$-\frac{1}{5}+\frac{4}{25}\pm0\pm0+\frac{1}{25}$	
	$-25+50-25$	$-\frac{4}{25}\pm0-\frac{4}{5}+\frac{24}{25}$	
1	or $-1+2-1$	or $^*-1\pm0-5+6$	$1+2$
	$\dagger-1+1$	$-1+2-1$	
-1	$+1-1$	$-2-4+6$	
	$+1-1$	$-2+4-2$	
		$-8+8$	
		or $-1+1\dagger$	

$\therefore\ x-1$ is the G. C. M.

172. $2ax - 3y$. **173.** $x^3 + ax^2 - 5a^2x + 3a^3$. **174.** $x = \frac{3}{13}$.

175. $\frac{1286}{889}$. **176.** 35. **177.** 19. **178.** $0;\ -\frac{24}{25}$.

179. $x = \frac{653}{59};\ y = \frac{1102}{59}$. **180.** $\frac{411}{662}$. **181.** $\frac{9}{5};\ \frac{1}{5}$.

182. $3(1 \pm \sqrt{3})$. **183.** 4; 1. **184.** $\sqrt{\left\{24^2 - \left(\frac{77}{15}\right)^2\right\}}$.

185. $\frac{5 \pm \sqrt{417}}{2};\ \frac{5 \pm \sqrt{273}}{2}$. **186.** $\frac{4a^3}{(1+a)^2}$. **187.** 4; 3.

188. $2 \pm \sqrt{\frac{5 \pm \sqrt{-15}}{2}}$. **189.** $\frac{b \pm \sqrt{\{b(b-2a)\}}}{2}$. **190.** -3; 2.

191. $\pm\frac{3}{2}\{1 - \sqrt{(-3)}\}$. **192.** $\pm 5;\ \pm 3\sqrt{5}$. **193.** $0;\ \frac{\sqrt{3}}{2}$.

20—2

194. $9;\ 16;\ \dfrac{57 \pm 7\sqrt{65}}{2}$. **195.** $-\dfrac{a}{2}(1 \pm \sqrt{5})$. **196.** $\dfrac{ac^2}{b^2}$.

197. $3;\ -\dfrac{3}{4};\ \dfrac{81 \pm 9\sqrt{97}}{8}$. **198.** $-3;\ -5$. **199.** $2;\ \sqrt[3]{-b}$.

200. $\sqrt{\dfrac{1 \pm \sqrt{5}}{2}}$. **201.** $a;\ \dfrac{1}{a}$. **202.** $8;\ \dfrac{125}{64}$. **203.** $5;\ 2$.

204. $x = 1 - \sqrt{5};\ y = \sqrt{5} - 1$. **205.** $x = 4$ or $\dfrac{14}{3}$; $y = \dfrac{84}{65}$ or 1.

206. $x = \dfrac{653}{59};\ y = \dfrac{1102}{59}$. **207.** $x = 11$ or 5; $y = 5$ or 11.

208. $x = \dfrac{8 \pm 2\sqrt{51}}{7};\ y = 1 \pm \sqrt{\dfrac{51}{7}}$.

209. $x = \dfrac{17}{2}$ or 4; $y = \dfrac{7}{2}$ or 8.

210. $x = 4$ or $-1 \pm \sqrt{5}$; $y = 1$ or $1 \pm \sqrt{5}$.

211. $x = 2 \pm \sqrt{13};\ y = \dfrac{1}{4 \pm \sqrt{13}}$.

212. $x = 3;\ 2$ or $\dfrac{5 \pm \sqrt{-151}}{2}$; $y = 2;\ 3$ or $\dfrac{5 \mp \sqrt{-151}}{2}$.

213. $x = \left\{\dfrac{b}{2} \pm \dfrac{1}{2}\sqrt{\left(\dfrac{4a - b^3}{3b}\right)}\right\}^4;\ y = \left\{-\dfrac{b}{2} \pm \dfrac{1}{2}\sqrt{\left(\dfrac{4a - b^3}{3b}\right)}\right\}^4$.

214. $x = \dfrac{4a}{5};\ y = \dfrac{5a}{4}$.

215. $x = \dfrac{2(a + b) + 1 \pm \sqrt{(4a + 1)}}{4};\ y = \dfrac{2(a - b) + 1 \pm \sqrt{(4a + 1)}}{4}$.

216. $x = abc \cdot \dfrac{ab + ac - bc}{a^2b^2 + a^2c^2 - b^2c^2};\ y = abc \cdot \dfrac{ab + bc - ac}{b^2c^2 - a^2c^2 - a^2b^2}$.

217. If $b = \dfrac{1 + \sqrt{(4a + 1)}}{2};\ x = \dfrac{b \pm \sqrt{(a - 3b)}}{2};\ y = \dfrac{b \mp \sqrt{(a - 3b)}}{2}$.

218. $x = 8;\ y = 5$.

219. $x = \dfrac{3b^2 \pm \sqrt{\{3b(4a^3 - b^3)\}}}{6b};\ y = \dfrac{-3b^2 \pm \sqrt{\{3b(4a^3 - b^3)\}}}{6b}$.

220. In question read $x^2 + y^2$; $x = 6$; $y = 3$ or -2.

221. $x=5,\ 2$ or $\dfrac{7\pm\sqrt{(-303)}}{2}$; $y=2,\ 5$ or $\dfrac{7\mp\sqrt{(-303)}}{2}$.

222. $x=\left(\dfrac{a^{\frac{3}{2}}}{\sqrt{a}+\sqrt{b}}\right)^{\frac{1}{2}}$; $y=\left(\dfrac{b^{\frac{3}{2}}}{\sqrt{a}+\sqrt{b}}\right)^{\frac{1}{2}}$.

223. $x=\pm 2$; $y=\pm 2$. **224.** $x=\pm 5,\ \pm 3$; $y=\pm 3,\ \pm 5$.

225. $x=\dfrac{1}{2}(\sqrt[m]{3a}+\sqrt[n]{5b})$ or $\dfrac{1}{2}\{\sqrt[m]{(-a)}+\sqrt[n]{(-b)}\}$;

$y=\dfrac{1}{2}(\sqrt[m]{3a}-\sqrt[n]{5b})$ or $\dfrac{1}{2}\{\sqrt[m]{(-a)}-\sqrt[n]{(-b)}\}$.

226. $x=4$ or $\dfrac{5}{3}$; $y=2$ or $-\dfrac{1}{2}$.

227. $x=9$ or $\dfrac{-13+\sqrt{(-39)}}{2}$; $y=3$ or $\dfrac{-13-\sqrt{(-39)}}{2}$.

228. $x=4$; $y=1$. **229.** $x=6$; $y=4$.

230. $x=3,\ 2,\ -2,\ -3$; $y=2,\ 3,\ -3,\ -2$.

231. $x=3,\ 2,\ -\dfrac{5}{2},\ -\dfrac{7}{2},\ \dfrac{5}{2}\pm\sqrt{3},\ -3\pm\sqrt{3}$;

$y=2,\ 3,\ -\dfrac{7}{2},\ -\dfrac{5}{2},\ \dfrac{5}{2}\mp\sqrt{3},\ -3\mp\sqrt{3}$.

232. $x=\pm\dfrac{1}{\sqrt{3}}$; $y=\mp\dfrac{5}{\sqrt{3}}$. **233.** $x=\dfrac{4a}{5}$; $y=\dfrac{5a}{4}$.

234. $x=49$ or 25; $y=25$ or 49.

235. $x=\dfrac{ab}{a+b}\pm\dfrac{\sqrt{\{(a+b-ab)^2+4ab\}}}{a+b}$; $y=\dfrac{a-b}{a+b}$.

236. $x=3\sqrt{2}$; $y=\sqrt{2}$. **237.** $x=1$; $y=\sqrt{2}$.

238. $x=\pm 2,\ \pm\dfrac{1}{\sqrt{3}}$; $y=\pm 1,\ \mp\dfrac{5}{\sqrt{3}}$.

239. $x=\pm 1,\ \pm 2$; $y=\pm 2,\ \pm 1$. **240.** $x=64$; $y=8$.

241. $x=2,\ -\dfrac{1}{3}$; $y=3,\ -\dfrac{1}{2}$. **242.** $x=5,\ -3$; $y=3,\ -5$.

243. $x=\pm 9$; $y=\pm 4$. **244.** $x=\pm 3,\ \pm 1$; $y=\pm 1,\ \pm 3$.

245. $x=2$; $y=1$; $z=4$. **246.** $x=1$; $y=2$; $z=4$.

247. $x=10$; $y=5$; $z=3$.

248. $z=\dfrac{(b_{,}a_{,,}-b_{,,}a_{,})(a_{,}d-ad_{,})-(ba_{,}-b_{,}a)(a_{,,}d_{,}-a_{,}d_{,,})}{(b_{,}a_{,,}-b_{,,}a_{,})(ca_{,}-c_{,}a)-(ba_{,}-b_{,}a)(c_{,}a_{,,}-c_{,,}a_{,})}$.

249. $z = \dfrac{(e_1a_2 - e_2a_1)(b_1a_3 - b_3a_1) - (e_1a_3 - e_3a_1)(b_1a_2 - b_2a_1)}{(c_1a_2 - c_2a_1)(b_1a_3 - b_3a_1) - (c_1a_3 - c_3a_1)(b_1a_2 - b_2a_1)}$.

250. $x = \dfrac{1}{3}$; $y = \dfrac{1}{2}$; $z = \dfrac{1}{4}$. **251.** $x = -5$; $y = 7$; $z = 2$.

252. $z = \dfrac{(ce - bf)(b^2 + ac) - (bd + ae)(c^2 + ab)}{(c^2 + ab)(a^2 + bc) - (b^2 + ac)^2}$.

253. $x = 6$; $y = 12$; $z = 60$.

254. $y = \dfrac{(bk - al)(ac - b^2) - (cl - bm)(bc - a^2)}{(b^2 - ac)(ac - b^2) - (bc - c^2)(c^2 - ab)}$.

255. $x = 1$; $y = 2$; $z = 3$. **256.** $x = 1$; $y = 2$; $z = 4$.

257. $x = 1$; $y = 2$; $z = 5$. **258.** $x = 2$; $y = 7$; $z = 3$.

259. $x = 30$ or $\dfrac{71}{2}$; $y = \pm 29$ or $\dfrac{1}{2}\sqrt{(3626)}$; $z = 36$ or $-\dfrac{59}{2}$.

260. $v = \dfrac{a + d - 5b + 7c}{4}$; $x = \dfrac{a + b - 5c + 7d}{4}$;

$y = \dfrac{b + c - 5d + 7a}{4}$; $z = \dfrac{c + d - 5a + 7b}{4}$.

261. 32; 40. **262.** 6 cwt.; 9 cwt.; 8 cwt.

263. 4 hrs. 30 min. a.m.; 5 hrs. a.m. **264.** £1. 2*s.* 6*d.*

265. 7 hrs. 30 min.; 150 miles. **266.** 12. **267.** 400.

268. 530 gr. 490 gr. **269.** 24 days; 48 days. **270.** 131·25.

271. 1800. **272.** 1296. **273.** £2. 10*s.*; 13*s.* 4*d.*

274. $18\frac{70}{117}$; $20\frac{12}{13}$. **275.** 24750.

276. 260 miles; $2\frac{24}{27}$ miles per hour. **277.** 130.

278. 6 miles an hour; 20 miles.

279. 2*s.* $5\frac{2}{5}$*d.*; 7*s.* $4\frac{1}{5}$*d.*; 3*s.* $3\frac{1}{5}$*d.* **280.** 12.

281. Price of sulphur per cwt.

$$= x = \frac{ae + bf + cd \pm \sqrt{\{(ae + bf + cd)^2 - 4acde\}}}{2ad}.$$

Price of nitre per cwt. $= y = \dfrac{c - ax}{b}$.

282. 1; 3; 5; 7; 9. **283.** $2\frac{44}{203}$. **284.** 3 miles an hour.

285. 64 miles. The trains must travel in the same direction, the Bristol train overtaking the London train, and having started at 7 a.m.

286. *A* will have passed *B* three miles; *B* having turned back.

287. 22·3 miles. **288.** 32 hrs.; 24 hrs. **289.** 968; 704.

290. After 36 hours and 30 hours; 10 yards; $8\frac{1}{3}$ yards.

291. 7. **292.** £121; £154. **293.** 3·12.

294. $3\frac{2}{3}$ months. **295.** £1. 8*s*.

296. $\frac{k+4h\pm\sqrt{(k^2+16h^2-4hk)}}{6}$ hours;

$\frac{4h-k\pm\sqrt{(k^2+16h^2-4hk)}}{2}$ hours.

297. 5 p.m.; 6 p.m. **298.** 18 shillings.

299. -156; $105\sqrt{2}$. **301.** 5; $\frac{335}{49}$; $\frac{425}{49}$; $\frac{515}{49}$, &c.; 2500.

302. $(3n+2)n$. **303.** 30; 396. **304.** 73; 99.

305. $\frac{29}{3}$; $\frac{28}{3}$; 9; $\frac{26}{3}$; $\frac{25}{3}$. **306.** $-\frac{24}{5}$; $-\frac{38}{5}$; $-\frac{52}{5}$; $-\frac{66}{5}$.

307. 45. **308.** $(n+2)n$. **309.** -108; $-\infty$.

310. $3n\,.\,\frac{n+3}{8}$. **311.** $\frac{2205}{4}$. **312.** $3+3\frac{1}{3}+3+\frac{2}{3}+4+$ &c.

313. $-\frac{15}{2}$. **314.** $243\frac{1}{2}$. **315.** 24. **316.** 8.

317. $3+\frac{10}{3}+\frac{11}{3}+$ &c.; $5+\frac{16}{3}+\frac{17}{3}+$ &c.

318. 5; 7; 9; 11; 13.

319. 3 hrs. 25 m. **320.** 14 days. **321.** 4; 5; 6.

322. $S=\{2a+(n-1)b\}\frac{n}{2}$; $S_1=\{2a-(n-1)b\}\frac{n}{2}$;

$\therefore\ S-S_1=n(n-1)b$ and $S+S_1=2an$; $\therefore$ &c.

323. On the fifth day. **324.** 24 and 48 miles. **327.** 1536.

328. 49 and 1. **329.** -64. **331.** $\frac{\sqrt[3]{f}}{t}\cdot\frac{t^2-f^2}{\sqrt[3]{t}-\sqrt[3]{f}}$.

333. $\frac{11605}{6561}$. **334.** $\frac{3\sqrt[6]{3}}{\sqrt[6]{2}-1}$ and $8\sqrt[6]{3}$.

335. $\frac{30-15\sqrt{2}}{8}$ and $4-2\sqrt{2}$. **336.** $\frac{364}{81}$.

337. $-\frac{7}{8}\cdot\frac{\sqrt[4]{10}}{2+\sqrt[4]{8}}$ and $\frac{1}{8}\sqrt[4]{10}$. **338.** $62(\sqrt{2}-2)$.

339. $\frac{(1-p)\,3n.p^n+(1+2p)(p^n-1)}{(1-p)^2}$. **340.** $40(\sqrt{3}+1)$.

341. $\sqrt[3]{4}$; $\sqrt[3]{2}$; 1; $\sqrt[3]{\frac{1}{2}}$; $\sqrt[3]{\frac{1}{4}}$. **342.** $\frac{2-\sqrt{-1}}{5}$.

343. $\frac{13(3+1)}{27}$; $\frac{3+\sqrt{3}}{6}$. **344.** 1000; 3000 and 9000.

345. 63. **348.** 3276. **349.** 7.

352. $a : b :: c : d$, let a be the greatest, $\therefore$ d the least.

Let $\frac{a}{b}=m$, $\therefore \frac{c}{d}=m$ and $m>1$; $a+d=mb+d$ and $b+c=b+md$ $(mb+d)-(b-md)=(m-1)(b-d)$ which must be +, since $m>1$ and $b>d$, $\therefore a+d>b+c$.

354. Express in front of ordinary train $\frac{7^5-5^5}{2.3.5^3}-15$.

355. 25 and 100.

356. $x-y$; x and $x+y$ are in Arith. Progression; if they be also in Harm. Progression, we must have

$$\frac{1}{x-y}+\frac{1}{x+y}=\frac{2}{x};\ \therefore\ x^2=x^2-y^2;\ \therefore\ y^2=0;$$

$\therefore$ $x-y$; x and $x+y$ are identical.

Also $\frac{x-y}{x}=\frac{x}{x+y}$; $\therefore$ $x-y$, x and $x+y$ would be in Geom. Progression.

357. $\frac{1}{a}-\frac{1}{b}=\frac{1}{b}-\frac{1}{c}$, $\therefore \frac{a-b}{ab}=\frac{b-c}{bc}$;

$$\frac{1}{a}+\frac{1}{c-b}=\frac{c-b+a}{a(c-b)} \text{ and } \frac{1}{c}+\frac{1}{a-b}=\frac{a+c-b}{c(a-b)};$$

$$\therefore \frac{1}{a}+\frac{1}{c}+\frac{1}{c-b}+\frac{1}{a-b}=(a+c-b)\left\{\frac{1}{a(c-b)}+\frac{1}{c(a-b)}\right\}=0;$$

since $c(a-b)=a(b-c)$.

358. See Appendix. **359.** Divide by $(v + \cdot 5)$ thus:

$$\begin{array}{r|l} 1 & 1 - 5 \quad - 3 \quad \pm 0 \quad - 5 \quad + 2 \\ -\cdot 5 & - \quad \cdot 5 + 2\cdot 75 + 0\cdot 125 - 0\cdot 0625 + 2\cdot 53125 \\ \hline & 1 - 5\cdot 5 - 0\cdot 25 + 0\cdot 125 - 5\cdot 0625 + 4\cdot 53125 \end{array}$$

Hence the value of the given expression is 4·53125.

360. 158; 169; 56·92897104; 28·57983059. **361.** ·4161.

362. 2; ∞. **363.** 11·60768. **364.** – 1·1567768576.

365. 8·82489. **366.** 10·5712248832. **367.** – 12.

368. See Appendix. **369.** See Appendix.

370. $x^4 - 6x^3 + 6x^2 + 34x - 195 = 0$. **371.** $-13x^3$.

372. $\frac{b}{a}$; c and $-ad$. **373.** $+444x$. **374.** 70.

375. $x^4 - 5x^3 - 11x^2 + 149x - 230 = 0$. **376.** $+134x$.

377. 3 and – 4. **378.** 4 and – 7. **379.** 6 and – 2.

381. 3 and 2. **382.** $-14x^5$. **383.** See Appendix.

384. 1; 2; 3; – 1; – 3. **385.** ·3 is a root.

386. $2 \pm \sqrt{-5}$ and $2 \pm \sqrt{5}$. **387.** $n = 4$.

388. See Appendix.

389. $a + 1$ each equal to p; $b + 1$ each equal to q; and so on.

390. Three, each equal to 2; and two, each equal to $-\frac{1}{2}$.

391. Two pairs $\frac{1 \pm \sqrt{5}}{2}$ and – 2. **392.** Two pairs $4 \pm \sqrt{3}$.

393. – 2, – 2 and 5. **394.** Two pairs $\frac{3 \pm \sqrt{-3}}{2}$.

395. Three, each equal to – 3; and two, each equal to 2.

396. Two pairs $1 \pm \sqrt{-2}$.

397. Common root, 3; other roots 2 and 1; 6 and 5.

398. See Appendix. **399.** See Appendix.

400. $\begin{cases} x_{\prime}^3 - 11\cdot 91x_{\prime}^2 + 35\cdot 2827x_{\prime} + 29930773 = 0 \\ x_{\prime\prime}^3 + 4\cdot 5x_{\prime\prime}^2 - 5\cdot 25x_{\prime\prime} - 29\cdot 625 = 0. \end{cases}$

401. Five roots each equal to – 2.

402. $x_{\prime}^4 - \frac{130}{16}x_{\prime}^2 - \frac{306}{64}x_{\prime} - \frac{611}{256} = 0$. **403.** See Appendix.

405. See Appendix. **406.** See Appendix.

407. $\pm\sqrt{(-1)}, \dfrac{-5\pm\sqrt{21}}{2}; \dfrac{7\pm 3.\sqrt{5}}{2}, \pm\sqrt{(-1)}$.

408. Two pairs, (1) $\dfrac{-1\pm\sqrt{(-3)}}{2}$;

(2) $-\dfrac{2+\sqrt{3}\pm\sqrt{(3+4\sqrt{3})}}{2}, -\dfrac{2-\sqrt{3}\pm\sqrt{(3+4\sqrt{3})}}{2}$ and -1.

409. (1) $3\pm 2\sqrt{2}, 2\pm\sqrt{3}$; (2) $\pm\sqrt{(-1)}, \dfrac{5\pm\sqrt{21}}{2}, 1$.

410. $-3\pm 2\sqrt{2}, -2\pm\sqrt{3}, 1$. **411.** $-1, 3\pm 2\sqrt{2}, 2\pm\sqrt{3}$.

412. $3\pm 2\sqrt{2}, 2\pm\sqrt{3}, 1$. **413.** $4\pm\sqrt{15}, 3\pm 2\sqrt{2}, -1$.

414. $-2\pm\sqrt{3}, \dfrac{1\pm\sqrt{(-3)}}{2}, 1$.

415. $\dfrac{1+\sqrt{(-7)}\pm\sqrt{\{2\sqrt{(-7)}-22\}}}{4}, \dfrac{1-\sqrt{(-7)}\pm\sqrt{\{-2\sqrt{(-7)}-22\}}}{4}, 1$.

416. $1, \pm\sqrt{(-1)}, \dfrac{1\pm\sqrt{(-3)}}{2}$. **417.** See Appendix.

See Appendix. **419.** 4; and 6.

420. Four imaginary. **421.** Four imaginary.

422. See Appendix. **423.** 4.

424. When the given roots are eliminated, the resulting equation is $x^4-4x^2+7=0$, which can be solved as a quadratic, and which gives the roots $\pm\sqrt{\{2\pm\sqrt{(-3)}\}}$.

425. $\pm\sqrt{2}$.

426. The roots lie between 0 and 1; between 3·1 and 3·2, between 3·8 and 3·9; and between 5 and 6.

427. 2·4 and 2·7.

428. Two roots imaginary; one real root $-3·2842775377$.

429. Two imaginary; 2; $-·55402$.

430. Two imaginary $-1·074340759$.

431. Between 1 and 2; 2 and 3; -3 and -4; 1·282823.

432. 1·8514712. **433.** $-1\pm\sqrt{(-3)}$.

434. 1·21312775; 1·22952121; $-19·44264896$.

435. Two imaginary; $-3·396096121$.

436. *Two* imaginary; one between 0 and 1; 7·9877534345.

437. Two imaginary; $-\cdot75728$.

438. 3·58578643; between 5 and 6; 6 and 7.

439. 2·0591425; one between 0 and -1; two imaginary.

440. 1·1893274; 1·7205081 and two imaginary.

441. 1·71522526; by De Gua. **442.** Two pairs, $3 \pm \sqrt{5}$.

443. Between 1 and 2; 2 and 3; two imaginary.

444. (1) 1·4142136; $-1\cdot4142136$; two imaginary. (2) Two pairs, $3 \pm \sqrt{3}$.

445. Two pairs, $2 \pm 5\sqrt{(-1)}$; ·5 and ·5.

446. 1; $-2\cdot4288185$; two imaginary.

447. Two imaginary; one between -4 and -5; ·27188248.

448. ·24673212; $-\cdot41986821$; two imaginary.

449. $4+\frac{1}{2+}\,\frac{1}{2+}\,\frac{1}{2+}\,\frac{1}{2+}\,\frac{1}{2}$ and $4+\frac{1}{2+}\,\frac{1}{2+}\,\frac{1}{2+}\,\frac{1}{12+}\,\frac{1}{2}$;
or $x = 4\cdot4143$ and $x = 4\cdot4161$.

452. Irreducible. **453.** $x = 3$. **454.** 4·378765.

455. ·50794. **456.** 2·3275. **457.** 4·4641. **458.** 5·7652.

459. $x = 14$, $y = 0$; $x = 3$, $y = 5$.

460. 20 infantry regiments and 10 cavalry regiments.

461. $\frac{4}{5}$; $\frac{5}{6}$; $\frac{9}{11}$; $\frac{14}{17}$; $\frac{65}{79}$; $\frac{144}{175}$.

462. $\frac{2}{3}$; $\frac{5}{7}$; $\frac{12}{17}$; $\frac{29}{41}$; $\frac{70}{99}$; $\frac{169}{239}$; &c.

463. $\frac{1}{4}$; $\frac{21}{85}$; $\frac{64}{259}$; $\frac{213}{862}$; $\frac{1129}{4569}$; $\frac{2741}{10000}$.

464. $\frac{3}{4}$; $\frac{4}{5}$; $\frac{11}{14}$; $\frac{48}{61}$; $\frac{107}{136}$; $\frac{262}{333}$; $\frac{369}{469} = \frac{37}{47}$ nearly.

465. $\frac{7}{8}$; $\frac{15}{17}$; $\frac{22}{25}$; $\frac{59}{67}$; $\frac{81}{92}$; $\frac{140}{159}$; $\frac{1761}{2000}$.

466. $\frac{1}{6}$; $\frac{4}{25}$; $\frac{5}{31}$; $\frac{14}{87}$; $\frac{117}{727}$; $\frac{365}{2268}$; &c.

467. $\frac{9}{10}$; $\frac{10}{11}$; $\frac{39}{43}$; $\frac{400}{441}$. **467*.** $\frac{1}{2}$; $\frac{8}{15}$; $\frac{9}{17}$; $\frac{836}{1579}$.

468. $\frac{n(n+1)^2}{2}$. **469.** $n(n+1)\frac{2n+13}{6}$.

470. $\dfrac{n(n+4)(n+5)}{3}$. **471.** $15-3(5-3n)\,2^n$.

472. $3^{n+1}.(n-1)+3$.

473. $\dfrac{3.\{3^n.(2n-1)+1\}}{4}$. **474.** $\dfrac{n(n+1)(2n+1)}{6}$.

475. $\dfrac{n(n+1)(n+2)}{6}$. **476.** $\dfrac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$.

477. $\dfrac{n(2n+1)(2n-1)}{3}$. **478.** $\dfrac{3}{2(x-6)}-\dfrac{3}{5(x-3)}+\dfrac{1}{10(x+2)}$.

479. $\dfrac{6}{7(x-5)}-\dfrac{1}{5(x-3)}+\dfrac{12}{35(x+2)}$.

480. $\dfrac{3}{x-1}+\dfrac{5}{x-3}-\dfrac{2}{x+5}$.

481. $1-\dfrac{1}{11(3x-1)}-\dfrac{3}{x-2}+\dfrac{103}{11(x-4)}$.

482. $\dfrac{15x+3}{x^2-4x+5}+\dfrac{3}{x-2}$. **483.** $\dfrac{2x+3}{x^2-5x+7}-\dfrac{3}{x-2}$.

484. $\dfrac{4}{x-5}-\dfrac{9}{x-3}+\dfrac{42}{x+2}$. **485.** $\dfrac{3}{2(x-4)}-\dfrac{13}{6(x-2)}+\dfrac{5}{3(x+4)}$.

486. $\dfrac{64}{7(x-4)}-\dfrac{9}{x-5}-\dfrac{6}{7(x+2)}$.

487. $\dfrac{1}{6(x-1)}+\dfrac{287}{132(x-7)}-\dfrac{5}{4(x-3)}-\dfrac{11}{x+4}$.

488. $\dfrac{1}{x-3}+\dfrac{4}{x+7}-\dfrac{3}{2x+1}+\dfrac{2}{2x-1}$.

489. $\dfrac{4}{x-5}-\dfrac{2}{x^2-4x+13}$. **490.** $\dfrac{2x+1}{x^2-4x+5}+\dfrac{3}{x-2}$.

491. $\dfrac{257}{49(x-4)}+\dfrac{100}{7(x+3)}-\dfrac{110}{49(x+3)}$.

492. $\dfrac{3}{x-3}-\dfrac{4}{3(x-1)}-\dfrac{1}{2(x-4)}-\dfrac{1}{6(x+2)}$.

493. $\dfrac{7}{x-1}-\dfrac{3}{x+2}-\dfrac{5}{x+4}+\dfrac{12}{2x+3}$.

496. $$\frac{1}{x\sqrt{2}}\cdot\left\{1-\frac{1}{2}\cdot\frac{a^2}{x^2}+\frac{1.3}{2.2^2}\cdot\frac{a^4}{4x^4}-\frac{1.3.5}{2.3.2^3}\cdot\frac{a^6}{8x^6}+\&c.\ \ldots\ldots\right.$$
$$\frac{(-1)^{n-1}}{x\sqrt{2}}\cdot\frac{1.3.5\ldots(2p-3)}{\lfloor\underline{p-1}\,.\,4^{p-1}}\left(\frac{a}{x}\right)^{2p-1}.$$

497. $$v^{\frac{25}{2}}-\frac{5}{2}\cdot v^{\frac{15}{2}}\cdot a^3x^2+\frac{15}{8}\cdot v^{\frac{5}{2}}\cdot a^6x^4-\&c.\ \ldots\ldots$$
$$\frac{5.3.1.3.5\ldots(2p+9)}{\lfloor\underline{p-1}\,.\,2^{p-1}}\cdot v^{-(10p+35)}\cdot a^{3(p-1)}\cdot x^{2(p-1)}(-1)^{p-1}.$$

498. $$a^3x^{\frac{3}{2}}+\frac{3}{2}ax^{\frac{1}{2}}b+\frac{1.3}{2.2^2}\cdot a^{-1}\cdot x^{-\frac{1}{2}}b^2-\frac{1.3}{\lfloor\underline{3}\,.\,2^3}\cdot a^{-3}\cdot x^{-\frac{3}{2}}\cdot b^3+\ldots$$
$$\frac{1.3.3.5.7\ldots(2p-7)}{\lfloor\underline{p-1}\,.\,2^{p-1}}\cdot a^{5-2p}\cdot x^{\frac{5-2p}{2}}\cdot b^{p-1}\cdot(-1)^{p-1}.$$

499. $$a^3-\frac{3}{2}ax^2+\frac{3x^4}{8a}+\frac{x^6}{16a^3}-\ldots\ldots\frac{3.1.1.3.5\ldots(2p-7)}{\lfloor\underline{p-1}\,.\,2^{p-1}}\cdot\frac{x^{2n}}{a^{-2p+5}}.$$

500. $$a^{-\frac{3}{2}}+\frac{3}{4}a^{-\frac{7}{2}}x^2+\frac{21}{32}a^{-\frac{11}{2}}\cdot x^4+\frac{77}{128}a^{-\frac{15}{2}}x^6+\ldots\ldots$$
$$\frac{3.7.11\ldots(4p-5)}{\lfloor\underline{p-1}\,.\,4^{p-1}}\cdot a^{-\frac{4p-1}{2}}\cdot x^{2(p-1)}.$$

501. $$a^{-\frac{6}{5}}+\frac{3}{5}\cdot a^{-\frac{16}{5}}\cdot bx+\frac{3.8}{2.5^2}\cdot a^{-\frac{26}{5}}\cdot b^2x^2\ldots\ldots$$
$$\frac{3.8.13.18\ldots(5p-7)}{\lfloor\underline{p-1}\,.\,5^{p-1}}\cdot a^{\frac{4-10n}{5}}\cdot b^{p-1}\cdot x^{p-1}.$$

502. $$\frac{a}{c}+\frac{ax}{c^3}+\frac{1.3}{2}\cdot\frac{ax^2}{c^5}+\frac{1.3.5}{\lfloor\underline{3}}\cdot\frac{ax^3}{c^7}+\ldots\ldots$$
$$\frac{1.3.5.7\ldots(2p-3)}{\lfloor\underline{p-1}}\cdot\frac{ax^{p-1}}{c^{2p-1}}.$$

503. $$a^{-1}\frac{2}{3}a^{-4}y^2+\frac{2.5}{2.3^2}\cdot a^{-7}y^4+\frac{2.5.8}{\lfloor\underline{3}\,.\,3^3}\cdot a^{-10}\cdot y^6+\ldots\ldots$$
$$\frac{2.5.8\ldots(3p-4)}{\lfloor\underline{p-1}\,.\,3^{p-1}}\cdot a^{-(3p-2)}\cdot y^{2(p-1)}.$$

504. $a^{-\frac{13}{3}} + \frac{4}{3}a^{-\frac{25}{3}} . x + \frac{4 . 7}{2 . 3^2} . a^{-\frac{37}{3}} . x^2 + \ldots\ldots$

$$\frac{4 . 7 . 10 . 13 \ldots (3p-2)}{\lfloor\underline{p-1} . 3^{p-1}} . a^{-\frac{12p+1}{3}} . x^{p-1}.$$

505. $a^{\frac{1}{4}} . x + \frac{1}{4}a^{-\frac{11}{4}} . x^3 + \frac{1 . 5}{2 . 4^2} . a^{-\frac{23}{4}} . x^5 + \ldots\ldots$

$$\frac{1 . 5 . 9 . 13 \ldots (4p-7)}{\lfloor\underline{p-1} . 4^{p-1}} . a^{-\frac{12p-13}{4}} . x^{2p-1}.$$

506. $1 + \frac{2}{3}a^{-1} . y + \frac{2 . 5}{2 . 3^2}a^{-2} . y^2 + \frac{2 . 5 . 8}{\lfloor\underline{3} . 3^3} . a^{-3}y^3 \ldots\ldots$

$$\frac{2 . 5 . 8 \ldots (3p-4)}{\lfloor\underline{p-1} . 2^{p-1}} . a^{-(p-1)} . x^{p-1}.$$

507. $a^{\frac{1}{2}} + \frac{1}{2}a^{-\frac{3}{2}} . x^3 + \frac{1 . 3}{2 . 2^2} . a^{-\frac{7}{2}} . x^6 \ldots\ldots$

$$\frac{1.3.5.7 \ldots (2p-3)}{\lfloor\underline{p-1} . 2^{p-1}} . a^{-\frac{(4p-5)}{2}} . x^{3(p-1)}.$$

508. $x^{\frac{2}{5}} + \frac{1}{5}x^{-\frac{3}{5}} . y + \frac{1 . 6}{2 . 5^2} . x^{-\frac{8}{5}} . y^2 \ldots\ldots$

$$\frac{1 . 6 . 11 . 16 \ldots (5p-9)}{\lfloor\underline{p-1} . 5^{p-1}} . x^{-\frac{5p-7}{5}} . y^{p-1}.$$

509. $x^{\frac{2}{3}} + \frac{1}{3}x^{-\frac{10}{3}} . y + \frac{1 . 4}{2 . 3^2}x^{-\frac{22}{3}} . y^2 + \ldots\ldots$

$$\frac{1 . 4 . 7 \ldots (3p-5)}{\lfloor\underline{p-1} . 3^{p-1}} . x^{-\frac{14-12p}{3}} . y^{p-1}.$$

510. $a + \frac{3}{2}b + \frac{3 . 5}{2 . 2^2}a^{-1}b^2 \ldots\ldots\ \frac{3 . 5 . 7 . 9 \ldots (2p-1)}{\lfloor\underline{p-1} . 2^{p-1}} . a^{2-p} . b^{p-1}.$

511. $a + \frac{1}{3}a^{-1} . b^2 + \frac{1 . 4}{2 . 3^2}a^{-3}b^4 \ldots \frac{1 . 4 . 7 . 10 \ldots (3p-5)}{\lfloor\underline{p-1} . 3^{p-1}} . a^{-(2p-3)} . b^{2(p-1)}.$

512. $a^{-\frac{10}{3}} + \frac{5}{3}a^{-\frac{16}{3}} . bc^3 + \frac{5 . 8}{2 . 3^2}a^{-\frac{22}{3}} . b^2c^6 \ldots\ldots$

$$\frac{5 . 8 . 11 \ldots (3p-1)}{\lfloor\underline{p-1} . 3^{p-1}} . a^{-2 . \frac{3p+2}{3}} . b^{p-1} . c^{3(p-1)}.$$

513. $a^{\frac{1}{3}}x^{-\frac{1}{3}} - \frac{1}{3}a^{-\frac{5}{3}}x^{-\frac{4}{3}}.b^2 + \frac{1.4}{2.3^2}.a^{-\frac{11}{3}}x^{-\frac{7}{3}}.b^4 \ldots\ldots$

$$\frac{4.7.10\ldots(3p-5)}{\lfloor\underline{p-1}.3^{p-1}}.a^{-\frac{6p-7}{3}}x^{-\frac{3p-2}{3}}.b^{2(p-1)}.$$

514. $x^{-\frac{1}{2}}.y^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{5}{2}}.y^{\frac{5}{2}} + \frac{1.3}{2.2^2}.x^{-\frac{9}{2}}.y^{\frac{9}{2}} \ldots\ldots$

$$\frac{3.5.7.9\ldots(2p-3)}{\lfloor\underline{p-1}.2^{p-1}}.x^{\frac{3-4p}{2}}.y^{\frac{4p-3}{2}}.$$

515. No. 509.

516. $y^{-\frac{1}{4}} + \frac{1}{4}y^{-\frac{9}{4}}.z^3 + \frac{1.5}{2.4^2}.y^{-\frac{17}{4}}.z^6 \ldots\ldots$

$$\frac{1.5.9.13\ldots(4p-7)}{\lfloor\underline{p-1}.4^{p-1}}.y^{-\frac{8p-7}{4}}.z^{3(p-1)}.$$

517. $1 - \frac{x}{a} + \frac{1}{2}.\frac{x^2}{a^2} - \frac{1}{2}.\frac{x^3}{a^3} + \frac{1.3}{2.2^2}.\frac{x^4}{a^4} - \frac{1.3}{2.2^2}.\frac{x^5}{a^5} + \&c.$

518. $a^{\frac{14}{5}} + \frac{2}{5}a^{\frac{9}{5}}.y^3 + \frac{6.2^2}{2.5^2}.a^{\frac{4}{5}}.y^6 + \ldots\ldots$

$$\frac{6.11.16\ldots(5p-9).2^{p-1}}{\lfloor\underline{p-1}.5^{p-1}}.a^{\frac{19-5p}{5}}.y^{3(p-1)}.$$

519. $x^{\frac{1}{3}} + \frac{1}{3}a^{-3}.x^{-\frac{5}{3}}z + \frac{1.4}{2.3^2}.a^{-6}.x^{-\frac{11}{3}}z^2 + \ldots\ldots$

$$\frac{4.7.10\ldots(3p-5)}{\lfloor\underline{p-1}.3^{p-1}}.a^{3(1-p)}.x^{\frac{7-6p}{3}}.z^{p-1}.$$

520. $a^{\frac{4}{3}}.b^{\frac{1}{2}} + \frac{1}{3}a^{-\frac{2}{3}}.b^{\frac{3}{2}}.c + \ldots\ldots$

$$\frac{1.4.7.10\ldots(3p-5)}{\lfloor\underline{p-1}.3^{p-1}}.a^{-\frac{2(3p-5)}{3}}.b^{\frac{2p-1}{2}}.c^{p-1}.$$

521. $a^{\frac{1}{3}} - \frac{1}{3}a^{-\frac{2}{3}}.b^{\frac{1}{2}} - \frac{1.2}{2.3^2}.a^{-\frac{5}{3}}.b - \ldots\ldots$

$$\frac{1.2.5.8\ldots(3p-7)}{\lfloor\underline{p-1}.3^{p-1}}.a^{-\frac{3n-4}{3}}.b^{\frac{p-1}{2}}.$$

522. $$1+\frac{x}{a}+\frac{x^2}{2a^2}+\frac{x^3}{2a^3}+\frac{1\,.\,3}{2\,.\,2^2}\cdot\frac{x^4}{a^4}+\&c.$$

but if $a=3$ and $x=1$; $\sqrt{\frac{a+x}{a-x}}=\sqrt{2}$.

Hence $$\sqrt{2}=1+\frac{1}{3}+\frac{1}{2}\cdot\frac{1}{3^2}+\frac{1}{2}\cdot\frac{1}{3^3}+\frac{1}{8}\cdot\frac{1}{3^3}+\frac{1}{8}\cdot\frac{1}{3^4}$$
$$+\frac{5}{2\,.\,8}\cdot\frac{1}{3^6}+\frac{5}{2\,.\,8}\cdot\frac{1}{3^7}+\&c.$$

$$\begin{array}{r|l l r}
3 & 1\cdot & \dotfill & 1\cdot \\ \cline{2-2}
3 & \cdot333333 & \dotfill & \cdot333333 \\ \cline{2-2}
3 & \cdot111111 & \ldots\ldots\ \frac{1}{3^2}\times\frac{1}{2}\ \ldots\ldots & \cdot055556 \\ \cline{2-2}
3 & \cdot037037 & \ldots\ldots\ \frac{5}{8}\times\frac{1}{3^3}\ \ldots\ldots & \cdot023148 \\ \cline{2-2}
9 & \cdot012346 & \ldots\ldots\ \frac{1}{8}\times\frac{1}{3^4}\ \ldots\ldots & \cdot001543 \\ \cline{2-2}
3 & \cdot001372 & \ldots\ldots\ \frac{10}{4\,.\,8}\cdot\frac{1}{3^6}\ \ldots\ldots & \cdot000430 \\ \cline{2-2}
 & \cdot000457 & \ldots\ldots\ \frac{10}{4\,.\,8}\cdot\frac{1}{3^7}\ \ldots\ldots & \cdot000014 \\ \cline{4-4}
 & & & 1\cdot414024 \\ \cline{4-4}
\end{array}$$

523. $-\frac{13}{3^6}\,.\,x^{-\frac{32}{3}}\,.\,y^5$. **524.** 1956. **525.** 109600.

526. 24. **527.** 2520. **528.** 420. **529.** 244. **530.** 24.

531. 5040. **532.** 170. **533.** 645. **534.** 45. **535.** 15.

536. 4989600. **537.** 6720. **538.** 725760. **539.** 33.

540. $\frac{\lfloor\underline{90}}{\lfloor\underline{70}\,.\,\lfloor\underline{20}}$ days; and $\frac{\lfloor\underline{89}}{\lfloor\underline{70}\,.\,\lfloor\underline{19}}$ times. **541.** 325.

542. 28 days. **543.** 84. **544.** 63. **547.** 11.

548. 626. **549.** 11879. **550.** 5610; 5180. **551.** 14492.

552. 6. **553.** 120. **554.** 60. **555.** 4. **556.** 16.

557. 4095. **558.** 1840. **559.** 11940. **560.** $n-1$.

561. 190. **562.** 1405. **563.** 22. **564.** 672.

565. 399; 728; 1218. **566.** 25. **567.** 5525.

568. $\dfrac{n(n+1)(n+2)}{3}$. **569.** 2639; 5019.

570. 4960; 9455. **571.** 140. **572.** 5525; 2925.

573. 2639; 5019. **574.** 50. **575.** 5763.

577. Com. diff. $= \dfrac{n(n^2-1)}{6}$. **580.** 4970.

581. $m = \dfrac{n-1}{3}$; 2024; 3795. **582.** 20540. **583.** 330.

586. 4250. **589.** $a^z = n$. **592.** $a = \epsilon = 2{\cdot}718281828459$ &c.

596. $\log_{(2\sqrt{3})} 144 = 4$. **597.** 4. **598.** $\cdot 33\dot{3}$. **599.** 2·5849.

600. 0·2922560; $\bar{3}{\cdot}6512780$. **601.** $\log_a n = \dfrac{\log_e n}{\log_e a}$.

602. 10721. **603.** 80618. **604.** 1·760913; $\bar{2}{\cdot}937531$.

605. ·9542426; 1·4427; ·91025; ·16409.

606. ·447158; 2·89353. **607.** $\bar{1}{\cdot}90309$. **608.** 3·584963.

609. 0·39794. **610.** $\bar{3}69897$; 0.8115752.

611. ·69897; 1·2552725; 2·1303338; 8750613.

613. $n = 8$; $z = 2$.

615. If $n = 1$ and $z = 1$, $\dfrac{z}{2n+z} = \dfrac{1}{3}$; and we get $\log_e 2$ and then $\log_e 8 = 3 \log_e 2$. Again, if $n = 8$ and $z = 2$, $\dfrac{z}{2n+z} = \dfrac{1}{9}$ and we get $\log_e 10$. **616.** $\dfrac{z}{2n+z} = \dfrac{1}{6}$.

617. 3·54413. **618.** $x = {\cdot}8796686$; $y = {\cdot}26285$.

619. $x = -10{\cdot}73644$; $y = -19{\cdot}017$. **620.** $x = \dfrac{1}{2}$; $y = \dfrac{5}{2}$.

621. $x = -2{\cdot}91481$; $y = 5{\cdot}65678$.

622. $x = 6$; $x = \dfrac{\log c}{2(\log b + 2\log a)}$.

623.

$$x = \frac{7 \log b \,.\, \log d}{(5\log a + 3\log b)\log d - 8\log b \log c}.$$

$$y = \frac{28\,.\,(\log b)^2}{(5\log a + 3\log b)\log d - 8\log b\,.\,\log c}.$$

625. The 11th. **626.** 2·2872824. **627.** ·002431267.

628. ·00543928. **629.** ·108603. **630.** 6·19620102.

631. ·0005218504. **632.** ·0180448. **633.** 33·035.

634. 1·317372. **635.** 12·58341; $x = \frac{\log b}{\log a}$; $y = \frac{\log c}{\log\log b - \log\log a}$.

636. 3·080336. **637.** ·5095667. **638.** ·000000311481.

639. 1·469493. **640.** ·04950141. **641.** 43336970.

642. 1·739405. **643.** ·4403654. **644.** £519. 2*s.* 1*d.*

645. $\frac{3}{4}$. **646.** $\frac{81}{625}$. **647.** $\frac{3781}{42875}$.

672. $AK = \frac{-3 \cdot \sqrt{(r_1^2 - r^2)} \pm 8l^2 + r_1^2 - 9r^2}{2\sqrt{2}}$.

673. $ab \cdot \sqrt{\left(\frac{2}{a^2+b^2}\right)}$; $\frac{a(a+b)}{\sqrt{(a^2+b^2)}}$; $\frac{b(a+b)}{\sqrt{(a^2+b^2)}}$; $\frac{ab(a+b)}{a^2+b^2}$.

675. $\frac{a}{s}\{a \pm \sqrt{(2s^2+a^2)}\}$; $\frac{2as}{a \pm \sqrt{(2s^2+a^2)}}$.

676. Distance from angle $= \frac{2s}{3}$.

677. The segments being e and f, and the ratio $n:1$;
the sides are $\left(\frac{e^2-f^2}{n^2-1}\right)^{\frac{1}{2}}$ and $n\left(\frac{e^2-f^2}{n^2-1}\right)^{\frac{1}{2}}$.

678. $\frac{abc}{4\sqrt{\{s(s-a)(s-b)(s-c)\}}}$.

679. Length of line $= \left\{\frac{2a(a-2r)}{3 \pm \sqrt{5}}\right\}^{\frac{1}{2}}$ where a is the longest line which can be drawn to the circle of which the radius is r.

680. $a = \left[\frac{d}{2}\{d \pm \sqrt{(d^2-b^2)}\}\right]^{\frac{1}{2}}$.

681. Straight line meets base at distance $\frac{n-2}{2} \cdot b$ or 0 from foot of perpendicular on base.

682. $\left\{\frac{a(ab-d^2)}{b}\right\}^{\frac{1}{2}}$ and $\left\{\frac{b(ab-d^2)}{a}\right\}^{\frac{1}{2}}$.

683. Trigonometrical question; $p \cdot \frac{p\tan\alpha - s}{p + s\tan\alpha}$.

684. ± 20 or ± 15; ± 15 or ± 20. **709.** 33° and 15°.

711. $38^\circ\ 11'\ 49''\cdot 7$. **713.** $\frac{\sqrt{3}}{2}$; $\frac{\sqrt{2}}{2}$; $10\sqrt{2}$; $-10\sqrt{3}$.

715. $21^\circ\ 29'\ 9''\cdot 6$; $\cdot 375$.

716. $\frac{\sqrt{3}}{3}$; $\sqrt{3}$; 1; $\frac{4-(\sqrt{3}+1)\sqrt{2}}{2\sqrt{(2-\sqrt{3})}}$; $2-\sqrt{3}$; $2+\sqrt{3}$.

717. $114^\circ\ 37'\ 33''$. **720.** 5; $10^\circ\ 42'\ 18''$. **732.** $9\frac{3}{5}$.

734. For $\cos 23^\circ\ 30'$ read $\cos 22^\circ\ 30'$. **773.** $\pm\frac{\sqrt{(3\pm\sqrt{5})}}{2}$.

774. $x=\sin^{-1}\pm\frac{1}{3}\sqrt{(17\pm\sqrt{869})}$; $y=\cos^{-1}\pm\frac{2}{9}\sqrt{(17\pm\sqrt{869})}$.

775. $\cos\phi=\frac{1}{\sqrt{2}}$.

777. $\theta+\phi=\tan^{-1}(-2)$; $(\theta-\phi)=\tan^{-1}\left(-\frac{1}{2}\right)$; $\theta+\phi>\frac{\pi}{2}$; $\theta-\phi<0$.

778. $\tan x=\frac{n\cot\beta-m\cot a}{m+n}$. **779.** $\theta=15^\circ$.

780. $\theta=\tan^{-1}\frac{2}{\tan(a+\beta)-\tan(a-\beta)}$.

782. $\tan\theta=\frac{n-1}{n+1}\cdot\cot a$. **783.** $\frac{2\mu}{1+\mu^2}$.

784. $\tan\phi=\frac{1}{3}$ or $\frac{5}{27}$. **785.** $\frac{2a}{1-a^2+b^2}$.

786. $\sin^2 A=\frac{1}{2}\pm\frac{1}{10}\sqrt{21}$; $\sin^2 B=\frac{1}{2}\pm\frac{1}{5}\sqrt{6}$.

787. $A=45^\circ$; $B=30^\circ$. **789.** $71^\circ\ 28'\ 6''$; $50^\circ\ 56'\ 10''$.

790. 17·321; 90° and 30°.

791. $C=546\cdot 8625$; $A=76^\circ\ 44'\ 45''$; $B=64^\circ\ 33'\ 0''$.

793. $15^\circ\ 57'\ 55''\cdot 2$; $134^\circ\ 54'\ 14''\cdot 3$; $29^\circ\ 7'\ 50''\cdot 5$. **794.** 700·79.

795. 381·14. **796.** 288·7. **797.** 16·7 **799.** 4009·5.

800. 466·94. **802.** $PW=2132$; $PF=1107\cdot 3$.

803. 962·605. **804.** 4188·597.

806. $AC=1626\cdot 636$; $BE=1106\cdot 8562$. **808.** 608·584.

809. $36\sqrt{3}$. **810.** 438·36; 177·14.

811. $h.\left\{\frac{\cos\beta}{\sin(\beta-\alpha)}-\frac{\cos\gamma}{\sin(\gamma-\alpha)}\right\}$. **812.** Equal.

813. 555·765. **814.** 5·8426; 3·8477; 8·3034.

815. 51·08; 280·96. **816.** 390·27; 345·15.

817. 826·32; 9° 50′ 58″. **818.** 1967·4.

819. PA = 926·35; PB = 783·5; PC = 510·79. **820.** 287·2.

821. (A circle can be described about $ABCD$, ∴ CDB is a right angle), CD = 353·5.

822. 1897·2. **823.** 2208·6; 178·73.

824. 5·176; 10; 14·141. **825.** 219·31. **826.** 177·582.

827. 86·0904; 43·88534.

828. (Take the horizontal plane, passing through the lower wire), 25° 14′.

829. (By Simpson's rule), 1 r. 13·1312 po.

830. 2421024. **832.** 10 r. 0 p. 19·5536 sq. yds.

833. 44·85 acres. **834.** 3 $\sqrt{21}$. **835.** 625·17.

836. 13·926 ch.; 23.21 ch.; 32·494 ch. **837.** 5·78926.

838. 3; 33. **839.** 110·45.

840. At base 30°; at vertex 120°. **841.** 20.

842. £23. 6*s*. $2\frac{3}{4}d$. **843.** $\frac{\sqrt{7}}{4}$. **845.** ·2146025.

846. $\frac{na^2}{4}.\cot\frac{180^\circ}{n}$; 25 $\sqrt{3}$; 100; 172·05. **849.** 625.

850. 3·36 feet.

851. (Distance lost, is difference of quadrantal arc and radius).

852. 280. **854.** 119·37 sq. ft. **855.** 78·604; 26·376.

856. 1253·88. **857.** 61419. **858.** 369·8.

859. $200\sqrt{(3)}-150\sqrt{(3)}=50\sqrt{(3)}=\frac{1}{3}\{150\sqrt{(3)}\}$.

862. 11·454. **863.** 464·1. **864.** 172·966.

865. 20731. **867.** 761·811. **868.** 184·2.

871. 2827·4 sq. yds. **873.** 6·0769.

874. (The number of circular sections in section of rope, is the same as the number of circumscribing hexagons) $= \frac{2}{d^2\sqrt{3}}$.

875. 60; 30°; 30°; 60°. **876.** 271·69.

878. Height : diameter :: 1 : 6. **879.** 140·625 lbs.

882. 34·7812 cub. in.; 11·8338 cub. in. **883,** 2·82454 in.

884. $r = 4{\cdot}64$ in.; 869 cub. in. **885.** 61·552.

886. 2664·2 gall. **887.** 148·44 ft. **888.** 531·128 cub. ft.

889. 3·09. **890.** 25000. **891.** 5177·3.

892. $\frac{\pi r^3}{4} \cdot \sqrt{(3)}$. **893.** 17·314 lbs. **894.** 930597; 9·9934.

895. 1·8522 in. **896.** 7·32 tons. **897.** 37811·3.

898. $\frac{39\pi}{12}$. **899.** 1·488 in. **900.** 2·0124 oz.

902. 134·3029; 3·9274 cwt. **903.** 942·211 cub. in.

904. 207·88 lbs. **905.** 1 ton 14 cwt. 2 qrs. 10 lbs.

906. 121·86 lbs.

907. $\frac{D^3 - d^3}{\Delta^3 - \delta^3} = \frac{D - d}{\Delta - \delta} \times \frac{D^2 + Dd + d^2}{\Delta^2 + \Delta\delta + \delta^2} = \frac{D^2 + Dd + d^2}{\Delta^2 + \Delta\delta + \delta^2} = \frac{\text{frustum}}{\textit{frustum}}$.

908. Height of cone cut off by tangent plane of sphere $= \frac{3r}{2}$;

Volume of cone $= \frac{3}{8}\pi r^3$; volume of segment $= \frac{5}{24}\pi r^3$;

$\therefore$ volume of charge $= \frac{1}{6}\pi r^3 = \frac{1}{8} \times \frac{4}{3}\pi r^3 = \frac{1}{8}$ volume of shot.

909. 23·562 cub. in. **910.** 11145·1013. **911.** 294·8 lbs.

912. 390·88. **913.** 86·837 in. **914.** 2·427 in.

915. The segments of axis are $\frac{h}{3}\sqrt[3]{9}$; $\frac{h}{3}\sqrt[3]{9} \cdot (\sqrt{2} - 1)$ and $\frac{h}{3}(3 - \sqrt[3]{18})$.

922. 1551·4 sq. miles.

931. $A = 139°\ 58'\ 55''$; $B = 141°\ 28'\ 57''$.

936. 48° 11′ 58″; 81° 8′ 51″; 67° 21′ 29″.

937. 45° 20′ 38″; 61° 6′ 54″; 86° 38′ 56″.

939. $B = 31°\ 58'\ 49''$; $C = 144°\ 57'\ 42''$; $c = 113°\ 50'\ 56''$.

940. 11° 30′. **941.** 33° 0′ 55″.

943. 64° 20′; 73° 54′; 33° 40′. **944.** 118° 48′.

945. 35° 16′.

946. 64° 20′. **948.** 92° 42′ 10″; 29° 59′ 44″; 310·28 sq. miles.

951. 70° 32′; 258·8 sq. mi. **952.** 8·38 sq. in. **953.** 89·9 sq. ft.

955. 85° 28′ 26″. **956.** Lat. 63° 38′ 49″ S., Dec. – 45° 51′ 35″.

957. ± 62° 48′ 29″. **958.** 73° 5′ 54″.

959. 55° 57′ 35″·5; 47° 13′ 30″·5.

960. 28° 10′ 7″·1; – 4° 9′ 10″·1.

961. 305° 47′ 28″·65; 22° 25′ 17″.

962. 42° 10′ 12″·5; – 3° 46′ 55″.

963. 336° 12′ 39″·7; – 4° 45′ 10″.

964. $23^h\ 15^m\ 39^s$. **965.** $19^h\ 49^m\ 41^s$. **966.** $10^h\ 10^m\ 15^s$.

967. $12^h\ 10^m\ 18^s\cdot 6$. **968.** $+ 4^m\ 37^s$. **969.** $+ 8^m\ 53^s\cdot 6$.

970. $8^h\ 42^m\ 1^s\cdot 2$. **971.** $17^h\ 57^m\ 26^s\cdot 52$. **972.** $13^h\ 14^m\ 44^s\cdot 02$.

973. $27^h\ 15^m\ 32^s\cdot 14$. **974.** $20^h\ 9^m\ 39^s\cdot 63$. **975.** 18° 57′ 15″ E.

976. $7^h\ 31^m\ 49^s\cdot 87$. **977.** 58° 57′ 17″; $8^h\ 6^m\ 40^s\cdot 8$.

978. 41° 21′ 36″. **979.** – 58° 56′ 18″.

980. 20° 48′ 15″; $20^h\ 40^m\ 40^s\cdot 2$.

981. – 60° 36′ 40″; 43° 50′ 50″. **982.** 21° 32′ 14″.

983. 3° 58′ 16″; $56^h\ 38^m\ 50^s\cdot 4$ W.

985. 31° 27′ 59″·4; $10^h\ 55^m\ 10^s\cdot 4$ W. **986.** 64° 13′ 16″.

987. 108° 28′ 15″. **988.** $134^h\ 37^m\ 13^s\cdot 5$ E.

989. 185° 49′ 43″·5 W. **990.** 155° 41′ 36″ E.

991. 102° 40′ 15″ E. **992.** 27° 11′ 36″·33 W.

993. 115° 9′ 38″ E. **994.** 60° 7′ 19″; $14^h\ 58^m\ 14^s\cdot 3$.

995. Lat. 26° 21′ 11″, Alt. 29° 38′ 44″.

996. Az. 51° 52′ 10″ E. of N.; Lat. 40° 20′ 25″.

997. 59° 6′ 1″; 9° 3′ 20″ W. of S.

998. *(1)* $7^h\ 29^m\ 37^s$; 60° 11′ 34″; (2) $4^h\ 22^m\ 35^s$; 119° 48′ 26″.

999. 51° 54′ 42″. **1000.** 52° 23′ 15″ or 7° 47′ 47″.

1001. $4^h\ 16^m\ 43^s$. **1002.** $17^h\ 38^m\ 53^s$. **1003.** 9° 22′ 24″.

1004. N. 5° 20′ 36″·5 W. **1005.** 60° 53′ 14″. **1006.** 48° 4′ 56″.

1007. 27° 43′ 50″; 117° 7′ 10″·5 E. **1008.** S. 1° 3′ 26″ W.

1009. 49° 21′ 52″; 81° 49′ 3″ W. Bearing N. 105° 53′ 25″ E. Distance 516·88 miles.

1010. True dist. 20° 57′ 34″, Lon. $10^h\ 50^m\ 48^s{\cdot}7$ E.

1011. 35° 59′ 14″. **1012.** 72° 33′ 4″. **1013.** 80° 9′ 34″.

1014. 19° 49′ 10″. **1015.** 28° 8′ 24″. **1016.** 97° 45′ 4″.

1017. 120° 1′ 45″. **1018.** $9y + 10x + 13 = 0$; $-\frac{10}{9}$; $-\frac{13}{9}$; $-\frac{13}{10}$.

1019. $\frac{53}{34}\sqrt{17}$; $\tan^{-1}\cdot 25$. **1020.** $5y + 9x = -\frac{17}{4}$.

1021. 35. **1022.** $6y + 11x - 35 = 0$.

1023. $\left(\frac{2}{5};\ 0\right)$; $(-2,\ 0)$; $\left(-\frac{22}{35},\ \frac{72}{7}\right)$. **1024.** 7·6.

1025. $x - y\sqrt{3} = 5 + \sqrt{3}$. **1026.** $\frac{38}{5\sqrt{10}}$. **1027.** $(1;\ -1)$.

1028. $2y - x + 17 = 0$. **1029.** $\tan^{-1} -\frac{1}{2}$; $\tan^{-1} -2$; $\frac{7}{5}$; $\frac{7}{10}$.

1031. $2y - 7x + 25 = 0$. **1032.** $\frac{4}{\sqrt{5}}$. **1033.** $\tan^{-1}\cdot 778$.

1034. $\tan^{-1} -\frac{17}{6}$. **1037.** $\left(\frac{88}{41};\ \frac{110}{41}\right)$.

1038. $4y + 3x - 5a = 0$. **1039.** $4y - x + 5 = 0$. **1040.** $\frac{4}{11}\sqrt{17}$.

1041. 15; $3\sqrt{13}$. **1042.** $\left(\frac{19}{13};\ \frac{4}{13}\right)$; at right angles.

1043. $2\sqrt{2}$. **1045.** $\tan^{-1}\frac{9}{8}$. **1046.** $\left(\frac{28}{21};\ -4\right)\tan^{-1} -\frac{15}{17}$.

1047. Equilateral Δ; area $9\sqrt{3}$. **1048.** 8·375 in. **1049.** $\frac{62}{5\sqrt{2}}$.

1050. 5. **1051.** $\frac{39}{2}$. **1053.** $y = -3x$.

1054. $3x^2 - 58x + 3y^2 = 0$. **1055.** $\frac{7}{6}$; $\frac{1}{2}\sqrt{29}$; $\left(0, \frac{7}{6}\right)\left(1, -\frac{3}{2}\right)$.

1056. $r = \frac{b}{\sqrt{(1 + a^2)}}$. **1057.** $\frac{\sqrt{11}}{3}$; $\frac{1}{3}(2 \pm \sqrt{7})$.

1058. If base $= b$; perpendicular $= p$, and segment of base $= q$. The locus is $3x^2 + 3y^2 - 2(b + q)x - 2py - s^2 = 0$, a circle.

1059. $\left(x - \frac{47}{17}\right)^2 + \left(y - \frac{36}{17}\right)^2 = 26{\cdot}21$.

1060. $\left(x - \frac{9}{7}\right)^2 + \left(y - \frac{1}{14}\right)^2 = \frac{8297}{14^2}$. **1062.** b.

1063. $(3, -1)$; $\sqrt{13}$; $\left(-\frac{1}{12}, 1\right)$; $\frac{31}{12}$.

1065. $(AB)\ q = \frac{5}{3}x$; $(BC)\ 5y = -3x + 18$. **1066.** $8y - 6x = 25$.

1067. 2; $25y + 2(37 \mp 3\sqrt{41})x = 0$. **1068.** $\tan^{-1}\frac{3}{4}$.

1069. $y^2 + x^2 + 6y - 2x + \frac{49}{5} = 0$.

1070. $(1, 7)$; $(-{\cdot}5, -{\cdot}5)$; $\frac{1}{2}\sqrt{234}$.

1071. $-\left(\frac{2 \pm \sqrt{2}}{2}; -\frac{2 \pm 3\sqrt{2}}{2}\right)$. **1072.** $\pm\frac{3}{\sqrt{7}}$.

1073. If $AB = a$ and $BC = b$; locus $y^2 - 2x^2 - by + ax = 0$.

1075. $x^2 - 2ax + 4my = 0$. **1078.** $\sqrt{5}$; $\sqrt{3}$; $\frac{1}{5}\sqrt{10}$.

1079. Circle. **1080.** $y^2 - 12x + 36 = 0$. **1081.** $x^2 + y^2 = l^2$.

1083. $4x^2 + 4y^2 = a^2$; radius $\frac{a}{2}$.

1084. (1) The ellipse becomes a circle; (2) it becomes a straight line. **1086.** $\frac{\pm a^2}{(a^2 + b^2)^{\frac{1}{2}}}$; $\frac{\pm b^2}{(a^2 + b^2)^{\frac{1}{2}}}$. **1087.** $\pm\frac{2ae}{1 + e^2}$.

1089. $8a\sqrt{2}$. **1090.** $-\frac{a^2 \pm a\sqrt{(a^2 + b^2)}}{b}$; $2\{a\sqrt{(a^2 + b^2)} - a^2\}$.

1091. Altitude of parabola being h; bisecting line cuts it at distance $\frac{h}{\sqrt[3]{4}}$ from highest point. **1093.** $\frac{a(2b-a)}{b}$.

1094. $y^2 = \frac{4x}{3}$. **1096.** Circle.

1098. If $(h; k)$ $(h'; k')$ be two points in a parabola (of which the latus-rectum is m) from which normals, perpendicular to one another, can be drawn, $kk' = -4m^2$. From the equations to the normals we obtain $2y = k + k'$, and $h + h' = 2x - \frac{y^2}{m} - 4m$. Squaring the first of these, and substituting, we get, finally, $y^2 = m(x - 3m)$, a parabola having the same axis as the original, and a latus-rectum one-fourth of that of the given curve; its vertex being at a distance $3m$ from that of the original. The result may also be obtained by means of the equation to the normal in terms of the tangent of its inclination to the axis. **1103.** $\frac{20}{\sqrt{(2+\sqrt{3})}}$; $2{\cdot}67949$.

1104. $P : Q : W :: 1 : \sqrt{3} : 2$. **1105.** $-{\cdot}475$.

1106. $\tan^{-1}\frac{4}{3}$; $\tan^{-1}\frac{3}{4}$. **1107.** $67{\cdot}2$ lbs.

1108. $4AD$ acting at E; by constructing the several parallelograms the proposition is obvious. **1109.** CA.

1110. If θ be the angle between P and R, $\sin\theta = \frac{Q}{P}.\sin\alpha$; also $R = -Q\cos\alpha \mp \sqrt{(P^2 - Q^2\sin^2\alpha)}$.

1111. $52{\cdot}44$; $\tan\theta = -6{\cdot}4$.

1112. $32{\cdot}247$ lbs. inclination of pressure to horizon $= 71^\circ\ 10'\ 30''$.

1113. $AC = \frac{3}{4}AB$; $\sin C = \frac{3 \pm \sqrt{137}}{16}$; $t = \frac{3}{4}\sqrt{(110 \pm 6\sqrt{137})}$.

1114. $R^2 = P^2 + Q^2 + 2PQ\cos\alpha$; $\sin\theta = \frac{P}{R}.\sin\alpha$.

1115. $\sqrt{3}$; $\theta = 90^\circ$. **1116.** $f_2 = \frac{f_1}{\sqrt{2}}$; $f_3 = \frac{2-\sqrt{2}}{\sqrt{2}}.f_2$.

1117. $\frac{14}{\sqrt{6}}$ cwt. **1118.** $\tan\theta = 3{\cdot}27$.

1119. 1682 lbs. vertically downwards.

1120. At A, $P = W\sqrt{2}$. At B, $P = W.\sqrt{\{2(4+\sqrt{3})\}}$;

$\theta = \tan^{-1}\frac{1+\sqrt{3}}{2}$. At C, $P = 2W$; $\theta_{,} = 60°$.

1125. 12·5 lbs. **1126.** 9·58 lbs.; 1·69 lbs.; 9·444 lbs.; ·556 lbs.

1127. 10·805 cwt.; 21·938 cwt.

1128. At A, $R = P\sqrt{(4+\sqrt{3})}$; $\theta = \tan^{-1}\frac{2+\sqrt{3}}{3}$.

At B, $\theta_{,} = \tan^{-1}\frac{P+Q}{3P+Q}\sqrt{3}$. At C, $\theta_{,,} = \frac{2Q-(P+Q)\sqrt{3}}{P+Q}$.

1129. Incl. to vertical $= \cos^{-1} - \frac{1}{4}\sqrt{2}$. **1130.** 1 cwt.

1131. $t = 16$; $t_{,} = 12$; $w = 12{\cdot}8$; $w_{,} = 7{\cdot}2$. **1132.** 9·68 cwt.

1133. Tension $AE = \frac{W}{2}.\tan\alpha.\operatorname{cosec}\theta$;

$$\text{pressure} = \frac{W}{2}.\sec\alpha.\sin(\alpha+\theta)\operatorname{cosec}\theta.$$

1134. 32·73 yds. from centre of stream.

1135. The strings must be vertical, and they will then sustain an additional weight of 4 lbs. if suspended at a distance $\frac{a}{4}$ from the stronger string.

1136. Tension of AC = tension of $BD = \frac{W}{2}$; tension of CE = tension of $ED = \frac{W}{\sqrt{3}}$; compression of $CD = \frac{W}{2\sqrt{3}}$. If the string were continuous, the bar would be forced upwards by a pressure on each side equal to $\frac{W}{4}(2-\sqrt{3})$.

1137. 5 : 8. **1138.** 6·276 cwt.; 37·4192.

1140. $\bar{x} = \frac{3a}{5}$; $\bar{y} = \frac{3a\sqrt{3}}{5}$. **1145.** ·589 in.

1146. $\bar{x} = \frac{5a}{12}$; $\bar{y} = \frac{5b}{12}$.

1148. From 1, $\frac{1}{3}\sqrt{13}$; from 2, $\frac{1}{3}\sqrt{10}$; from 3, $\frac{1}{3}\sqrt{5}$.

1149. $\frac{11a}{16}$; $\frac{4a}{5}$ from base of cylinder; $1 : \sqrt{3}$.

1150. 61·76 in. **1151.** $\frac{m}{18} + \frac{6 \times 16 \times 17m - 771b}{6 \times 2^4 \times 18} \times \frac{b\pi r^2}{C}$.

1153. 0° and 60°. **1154.** $\tan\theta = \frac{b-a}{b+a}\tan\alpha$.

1155. 73·21 lbs. **1157.** Dist. from opposite angle $\frac{19a}{21\sqrt{2}}$.

1158. If the angular points and the points of bisection be joined, those lines will bisect the sides of the interior triangle. Whence the centre of gravity of the latter may be proved to be identical with that of the original triangle.

1159. Distance from side $a = \frac{7}{18} b$.

1160. Common side on the edge.

1161. Distance from side of square ·7384a.

1162. 15·4 in. **1163.** 4·8 ft.; 50 lbs.

1164. Must balance on circumf. of common base, ∴ $r : h :: 1 : \sqrt{3}$.

1165. 40.

1166. Draw through the centres of gravity lines parallel to the bases of the triangles. These will form a regular figure: whence the property in question may be deduced.

1167. Taking the sides as axes,

$$\bar{x} \text{ or } \bar{y} = \frac{W + 3\,(B \text{ or } C)}{3\,(W + A + B + C)} \cdot (c \text{ or } b)$$

$$= \frac{c}{3} \text{ or } \frac{b}{3}; \text{ when } A = B = C.$$

1168. At point of bisection of the line drawn parallel to axes, at a distance $\frac{ar^2}{R^2 - r^2}$ from axis of exterior.

1169. $\tan\theta = \frac{92r}{95h}$. **1170.** Distance from vertex 7·609 in.

1171. $\frac{69}{14}$ from common base.

1172. 6·6 lbs.; 4·6 lbs.; 8·8 lbs. **1173.** $h = r\sqrt{3}$.

1174. $\theta = \tan^{-1}\frac{3}{8}$. Dist. of point from circumf. of base is $\tan^{-1}\frac{48}{55}$.

1175. $\frac{243+13m}{16(63+m)}\,.\,h$ from surface of water.

1177. 22 lbs. If AC and AB be taken as axes $\bar{x} = \frac{45}{22}$; $\bar{y} = \frac{5}{11}$.

1178. 6·21 and 23·79; 120°. **1179.** 3 cwt. 1 qr.

1180. 14 in. from weight 2. **1181.** 12 cwt. **1182.** 20$\frac{4}{7}$ in.

1183. $\tan^{-1}\frac{17}{24}\sqrt{3}$ and $60° - \tan^{-1}\frac{17}{24}\sqrt{3}$; $\frac{17}{3}$ in. from angle.

1184. Distance from point of suspension of P is $\tan^{-1}\frac{Q\sin\beta}{P+Q\cos\beta}$.

1185. 70° 12′ and 19° 48′. **1186.** $\left\{\sqrt{\left(\frac{2W+p}{P}\right)}-1\right\}a$.

1187. 45 lbs. **1188.** 4·02 lbs. **1189.** 56 lbs.

1190. $W + nw + P$ or $2^nP + (2^n - 1)w$.

1191. 33 lbs. exclusive of pulleys. **1192.** $\frac{wt}{g-t}$.

1194. $\frac{Mm + ft\cos a - Cg}{g}$. **1195.** $\frac{w_{,}l_{,} + wl}{l_{,}}$; $\frac{wll_{,}}{w_{,}l_{,} + wl}$.

1196. 2 cwt. and 5·016 cwt. **1199.** 569 lbs.

1200. $Q = \frac{W\sin a - P\,.\cos i}{\cos\gamma}$. **1201.** 3·8203 cwt.; 9·6732 cwt.

1202. 113·2. **1203.** h. cot 15°. **1204.** 1·55 cwt.

1205. 13·467 lbs. **1206.** 841·5. **1207.** $Q = 3P$; $R = 4P$.

1208. $W = 5P$; $W = 10P$; $W = 12P$. **1209.** 8·415 tons.

1210. 4·4563 lbs. **1211.** $\tan^{-1} 1{\cdot}33\dot{3}$. **1212.** 47·124 tons.

1213. 38·27 cwt. **1214.** ·052 cwt. **1215.** 215·42 cwt.

1216. 724. **1217.** $\frac{4r^2 + h^2}{3hr}$. **1218.** 169·68 lbs.

1219. $\cot^{-1}\cdot\dot{3}3$. **1220.** Height $= 2r\cot a$.

1221. $H = W \cdot \dfrac{ar}{\sqrt{(r^2+e^2)}}$. **1222.** $\dfrac{Wl}{4}\sec a$.

1223. Distance of one of the weights from highest point

$$= r \cdot \cot^{-1}\frac{2+\cos a}{\sin a}.$$

1224. $\dfrac{12P}{5}$. **1225.** $\cos\theta = \dfrac{(a^2+c^2)\,W^2b^2 - Qa^2c^2}{2ac}$.

1226. Greatest tension (when beam leaves wall)

$$= \frac{d\sqrt{(4l^2-d^2+a^2)}}{2l\sqrt{(4l^2-d^2)}}.$$

1227. $\tan^{-1}\dfrac{a-2c}{b}$. **1228.** $2^{n-1} \cdot p + (2^{n-1}-1)\,w$.

1229. 942·477 lbs. **1230.** $6\frac{4}{11}$. **1231.** 2° 42′.

1232. $\dfrac{2\pi l}{n}$. **1233.** 5760 lbs.

1234. $W \cdot \dfrac{2q \pm r}{2l \pm r} \cdot \dfrac{c}{2\pi R} \cdot \dfrac{h \pm \mu c}{c \pm \mu h}$. **1235.** 7·3 feet.

1236. 132·44 tons. **1237.** $P = \dfrac{W}{20000}$. **1238.** 1 : 6·88.

1239. 2847 tons. **1240.** $\left(\dfrac{w}{2}+\dfrac{W}{3}\right) \cdot \dfrac{1}{\sqrt{3}}$. **1241.** $\dfrac{W}{2}\sqrt{7}$.

1242. $\sin\dfrac{\theta}{2} = \dfrac{w\,(a+b)}{2aW}$. **1244.** $\dfrac{W}{3}\cot a$.

1247. Strain at each hinge $= \dfrac{W}{4b} \cdot \sqrt{(4b^2+9a^2)}$; $\tan\theta = \dfrac{2b}{3a}$.

1248. $\dfrac{12}{5}\sqrt{3}$. **1249.** $4\frac{5}{6}$ cwt.

1250. Inclination to face of plane $= \dfrac{\pi}{2} - 2a$.

1251. $\cos A = \dfrac{3361}{4840}$. **1253.** 4·814.

1254. $\cos^{-1}\cdot 838$; 52·22 lbs. **1255.** $W \cdot \sin\beta \cdot \operatorname{cosec}(a+\beta)$.

1257. 9·265 cwt. **1258.** Vertical angle $= 2\tan^{-1}\mu$.

1259. $2(1-\mu^2)\tan^3\theta - 4\mu\tan^2\theta - \sec^2\theta = 0$. **1260.** $W \cdot \sin 2i$.

1261. The compound solid will first begin to topple over; and when the inclination of the base of the cylinder is $\tan^{-1}\cdot 4$ the cone will topple from the cylinder. **1263.** $\tan^{-1}\cdot 63$.

1264. 5·905. **1265.** 62 cwt.; 72·739 cwt.

1266. 8·164 cwt. **1267.** It will fall over.

1268. 353·5 lbs. **1269.** ·086.

1270. The centre of gravity is not the centre of figure. If P be the point of contact with the plane, C the centre of figure and G the centre of gravity; the sphere will slide without rolling if $GPC > i$ and $< e$; and it will roll without sliding when $GPC < i$ and $i < e$: e being the limiting angle and i the inclination of the plane.

1271. $$P = W.\frac{\sin(i-e)\cos e - \sin(i+e)\cos(a+e)}{\cos^2 e - \cos(a-e)\cos(a+e)};$$
$$Q = W.\frac{\sin(i-e).\cos(a-e) - \sin(i+e)\cos e}{\cos(a-e)\cos(a+e) - \cos^2 e}.$$

1272. 30°. **1273.** $\tan i = 3\mu$. **1275.** $\mu = \frac{P-P'}{P+P'}.\tan i$.

1276. $2e$. **1277.** ·43. **1278.** $b\frac{1-\mu\mu'}{2\mu b + a(1+\mu\mu')}$.

1279. $\frac{2W}{9}\sqrt{21}$; $\tan^{-1}\frac{\sqrt{3}}{9}$. **1280.** 11·0344 feet.

1282. $\tan^{-1}\frac{1-\mu\mu'}{2\mu}$; this inclination will be the same for either side if the position of the centre of gravity be symmetrical for all sides, i. e. if the board be uniform, but not otherwise. When the inclination is 45°, $\mu = \frac{1}{2+\mu'}$.

1283. Segments by centre of gravity are as 43 : 57 nearly.

1284. $\mu = \frac{\sqrt{(31)}-4}{5}$. **1285.** Not higher than 18·75 feet.

1287. $\cos^{-1}(\mu.\cot a)$. **1288.** $\tan^{-1}.\frac{\mu r\sqrt{3}}{r\sqrt{(3)}+\mu^2 r - 1}$.

1289. 93·3 lbs. **1290.** 7·212 lbs. **1291.** $\tan^{-1}\frac{2w+W}{w}.\mu$.

1294. 3·7 lbs. **1295.** $1\frac{8}{35}$ lb. **1296.** 10·615 and 13·385.

1298. $\frac{2r}{\pi}$ from centre. **1299.** $\frac{3}{4}y$.

1300. Distance from centre $= \frac{c \,.\, r}{a}$. **1301.** $\frac{16m}{5}\pi a^2$.

1302. $\frac{4}{3}\pi ab^2$. **1303.** $\bar{x} = \bar{y} = \bar{z} = \frac{3}{8}r$. **1304.** $\frac{3}{4}b$; $\frac{3}{4}a$.

1305. $3\frac{1}{6}$ cwt.; tension would be converted into compression.

1307. Beam gives way at 8·38 tons; wire at 12·6 tons.

1308. 3·7138 tons. **1309.** 3·39 feet. **1310.** 23712·216.

1311. 59°·14; 60″. **1312.** 3·76. **1313.** ·008 in.

1314. 65·29. **1315.** 16756 lbs. **1316.** 57 tons 11·65 cwt.

1317. ·004012. **1318.** 22·4 in. circumference; 3 tons 56 lbs.

1319. $X = \int_0^l \frac{w}{m} y \, dy = \frac{wl^2}{2m}$. **1320.** Circumference = 4·15 in.

1321. 2·98 miles. **1322.** ·85 in. inside.

1323. Diameter increased 1 : $\sqrt{2}$. **1324.** 89600.

1325. 47 minutes. **1326.** 61·3. **1327.** 61·8.

1328. 663 cub. feet. **1329.** 34·16 H. P.; 222·8 H. P.

1330. 427·8. **1331.** $\cdot 008 \left\{ \frac{A}{a} . (2W + a) - W \right\}$.

1332. 61·09 cub. feet. **1333.** 41 H. P. **1334.** $\frac{42000K}{pln}$.

1335. 5·1 H. P. **1336.** 85·3 H. P. **1337.** 3·67 feet.

1338. $2\sqrt{\frac{rR}{plm}}$. **1340.** 1500 tons. **1341.** 43·2 bushels.

1342. 160·7 bushels. **1343.** 361·4 bushels.

1344. $\frac{f_1w_1 + f_2w_2 + f_3w_3 + \&c.}{528}$; $\frac{9000(f_1w_1 + f_2w_2 + f_3w_3 + \&c.)}{B}$.

1345. 230400. **1346.** 161280. **1347.** 2·79 tons.

1348. 1·1 ton. **1349.** 2083 millions. **1350.** 35733.

1354. 38640. **1355.** 57·14. **1357.** 69 miles an hour.

1358. $\tan^{-1} \cdot 5$; 768 feet; No, in a parabola.

1359. $\sin^{-1}\frac{6+2\sqrt{29}}{25}$; $\frac{75}{(3+\sqrt{29})}$ minutes.

1360. $83\cdot 3\dot{3}$ and $416\cdot 6\dot{6}$ from starting points.

1361. 46·875 miles an hour; 18·75 miles an hour.

1362. 115920. **1363.** 16·1. **1364.** 567 lbs.; 850·5 lbs.

1365. 38400 tons at 1 foot per second. **1366.** 4008·85 feet.

1367. 13600 lbs. **1368.** 200 oz.; 412·16 oz.

1370. $\frac{\sqrt{\{6(w+p)pga\}}}{6(2w+p)}$. **1371.** 60° with line; 10 miles an hour.

1372. 44 feet from line of original relative motion; oblique velocity 44. **1373.** 11·67 feet per second.

1374. 24·64 feet. **1375.** 5·656 miles an hour; S.E.

1376. 29° 13′; 8·84 feet. **1377.** 8·48 miles an hour.

1378. Both bodies being subjected to the same velocity of motion by stream, their relative motion is unaffected by it, and it may be disregarded. Angle with bank $= 60°$.

1379. 101·62 feet from funnel. **1380.** 33·9 feet 'aft.'

1381. The motions are reversed, and each body moves with e times its original velocity.

1382. 6·9 feet. **1383.** 14·44 oz.

1384. Velocities $\sqrt{61}$ and $\sqrt{91}$ at angles $\cot^{-1} 9\sqrt{3}$ and $\cot^{-1}\frac{9\sqrt{3}}{11}$ with tangent at impact.

1385. $A : B :: 4 : 3$. Vel. of $B = \frac{14}{3}$ in direction of A's motion.

1386. 64·506. **1387.** Vel. $= 10\cdot 45$; $\tan^{-1}\frac{25+16\sqrt{3}}{45\sqrt{3}}$.

1388. Spheres will recede on lines symmetrical with original lines of motion. **1389.** $2e-1 : 3$.

1390. Angle of incidence $= \tan^{-1}\frac{e(1+e)\tan a}{1-e\tan^2 a}$.

1392. Velocity of $C = \frac{1+e}{2} \cdot (a^2 + b^2 + 2ab \cos C)^{\frac{1}{2}}$;

angle with $AC = \sin^{-1} \frac{c \,.\, \sin A}{\sqrt{(4b^2 + c^2 - 4bc \cos A)}}$.

1393. 11·02 feet from plane. **1394.** 4 seconds; impossible.

1395. $\frac{be \,.\, (1+e)}{1+e+e^2}$. **1396.** 21·33 feet. **1397.** 30 feet.

1398. 21·77; 39·77. **1399.** $\frac{a}{g} \cdot \{a + gt \pm \sqrt{(a^2 + 2agt)}\}$.

1400. 179·2. **1401.** 150 feet. **1402.** 103·1.

1403. 100·3. **1404.** 2·16 sec.; 1·467. **1405.** 100·625 feet.

1406. 205 feet below starting point.

1407. ·878 foot; $s \,.\, \frac{1+e^2}{1-e^2}$. **1408.** 20·23 miles per hour.

1409. 4·443 below top of cliff; A will be falling, and B rising.

1411. 375 feet. **1412.** 7·3 sec. and 2·7 sec.

1413. Height of tower 193·2; greatest height 257·6; 6 sec.

1414. 115; 250; 4·15 sec. **1415.** 135·2; 76·2 feet.

1416. 4·98 sec.; 80·252. **1417.** 229·056 feet from base.

1418. 49·8 miles an hour; $6^{m}. 51^{s}.5$.

1419. Acceleration $= \frac{P-Q}{P+Q} \cdot g$; tension $= \frac{2PQ}{P+Q}$.

1420. $\frac{7l}{6}$. **1421.** $\frac{8g}{7}$. **1422.** 32·2; 3·75 oz.

1423. 8·12. **1424.** $\frac{16}{3}$ cwt. **1425.** $2\sqrt{2g}$.

1426. 4·91 oz. **1427.** 4·69 sec.; 21·44. **1429.** 60·6.

1430. 1·77 sec.; 9·022 feet. **1431.** 35·35.

1432. 2 tons; 890·76 yards. **1433.** 16·1.

1434. 47·1 miles per hour. **1435.** 39·75.

1442. 3434·7 feet; 28·7 sec.

1446. $\tan a = \frac{h'^2k - h^2k'}{hh'(h' - h)}$; $V^2 = \frac{gh'(h' - k)}{2h(h'k - hk')}\sec^2 a$;

$t^2 = \frac{2}{g} \cdot \frac{h'k(h - h') - h^2(k' - k)}{h(h' - h)}$.

1447. The bodies will all be at equal distances $\frac{1}{2}gt^2$ vertically below the extremities of equal lines drawn from the point of projection in the directions of projection.

1448. 3084 yards. **1450.** 7764 feet; 3·66 inches.

1451. Resolve parallel to plane. **1452.** 1630·7 feet.

1454. 14·314 feet. **1455.** $-4^\circ\,54'$; ·503 inch.

1456. If side $= 2a$ and base $= 2c$; velocity $= \frac{9a^2g}{4} \cdot \sqrt{\left(\frac{3}{4a^2 - c^2}\right)}$.

1457. $t = \frac{1}{g\cos i} \cdot \{V. \sin(a - i) \pm Vr^2. \sin^2(a - i) - 2ag\cos i\}$.

1458. The greatest distance is the same in each trajectory, viz. $\frac{v^2\sin^2(a - i)}{2g\cos i}$. The times are also equal, therefore the velocities are in A. P.; $n = \frac{1}{2}\cot(a - i)\cot i$.

1460. $\frac{t'}{t} = \cot a$. **1461.** $x^2 = \frac{15v^2g}{4} \cdot y$.

1462. ·79 inch. **1463.** 4·0678 lbs.; 1574·9 yards.

1464. 618 feet. **1466.** It returns to A.

1467. ·5 and 1 : 2. **1468.** 67·1 below the obstacle.

1469. $\sqrt{(3gh)}$.

1470. g feet below, and g feet horizontally from starting point.

1471. 3183·3 yards.

1473. Below; error $= \frac{2v^2}{g} . \sin^2 a . \tan i . \sec i$.

1474. $BA > BC$; $\therefore BAC < BCA = CAH$.

1475. 13416·6 feet. **1476.** $\frac{4h^2 - a^2}{4h}$. **1477.** $-2^\circ\,23'$.

1478. $\frac{2374}{1187}$ sec. **1479.** 12·27.

1480. If n', p' be the actual numbers of vibrations, n and p being the true numbers, then $n'-n : p'-p :: n : p$. But if α and β be the corrections of the pendulums $\frac{n'-n}{n} = \frac{\alpha}{2l}$ and $\frac{p'-p}{p} = \frac{\beta}{2l'}$, nearly; whence $\alpha : \beta :: l : l'$.

1481. Time of revolution 1·42 hours. **1483.** 3 : 5.

1484. 8·34 feet from ceiling. **1485.** 3980 feet; 144·85 feet.

1486. 4000 miles. **1488.** 8·64 sec. **1489.** 9·787 inches.

1490. $18^m\ 1^s$. **1491.** 39·15 inches; 4·32 sec.

1492. 1292·5 feet. **1493.** 24° 25′. **1494.** 17 times.

1495. 3·036 inches.

1496. If G be the measure of the force of gravity, when the mile and hour are taken as units; and θ the required angle: $\tan\theta = \frac{m^2}{G}$; tension $= W\sqrt[4]{\left(\frac{m^4}{G^2}+1\right)}$; number of oscillations per minute $= \frac{60}{\pi}\sqrt[4]{\left(\frac{m^4+G^4}{A^2}\right)}$, where $A = \frac{a}{5280}$.

1497. $1 : \sqrt{2}$. **1498.** 344 yards. **1499.** ·112.

1500. 12096 feet. **1501.** $\sqrt{2} : 1$. **1502.** $\cos^{-1}\cdot 25$.

1503. $\frac{10}{9}\cdot\frac{s_1v_1+s_2v_2+s_3v_3}{v_1+v_2+v_3}$. **1504.** $\frac{288(W-w)}{125}$.

1506. $\frac{np-mp'}{(p-1)(m+n)}$. **1507.** $w = 4w_{,}$ **1508.** 231 : 89.

1509. 1 : 17·28. **1510.** 32·5 oz. **1511.** $\frac{b}{p^2+b^2}$.

1512. 27 : 10. **1513.** Copper 443·4 lbs.; tin 56·6 lbs.

1514. $\sqrt{\left\{\frac{a^2+(a+b)^2}{2}\right\}} - a$. **1515.** 678·8 grs. **1517.** 35 : 3.

1518. 5 : 17. **1519.** 1 : 7. **1520.** 527 : 623.

1521. $4\sqrt{2}-3 : 2\sqrt{6}$. **1522.** $\frac{2h}{3}$. **1523.** $l : r$.

1524. When the depth of the centre is h; $\frac{3\pi h - 4r}{3\pi h + 4r}$; $\frac{4r}{\pi}$.

1525. $1+\sqrt{3} : 2+\sqrt{3}$. **1526.** $1 : 2$. **1527.** $r-\frac{l}{8}$.

1528. $\frac{a^3}{2}$; $\frac{2}{3}a$. **1529.** $\frac{5a^3}{8}.\rho$.

1530. If side $= 2a$, and base $= 2b$; ratio $= \frac{4}{27}.\frac{14c^2-5b^2}{4c^2-b^2}$; tension

$$= 2W\frac{c^2-b^2}{14c^2-5b^2}.$$

1531. $\frac{3(s+s_1)}{3s-s_1}$. **1532.** 6·1728 lbs.

1533. $y=\frac{p(2x-3b)}{3b(x-2b)}.x$. **1534.** 8·67 feet. **1535.** $\frac{2b}{a}.\rho$.

1536. ·15. **1537.** 120 lbs. in direction of axis $\frac{3}{10}.\frac{28\sqrt{5}-5}{43}$.

1538. 416·7 lbs. **1539.** ·4653 inch. **1540.** 7168 sq. feet.

1541. Hemisphere in fluid; neutral. **1542.** 6·7; 5·924.

1543. 10·665 inches. **1544.** 13 tons 1·9 cwt.

1545. 1 ton 15·6 cwt. **1546.** 4 tons 7·7 cwt.

1547. $\frac{v}{3}(2+3at)$. **1548.** 4·65 feet. **1549.** 82·83 feet.

1550. Volume increased by $\frac{37v}{260}$; 4685·1 feet.

1551. 399·21 feet. **1552.** 2·5 sec. **1553.** 36·8 feet.

1554. 107°. **1555.** $\frac{9^{10}}{10^{10}}\rho$; 933°·45. **1556.** 39 lbs.

1557. $\frac{7}{3}$. **1558.** $\frac{4\sqrt{2}}{1+x^4}$.

1559. $e^x.x^2.\{(x+3)\tan x + x\sec^2 x\}$. **1562.** $\frac{1}{(1-x)(1-x^2)^{\frac{1}{2}}}$.

1563. $\frac{x^3}{(1+x^2)^{\frac{1}{2}}}$.

1564. $\dfrac{2x}{a^2-b^2}.\left\{1+2\sqrt{\left(\dfrac{x^2+b^2}{x^2+a^2}\right)}+2\sqrt{\left(\dfrac{x^2+a^2}{x^2+b^2}\right)}\right\}.$

1565. $\dfrac{x-(1-x^2)^{\frac{1}{2}}}{(1-x^2)^{\frac{1}{2}}.\{1+2x(1+x^2)^{\frac{1}{2}}\}}.$

1566. $2x.\dfrac{2a^2-x^2}{(a^2-x^2)^{\frac{3}{2}}}.(a^2+x^2)^{\frac{1}{2}}.$

1567. $\dfrac{x+2}{2(x+1)^{\frac{1}{2}}.\{x+(x+1)^{\frac{1}{2}}\}^2}.$ 1568. $\dfrac{2x(x-2)}{(x^2+x-1)^2}.$

1569. $\dfrac{nax^{n-1}}{(a+x)^{n+1}}.$ 1570. $(a^2+x^2)^{-\frac{3}{2}}.$ 1571. $x^3(a+bx^2)^{\frac{1}{2}}.$

1572. $\dfrac{1-(1-x^4)^{\frac{1}{2}}}{x(1-x^4)^{\frac{1}{2}}}.$ 1573. $\dfrac{3x^5}{1+x^3}.$ 1574. $\dfrac{(x^2-a^2)^{\frac{1}{2}}-x}{x^2-a^2}.$

1575. $\dfrac{\log x}{(1-x)^2}.$ 1576. $\dfrac{1}{x}-\sin x.$

1577. $x^{m-1}e^{\sin x}.(m+x\cos x).$ 1578. $\dfrac{2}{1+x^2}.$

1579. $n\sin^{n-1}x.\sin(n+1)x.$ 1580. $-\dfrac{2nx^{n-1}}{x^{2n}+1}.$

1581. $e^x(\cot x+\log\sin x).$ 1582. $-\dfrac{1}{\sin^3 x}.$

1583. $\dfrac{1}{a-b\cos x}.$ 1584. $\dfrac{1+x^2}{1-x^2+x^4}.$

1585. $\dfrac{\sin 2x}{\sqrt{\{2(\cos 2\alpha-\cos 2x)\}}}.$

1586. $e^x.(\log\sin x+2\cot x-\operatorname{cosec}^2 x).$

1587. $\dfrac{\sin x}{(1-x^2)^{\frac{1}{2}}}+\sin^{-1}x.\cos x.$ 1588. $\dfrac{e^{\sqrt{x}}.\operatorname{cosec} 2e^{\sqrt{x}}}{\sqrt{x}}.$

1589. $\dfrac{1}{2(x^2-x+1)}.$ 1590. $2(a^2-x^2)^{\frac{1}{2}}-\dfrac{2a^2}{a^2-x^2}-\dfrac{e^{\sqrt[3]{x}}}{x^{\frac{2}{3}}}.$

1591. $\dfrac{1}{x^2+x+1}.$ 1592. $\dfrac{1}{x(1+x+x^2+x^3)}.$

1594. $-\frac{x^2}{2} - \frac{2x^4}{\underline{|4}} - \&c.$ **1595.** $\frac{x^2}{2} - \frac{x^4}{\underline{|4}} + \frac{x^6}{\underline{|6}} + \&c.$

1596. $\frac{\pi}{2} - x - \frac{1}{\underline{|3}} x^3 - \frac{3 . \underline{|4}}{2^3 . \underline{|5}} x^5 - \frac{5 \underline{|6}}{2^4 . \underline{|7}} x^7 - \&c.$

1597. $1 + x - x^2 - \frac{4x^3}{3} - \&c.$ **1598.** $x + x^2 + \frac{5x^3}{\underline{|3}} + \&c.$

1599. $x - \frac{x^3}{3} + \frac{x^5}{5} - \&c.$

1601. $\sin^{-1} x + \frac{h}{(1-x^2)^{\frac{1}{2}}} + \frac{h^2}{2(1-x^2)^{\frac{3}{2}}} x + \frac{(1+2x^2)h^3}{2 . 3(1-x^2)^{\frac{5}{2}}} . x^2 + \&c.$

1602. $\cos^{-1} x - \frac{h}{(1-x^2)^{\frac{1}{2}}} - \frac{h^2}{2 . (1-x^2)^{\frac{3}{2}}} x - \frac{(1+2x^2)h^3}{\underline{|3}(1-x^2)^{\frac{5}{2}}} x^2 - \&c.$

1603. $n \log x + n\frac{h}{x} - n\frac{h^2}{2} + n\frac{h^3}{\underline{|3}} - \&c.$

1604. $\tan x + h \sec^2 x + h^2 \tan x . \sec^2 x + \&c.$

1605. $\tan^{-1} x + \frac{h}{1+x^2} - \frac{h^2}{(1+x^2)^2} . x + \frac{h^3}{3(1+x^2)^3}(3x^2 - 1) - \&c.$

1607. Vertical angle $= \tan^{-1} \frac{1}{\sqrt{2}}$.

1608. If a be the distance of the given point from the vertex; $x = a - 2m$; if $a < 2m$, x is negative and therefore impossible. At vertex the radius of curvature is $2m$; hence, for this and all smaller values of x, there is, in the true sense, no minimum value of the distance.

1609. Four miles from nearest point.

1610. Ratio of altitudes 1 : 3.

1611. Base of rectangle $= \frac{1}{\sqrt{3}} \times$ (base of parabola).

1612. $h : r :: 1 : \sqrt[3]{2}$. **1613.** Height $= \frac{4}{3}$ (radius of sphere).

1614. Course N. 53° 8′ W. **1615.** $2\pi\left(1-\frac{1}{3}\surd 6\right)$.

1616. Distance from $A=\frac{AC}{\surd 2}$.

1617. If co-ordinates of given point referred to given lines be a and b, the segments are $b+\surd(ab)$ and $a+\surd(ab)$.

1618. If ADQ and QP be rectangular co-ordinates of P, $AB=2r$ and $AD=a$; $AQ=\frac{ar}{2(a-r)}$.

1619. $\frac{a}{b}$. **1620.** $P(\surd 2-1)$.

1621. Subtangent $3a$; asymptote $y+x-\frac{2a}{3}=0$.

1622. $y-y'=\frac{2h(ah-h^2)^{\frac{1}{2}}}{a^2}.(x-h)$. **1623.** $y=x+\frac{a}{3}$.

1624. $-\frac{3a}{2}$; $\frac{(3a^2+b^2)^{\frac{3}{2}}}{8ab}$. **1625.** $2m$; $4m\surd 2$.

1626. $4a$; $2a\surd 2$.

1627. $\frac{17^{\frac{3}{2}}}{16}a$; $\alpha=\frac{3x^2+y^2}{2x}$, $\beta=\frac{3y^2+x^2}{2y}$, $\therefore\ \alpha+\beta=\frac{(x+y)^3}{2a^2}$, $(\alpha-\beta)=\frac{(y-x)^3}{2a^2}$; from these find the values of x and y, take the product, and substitute.

1628. The asymptotes are $y=\pm ax$; $\rho=-a^2b$.

1629. Curve cuts axes at points $(0;\ a^{\frac{2}{3}})$ and $(a^{\frac{2}{3}};\ 0)$ which are points of inflexion. The asymptote is $y=-x$.

1630. Two symmetrical asymptotes at right angles to one another: four infinite and opposite branches, two of which pass through the origin; the other two meet the axis of x at the point $(-4a;\ 0)$. $\rho=2a$.

1631. An asymptote parallel to axis of x, at a distance a; two infinite branches towards positive y's; points of inflexion $\left(\frac{a}{3};\ \frac{a}{4}\right)$ and $\left(-\frac{a}{3};\ \frac{a}{4}\right)$.

1632. Two infinite branches; one towards positive x and negative y; the other towards negative x and positive y. The other branches run into one another at the origin which is a point of inflexion. Greatest ordinate at $x=\frac{1}{\sqrt{3}}$; $\rho=\frac{1}{6}\sqrt{3}$.

1633. $-\frac{1}{3(a+3x^4)^{\frac{1}{2}}}$. **1634.** $(2rx-x^2)^{\frac{1}{2}}$.

1635. $\frac{2}{\sqrt{(4ac-b^2)}}\cdot\tan^{-1}\frac{2cx+b}{\sqrt{(4ac-b^2)}}$; if $4ac>b^2$;

or $\frac{1}{\sqrt{(b^2-4ac)}}\cdot\log\frac{2cx+b-(b^2-4ac)^{\frac{1}{2}}}{2cx+b+(b^2-4ac)^{\frac{1}{2}}}$; if $b^2>4ac$.

1636. $\log\frac{x-1+\sqrt{(3x^2-2x+1)}}{x}$. **1637.** $\log\frac{x}{x+1}-\frac{2}{x+1}$.

1638. $\log\frac{(x-3)^2(x+2)^3}{x^5}$. **1639.** $\frac{3}{2}(a^4-x^4)^{-\frac{1}{2}}$.

1640. $\frac{2}{3}\cdot\frac{x^2-4x-8}{(1+x)^{\frac{1}{2}}}$. **1641.** $\log\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$.

1642. $\frac{(x^2-a^2)^{\frac{1}{2}}(x^2+2a^2)}{3}$. **1643.** $-\frac{x^4}{4}\cdot\frac{3x^2+2}{(1+x^2)^2}+\frac{1}{2}\log(1+x^2)$.

1644. $-\frac{1}{2b(a+bx)^2}$. **1645.** $\frac{a^2b^2}{2}\sin^{-1}\frac{x}{a}+\frac{b^2}{2}\cdot x(a^2-x^2)^{\frac{1}{2}}$.

1646. $\log\frac{x}{x+\sqrt{(1+x^2)}}$. **1647.** $-\frac{1}{3}(x^2+2a^2)(a^2-x^2)^{\frac{1}{2}}$.

1648. $-\frac{1}{2}(3-2x^2)^{\frac{5}{3}}$. **1649.** $a\sin^{-1}\frac{x}{a}+(a^2-x^2)^{\frac{1}{2}}$.

1650. $(a+bx^n)^m$. **1651.** $\log(x-2)-\frac{2}{x-2}$.

1652. $\frac{2}{5}\log(x+3)+\frac{3}{5}\log(x-2)$.

1653. $\frac{2(a+bx)}{b^4}\cdot\left\{\frac{(a+bx)^3}{7}-\frac{3a}{5}(a+bx)^2+a^2(a+bx)-a^3\right\}$.

1654. $\frac{x-a}{2}\cdot(2ax-x^2)^{\frac{1}{2}}+\frac{a}{2}(4a-3)\operatorname{vers}^{-1}\frac{x}{a}$. **1655.** $-\frac{(a^2+x^2)^{\frac{1}{2}}}{a^4x}$.

1656. $\log\frac{(x+1)^{\frac{1}{3}}}{x^{\frac{3}{2}}\cdot(x-2)^{\frac{7}{6}}}$. **1657.** $r^2\operatorname{vers}^{-1}\frac{y}{r}-(2ry-y^2)^{\frac{1}{2}}$.

1658. $\frac{1}{\sqrt{c}}\cdot\log[2cx+b+2\sqrt{\{c(cx^2+bx+a)\}}]$.

1659. $\frac{5}{2}a^3\operatorname{vers}^{-1}\frac{x}{a}-\frac{2x^2+5ax+15a^2}{6}\cdot(2ax-x^2)^{\frac{1}{2}}$.

1660. $-\left\{\frac{1}{3(2ax+x^2)}-\frac{2}{3a^2}\right\}\cdot\frac{x+a}{a^2(2ax+x^2)^{\frac{1}{2}}}$.

1661. $\frac{\log x}{1-x}+\log\frac{1-x}{x}$. **1662.** $\frac{1}{6}x^6\log x-\frac{1}{36}x^6$.

1663. $\frac{x^{n+1}}{n+1}\left(\log x-\frac{1}{n+1}\right)$. **1664.** $\frac{1}{(1-n)(\log x)^{n-1}}$.

1665. $e^x.(x^2-2x+2)$. **1666.** $\log\left(\frac{e^x-1}{e^x+1}\right)^{\frac{1}{2}}$.

1667. $e^{\sqrt{x}}.(2x^{\frac{3}{2}}-6x+12x^{\frac{1}{2}}-12)$.

1668. $\frac{1}{2}\tan^2 x$. **1669.** $\sec x+\cos x$.

1670. $\frac{1}{6}\tan^6 x-\frac{1}{4}\tan^4 x+\frac{1}{2}\tan^2 x+\log.\cos x$.

1671. $\frac{2x^2-1}{4}\sin^{-1}x+\frac{x}{4}(1-x^2)^{\frac{1}{2}}$.

1672. $-\frac{x^2}{4}-\frac{x}{2}(1-x^2)^{\frac{1}{2}}\sin^{-1}x+\frac{1}{4}(\sin^{-1}x)^2$.

1673. $\frac{1}{5}\sin^5 x-\frac{1}{7}\sin^7 x$. **1674.** $\frac{x}{2}+\frac{1}{4}\sin 2x$.

1675. $\tan x\left(1+\frac{1}{3}\tan^2 x\right)$.

1676. $\frac{1}{2}\tan x\sec x+\frac{1}{2}\log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$.

1677. $\frac{a}{2}\left(\log\tan\frac{x}{2}-\cot x\,.\,\operatorname{cosec} x\right)$.

1678. $\frac{1}{3}\tan^3 x-\tan x+x$. **1679.** $\frac{\pi a^3}{4b^{\frac{1}{2}}}$.

1680. $\int\frac{ds}{dx}\,.\,dx=2\,(2ax)^{\frac{1}{2}}$. **1681.** $\frac{8m^2}{3}$; $\frac{h}{3}\pi\,(a^2+ab+b^2)$.

1682. $\frac{ab^2}{2}\,.\log\frac{a+(a^2-b^2)^{\frac{1}{2}}}{b}-\frac{a^2}{2}(a^2-b^2)$.

1683. $\int y\,dx=a\operatorname{vers}^{-1}\frac{x}{a}+(2ax-x^2)^{\frac{1}{2}}+C$;

$$\therefore\int_0^a y\,dx=a\left(\frac{\pi}{2}+1\right)=2\cdot5708a.$$

1684. $\int y\,dx=\frac{a^2}{2}\log\frac{a+x}{a-x}+C$; $\therefore\int_0^{\frac{a}{2}}y\,dx=\frac{a^2}{2}.\log 3$.

1685. $\int y\,dx=\frac{3}{5}(x-a)^{\frac{5}{3}}.\left(\frac{5}{8}x+\frac{3}{8}a\right)+C$; $\therefore\int_0^a y\,dx=\frac{9}{40}a^{\frac{8}{3}}$.

1686. $\int y\,dx=r\operatorname{vers}^{-1}\frac{x}{r}+(2rx-x^2)^{\frac{1}{2}}+C$; $\therefore\int_0^r y\,dx=r^2(\pi+2)$

= area of generating circle + $\frac{1}{2}$ square on diameter.

1687. $\int y\,dx=\frac{x^2}{2}\left(1-\frac{x^2}{2}\right)+C$; $\therefore\int_0^1 y\,dx=\frac{1}{4}$.

1688. $\int y\,dx=a^2.\log(4a^2+x^2)+a^2.\tan^{-1}\frac{x}{2a}+C$;

$$\therefore\int_0^{2a}y\,dx=a^2\left(\log 2+\frac{\pi}{4}\right).$$

1689. $\int y\,dx=\frac{2}{3}x^{\frac{1}{2}}.\,(3a-x)+C$; $\therefore\int_0^a y\,dx=\frac{4}{3}a^{\frac{3}{2}}$.

1690. $\int \pi x^2 dy = -\pi\left(\frac{a}{3}+\frac{x}{5}\right)x^{\frac{3}{2}}+C$; $\therefore$ volume $= -\frac{8}{15}\pi a^{\frac{5}{2}}$.

1691. $\int xy\,dx = \frac{b}{a}\left(\frac{x^2}{3}-\frac{ax}{6}-\frac{a^2}{2}\right).(2ax-x^2)^{\frac{1}{2}}+\frac{a^2}{2b}\operatorname{vers}^{-1}\frac{x}{a}$;

$$\therefore \int_a^{2a} 4\pi xy\,dx = \frac{\pi ba^2}{3}(3\pi+4).$$

1692. $FE = \frac{a}{b}(b^2-h^2)^{\frac{1}{2}}$;

$$\therefore \int (y-h)\,dx = \frac{ab}{2}\sin^{-1}\frac{x}{a}+\frac{b}{2a}.x(a^2-x^2)^{\frac{1}{2}}-hx;$$

$$\therefore \int_0^{\frac{a}{b}(b^2-h^2)^{\frac{1}{2}}}(y-h)\,dx = \frac{ab}{2}.\cos^{-1}\frac{h}{b}-\frac{ah}{2b}.(b^2-h^2)^{\frac{1}{2}} = \text{area } DFE,$$

Vol. generated by DFE about $FE = \int_0^{\frac{b}{a}(b^2-h^2)^{\frac{1}{2}}} \pi(y-h)^2\,dx$,

$$\int \pi(y-h)^2\,dx = \int \pi y^2\,dx - \pi\int 2hy\,dx + \pi h^2 x$$

$$= \pi\int\frac{b^2}{a^2}(a^2-x^2)\,dx - 2\pi h.\int\frac{b}{a}(a^2-x^2)^{\frac{1}{2}}dx + \pi h^2 x$$

$$= \pi.b^2x - \pi\frac{b^2}{3a^2}x^3 - 2\pi h.\frac{b}{a}.(\text{circ. area, rad. } a) + \pi h^2 x$$

$$= \pi.(b^2+h^2).x - \pi\frac{b^2}{a^2}.\frac{x^3}{3} - 2\pi h.\frac{b}{a}\,(\text{circ. area, rad. } a)$$

$$= \pi(b^2+h^2)x - \pi\frac{b^2}{3a^2}x^3 - \pi hab\sin^{-1}\frac{x}{a} + \pi h\frac{b}{a}x(a^2-x^2)^{\frac{1}{2}}.$$

1694. $\pi a^3\left\{\frac{\pi}{4}-\frac{1}{2}\log 2\right\}$.

1695. $2\pi m(b^2-a^2)$; $\frac{8}{3}\pi m^{\frac{1}{2}}\{(b+m)^{\frac{3}{2}}-(a+m)^{\frac{3}{2}}\}$.

1696. $\int xy\,dx = -\frac{1}{3}(r^2-x^2)^{\frac{3}{2}}+C$; $\int y\,dx = \frac{r^2}{2}\sin^{-1}\frac{x}{r}+\frac{x}{2}(r^2-x^2)^{\frac{1}{2}}$;

$$\therefore \bar{X} = \frac{4r}{3\pi}, \text{ and from symmetry } \bar{Y} = \frac{4r}{3\pi}.$$

1697. $\int \frac{r^2}{3} \cos\theta \,.\, d\theta = \frac{r^2}{3} \,.\, \sin\theta$; $\int \frac{r}{2}\, d\theta = \frac{r}{2}\,\theta$; $X = \frac{2}{3} r \,.\, \frac{\sin a}{a}$;

$$\text{if } a = \frac{\pi}{2}, \text{ for semicircle, } X_1 = \frac{4r}{3\pi}.$$

1698. $\int 2\,xy\,dx = -\frac{2}{3}(r^2 - x^2)^{\frac{3}{2}} + C$; $\int 2y\,dx = r^2 \sin^{-1}\frac{x}{r} + x(r^2 - x^2)^{\frac{1}{2}}$. Taking these between the limits $x = a$ and $x = r$ and reducing, we get $X = \frac{2p^3}{3r^2a - 3ap}$ if $p = \frac{1}{2}$ base; or, if h be the height of the segment, $a = r - h$;

$$\therefore\ X = \frac{2}{3} r \,.\, \frac{\left(\frac{p}{r}\right)^3}{a - \frac{p}{r} \,.\, \frac{r-h}{r}} = \frac{2}{3} r \,.\, \frac{\sin^3 a}{a - \sin a \,.\, \cos a}.$$

1699. $\int xy\,dx = \frac{4}{5} m^{\frac{1}{2}} x^{\frac{5}{2}}$; $\int y\,dx = \frac{4}{3} m^{\frac{1}{2}} x^{\frac{3}{2}}$; $\therefore\ X = \frac{3a}{5}$.

1700. $\int x \frac{ds}{dx} \,.\, dx = -r(r^2 - x^2)^{\frac{1}{2}} + C$; $\int \frac{ds}{dx} \,.\, dx = r \sin^{-1}\frac{x}{r}$. Hence, if the limits of integration be 0 and r, we have for semicircular arc, $X = \frac{2r}{\pi}$. And, if the limits be b and r,

$$X_1 = \frac{(r^2 - b^2)^{\frac{1}{2}}}{\cos^{-1}\frac{b}{r}} = \frac{\text{rad.} \times \text{chd.}}{\text{arc}}.$$

1701. $\int \pi x^2 y\,dy = \frac{y^6}{96m^2}$; $\int \pi x^2\,dy = \frac{y^5}{80m^2}$; $\therefore\ Y = \frac{5}{6} VA$.

1702. If density at unit of distance $= \rho$;

$$\text{at distance } x \text{ density} = \rho x;\ \therefore\ X = \frac{\int_0^a \rho x^2\,dx}{\int_0^a \rho x\,dx} = \frac{2a}{3}.$$

1703. Density at distance $x = \frac{\rho}{x}$; therefore weight of elementary disc $= \pi\rho \,.\, \frac{y^2}{x}\, dx$; therefore moment of elementary disc $= \pi\rho y^2 dx$; hence $X = \dfrac{\int_0^h \pi\rho y^2 dx}{\int_0^h \pi\rho \frac{y^2}{x} dx} = \frac{2}{3} h$.

1704. $\frac{n-1}{n-4} \,.\, h$. **1705.** $\frac{a}{32} \,.\, \frac{67 - 96 \,.\, \log 2}{3 \log 2 - 2}$.

1706. $\dfrac{2a^4}{3\,(a^2+b^2)^{\frac{1}{2}} \,.\, \{(a^2+b^2) \cos^{-1} \frac{b}{(a^2+b^2)^{\frac{1}{2}}} - ab\}}$.

1707. $\frac{n+2}{n+3} \,.\, h$ on the line $(=h)$ drawn from the vertex to the point of bisection of the base.

1708. By last question, distance of centre of gravity of elementary sector from centre of circle $= \frac{n+2}{n+3} \,.\, r$. Also weight of elementary sector $= \rho \,.\, \frac{r^{n+1}}{n+2}\, d\theta$; ∴ moment of elementary sector about radius $= \rho \,.\, \frac{r^{n+2}}{n+3} \cos\theta \,.\, d\theta$;

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\rho r^{n+2}}{n+3} \cos\theta \,.\, d\theta = \rho \frac{r^{n+2}}{n+3} \text{ and } \int_0^{\frac{\pi}{2}} \frac{\rho r^{n+1}}{n+2} \,.\, d\theta = \frac{\rho r^{n+1}}{n+2} \,.\, \frac{\pi}{2};$$

$$\therefore X = Y = \frac{n+2}{n+3} \,.\, \frac{2r}{\pi}.$$

1709. $M \,.\, \frac{a^2}{3}$. **1710.** $M \frac{r^2}{2}$. **1711.** $M \,.\, \frac{a^2}{24}$.

1712. $\frac{M}{2a} \int_{-a}^{+a} r^2 \sin^2\theta \, d\theta = M \frac{r^2}{2} \left(1 - \frac{\sin 2a}{2a}\right)$.

1713. $M 2r^2 \,.\, \left(1 - \frac{\sin a}{a}\right)$. **1714.** $M \frac{3r^2}{2}$. **1715.** $M \frac{r^2}{4}$.

1716. $M\frac{r^2+r_{,}^2}{2}$. **1717.** $M\frac{r^2}{2}$. **1718.** $M\frac{2r^2}{5}$.

1719. By 1710 moment of lamina about axis $= m.\frac{r_{,}^2}{2} = \frac{m}{2}.\left(\frac{r.x}{h}\right)^2$,

and $m = M.\frac{\pi r^2.dx}{\frac{\pi r^2 h}{3}} = M.\frac{3x^2}{h^3}dx$;

$$\therefore \Sigma\left(\frac{m}{2}.\frac{r^2x^2}{h^2}\right) = \int_0^h \frac{3}{2}.\frac{Mr^2}{h^5}x^4dx = M.\frac{3r^2}{10}.$$

1720. Let moment of sphere radius $r = MK^2$,

.... $r_{,} = M_{,}K_{,}^2$,

............shell (radii r and R) $= M_{,,}K_{,,}^2$,

then $M_{,}K_{,}^2 + M_{,,}K_{,,}^2 = MK^2$, but from 1718 $K_{,}^2 = \frac{2r_{,}^2}{5}$, and $K^2 = \frac{2r^2}{5}$; $\therefore M_{,,}K_{,,}^2 = M.\frac{2}{5}r^2 - M_{,}.\frac{2}{5}r_{,}^2$, but M, $M_{,}$ and $M_{,,}$ are as r^3, $r_{,}^3$ and $r^3 - r_{,}^3$; $\therefore (r^3 - r_{,}^3)K_{,,}^2 = \frac{2}{5}(r^5 - r_{,}^5)$; $\therefore K_{,,}^2 = \frac{2}{5}.\frac{r^5 - r_{,}^5}{r^3 - r^3}$.

1721. $\frac{r^2 + r_{,}^2}{2}$.

1722. Expand 1720 and find the limit when $r = r_{,}$; $MK^2 = \frac{2}{3}Mr^2$.

1723. Mr^2.

1724. If the elementary mass m be referred to the intersection of the diagonals by lines parallel to the sides, and if r be its radius of gyration; $mr^2 = mx^2 + my^2 + 2mxy\cos\alpha$, and therefore $MK^2 = \Sigma(mx^2) + \Sigma(my^2) + 2\Sigma(mxy)\cos\alpha$, but

$$m = M.\frac{dx.dy}{4ab\sin\alpha}\sin\alpha = \frac{M}{4ab}.dx.dy;$$

$$\therefore\ MK^2=\int_{-a}^{+a}\int_{-b}^{+b}\frac{M}{4ab}.\,x^2\,dx\,dy+\int_{-a}^{+a}\int_{-b}^{+b}\frac{M}{4ab}.\,y^2\,dx\,dy$$

$$+\,2\int_{-a}^{+a}\int_{-b}^{+b}\frac{M}{4ab}\,.\,xy\cos\alpha\,.\,dx\,dy$$

$$\int_{-a}^{+a}\int_{-b}^{+b}\frac{M}{4ab}\,x^2\,dx\,dy=\frac{M}{4ab}\,.\,\frac{2a^3}{3}\,.\,2b=M\,\frac{a^2}{3}$$

$$\int_{-a}^{+a}\int_{-b}^{+b}\frac{M}{4ab}.\,y^2\,dx\,dy.=M\,\frac{b^2}{3};\ \text{and}\ \int_{-a}^{+a}\int_{-b}^{+b}\frac{M}{4ab}\,xy\,dx\,dy\cos\alpha=0;$$

$$\therefore\ MK^2=\frac{M}{3}\,(a^2+b^2).$$

1725. Moment about axis of parabola $=M\,\dfrac{b^2}{5}$, and moment about tangent at vertex $=M\,\dfrac{6a^2}{7}$; therefore required moment

$$=M\left(\frac{6}{7}\,a^2+\frac{1}{5}\,b^2\right).$$

1726. If intersections of diagonals of opposite faces be joined, these will be the three principal axes, and therefore, as regards these axes, $\Sigma\,(mxy)$, $\Sigma\,(mxz)$ and $\Sigma\,(myz)$ are severally zero. Also, moment of side about edge = moment of edge about edge $=M\,\dfrac{a^2}{3}$; therefore moment of cube about edge (= moment of side about perpendicular edge) $=\dfrac{2a^2}{3}$; therefore moment of cube about one of the principal axes $=M.\dfrac{2a^2}{3}-M\,.\,\dfrac{a^2}{2}=M.\,\dfrac{a^2}{6}$. And diagonal makes with the principal axes, the angles $\cos^{-1}\dfrac{1}{\sqrt{3}}$; hence the general form $I=A\cos^2\alpha+B\,.\cos^2\beta+C\,.\cos^2\gamma=3\,A\cos^2\alpha$ gives, moment about diagonal $=3\,M\,\dfrac{a^2}{6}\times\dfrac{1}{3}=M\,\dfrac{a^2}{6}$.

1727. Since the line drawn through centre of cube parallel to diagonal of face makes angles of 45°; 45° and 90° with the principal axes, moment about that line

$$= M\frac{a^2}{6}(\cos^2 45 + \cos^2 45 + \cos^2 90) = M\frac{a^2}{6}.$$

Hence moment about $OQ = \frac{Ma^2}{6} + \frac{Ma^2}{4} = M.\frac{5a^2}{12}$.

1728. See 1726.

1729. Find the moments, $M\frac{3b^2+2a^2}{20}$; $M\frac{3b^2+2a^2}{20}$; $M.\frac{3b^2}{10}$ about the three principal axes, and apply the general form as in 1726; $M.\frac{3b^2}{20} \times \frac{b^2+ba^2}{a^2+b^2}$: a being the altitude and b the radius of the base of the cone.

1730. Mass of element $= M.\frac{3x^2}{a^2b}dy$;

$$\therefore \text{ moment} = \frac{3M}{4a^2b}, \int_{-b}^{+b}\frac{x^4}{2}dy;$$

substitute for x^4 from equation to ellipse, and we get moment about $2b = M.\frac{2}{5}a^2$.

1731. Taking elliptical section parallel to axis $2a$ and calling the minor axis of this section z, its mass will be represented by $M.\frac{3xz}{4abc}dy$. Also moment of ellipse about an axis through its centre perpendicular to its plane will be found to be Mass $\times \frac{a_,^2+b_,^2}{4}$. Hence moment of ellipsoid about $2b = \frac{3M}{16abc}\int_{-b}^{+b}(x^2+z^2)\,xzdy$. Substituting the values of x and z derived from the equations $x^2 = \frac{a^2}{b^2}(b^2-y^2)$ and $z^2 = \frac{c^2}{b^2}(b^2-y^2)$ we get ultimately:—

$$\text{moment about } 2b = \frac{a^2+c^2}{5}.$$

1732. $\frac{\pi s}{2} \cdot \sqrt[4]{\frac{25}{3g^2}}$. 1733. $\pi \cdot \sqrt[4]{\frac{8a^2}{9g}}$. 1734. $\pi \cdot \sqrt{\frac{7a}{5g}}$.

1735. $\pi \cdot \sqrt{\left\{\frac{23r^2 + 2h^2}{5g \cdot \sqrt{(h^2 + 16r^2)}}\right\}}$. 1736. By 1713 and 1700.

1739. If resistance for unit of velocity $= k$; $\frac{dv}{dt} = \frac{w - kv^2}{w} g$;

$$\therefore t = \frac{1}{2g}\sqrt{\frac{w}{k}} \cdot \log_e \frac{\sqrt{w} + v\sqrt{k}}{\sqrt{w} - v\sqrt{k}}.$$

1740. $v = \frac{ds}{dv} \cdot \frac{w - kv^2}{w} \cdot g$; $\therefore s = \frac{Vw}{2kg} \cdot \log_e \frac{w}{w - kv^2}$.

1741. $\frac{dt}{dv} = -\frac{1}{kv}$; $\therefore t = \frac{1}{k} \log \frac{r}{v}$; $s = \frac{V}{k}(1 - e^{-kt})$.

1742. $\frac{w}{2kg} \cdot \log_e\left(1 - \frac{k}{w} V^2\right)$. 1743. $\frac{w}{kg} \cdot \log_e\left(\frac{kgVt}{w} + 1\right)$.

1744. $t = \frac{2V \cdot \sin a}{g}$; and by 1743 $r = \frac{w}{kg} \cdot \log_e\left(\frac{kgV \cos a \cdot t}{w} + 1\right)$;

$$\therefore r = \frac{w}{kg} \cdot \log_e\left(\frac{kV^2}{w} \sin 2a + 1\right).$$

TABLES.

MEASURES OF WEIGHTS.

ENGLISH.

TROY WEIGHT.

grs.
24 = 1 dwt.
480 = 20 = 1 oz.
5760 = 240 = 12 = 1 Tr. lb.

AVOIRDUPOIS WEIGHT.

drs.
16 = 1 oz.
256 = 16 = 1 lb.
7168 = 448 = 28 = 1 qr.
28672 = 1792 = 112 = 4 = 1 cwt.
573440 = 35840 = 2240 = 80 = 20 = 1 ton.

FRENCH.

1 Milligramme	=	·001	gramme	=	·0154326	grains English.
1 Centigramme	=	·01	gramme	=	·154326	
1 Decigramme	=	·1	gramme	=	1·54326	
1 Gramme	=	1·	gramme	=	15·4326	
1 Decagramme	=	10·	grammes	=	154·326	
1 Hectogramme	=	100·	grammes	=	1543·26	
1 Kilogramme	=	1000·	grammes	=	15432·6	
1 Myriagramme	=	10000·	grammes	=	15432·6	

MEASURES OF LENGTH.

English.

inches.
12 = 1 foot.
36 = 3 = 1 yard.
198 = 16½ = 5½ = 1 rod.
792 = 66 = 22 = 4 = 1 chain.
7920 = 660 = 220 = 40 = 10 = 1 furlong.
63360 = 5280 = 1760 = 320 = 80 = 8 = 1 mile.

French.

1 Millimètre = ·001 mètre
1 Centimètre = ·01 mètre
1 Décimètre = ·1 mètre
1 Mètre = 1· mètre = 39·37009 English inches.
1 Décamètre = 10· mètres
1 Hectomètre = 100· mètres
1 Kilomètre = 1000· mètres
1 Myriamètre = 10000· mètres

MEASURES OF SURFACE.

English.

sq. inches.
144 = 1 square foot.
1296 = 9 = 1 square yard.
39204 = 272¼ = 30¼ = 1 pole.
1568160 = 10890 = 1210 = 40 = 1 rood.
6272640 = 43560 = 4840 = 160 = 4 = 1 acre.

Also; 1 acre = 4840 sq. yds. = $10 \times (22)^2$ sq. yds. = 10 sq. chains.

French.

1 Centiare = ·01 are
1 Déciare = ·1 are
1 Are = 1· are = ·0247105 English acre.
1 Décare = 10· ares
1 Hectare = 100· ares
1 Kiliare = 1000· ares
1 Myriare = 10000· ares

MEASURES OF CAPACITY.

English.

gills.
4 = 1 pint.
8 = 2 = 1 quart.
32 = 8 = 4 = 1 gallon.
64 = 16 = 8 = 2 = 1 peck.
256 = 64 = 32 = 8 = 4 = 1 bushel.
2048 = 512 = 256 = 64 = 32 = 8 = 1 quarter.
10240 = 2560 = 1280 = 320 = 160 = 40 = 5 = 1 load.

The Imperial gallon = 277·274 cubic inches = 10 lbs. water.

French.

1 Centilitre = ·01 litre
1 Décilitre = ·1 litre
1 Litre = 1· litre = 1·76068 English pint.
1 Décalitre = 10· litres
1 Hectolitre = 100· litres
1 Kilolitre = 1000· litres
1 Myrialitre = 10000· litres

The French unit of
length is the Mètre = 39·37009 Eng. inches.
weight is the Gramme = weight of Centimètre cubed of distilled water.
surface is the Are = 10 Mètres squared.
capacity is the Litre = 1 Décimètre cubed.
volume is the Stère = 1 Cubic Mètre.

FORMULÆ.

ARITHMETICAL PROGRESSION.

$$s=\{2a+(n-1)b\}\frac{n}{2};\ l=a+(n-1)b;\ s=n\frac{a+l}{2};\ b=\frac{l-a}{n-1}.$$

GEOMETRIC PROGRESSION.

$$s=a.\frac{r^n-1}{r-1};\ l=ar^{n-1};\ s=\frac{rl-a}{r-1};\ s_\infty=\frac{a}{1-r}.$$

PILES OF SPHERICAL SHOT.

Triangular $\dfrac{n(n+1)(n+2)}{6}$; Square $\dfrac{n(n+1)(2n+1)}{6}$;

Rectangular $\dfrac{n(n+1)(2n+1+3m)}{6}$.

PERMUTATIONS AND COMBINATIONS.

n things, r together, all different;

No. of Permutations $=n(n-1)\ldots(n-r+1)$,

No. of Combinations $=\dfrac{n(n-1)\ldots(n-r+1)}{\lfloor\underline{r}}$.

n things, all together; a alike, b alike, c alike; $\dfrac{\lfloor\underline{n}}{\lfloor\underline{a}\,.\lfloor\underline{b}\,.\lfloor\underline{c}}$.

BINOMIAL THEOREM.

$$(a+x)^n=a^n+na^{n-1}.x+\frac{n(n-1)}{2}a^{n-2}x^2+\frac{n(n-1)(n-2)}{\lfloor\underline{3}}.a^{n-3}.x^3\ldots$$

$$\ldots+\frac{n(n-1)(n-2)\ldots(n-r+2)}{\lfloor\underline{r-1}}.a^{n-r+1}.x^{r+1}.$$

Exponential Theorem.

$$a^x = 1 + Ax + \frac{A^2x^2}{2} + \frac{A^3x^3}{\underline{|3}} + \ldots\ldots \frac{A^nx^n}{\underline{|n}},$$

where $A = a - 1 - \frac{1}{2}(a-1)^2 + \frac{1}{3}(a-1)^3 - \&\text{c.} = \log_e a,$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{\underline{|3}} + \ldots\ldots \frac{x^n}{\underline{|n}},$$

$$e = 1 + 1 + \frac{1}{2} + \frac{1}{\underline{|3}} + \ldots\ldots \frac{1}{\underline{|n}} = 2{\cdot}718281828459,$$

$$\log_e \frac{1+x}{1-x} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\ldots \frac{x^n}{n}\right).$$

$$\log_e (n+z) = \log_e n + 2\,\frac{z}{2n+z} + \frac{1}{3}\cdot\left(\frac{z}{2n+z}\right)^3 + \frac{1}{5}\left(\frac{z}{2n+z}\right)^5 + \&\text{c.}$$

Plane Trigonometry.

$$\sin(A \pm B) = \sin A \,.\, \cos B \pm \cos A \,.\, \sin B,$$

$$\cos(A \pm B) = \cos A \,.\, \cos B \mp \sin A \,.\, \sin B,$$

$$\sin 2A = 2 \sin A \,.\, \cos A,$$

$$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1,$$

$$\sin A + \sin B = 2 \sin \tfrac{1}{2}(A+B) \cos \tfrac{1}{2}(A-B),$$

$$\sin A - \sin B = 2 \cos \tfrac{1}{2}(A+B) \sin \tfrac{1}{2}(A-B),$$

$$\sin(n+1)A = 2 \sin nA \,.\, \cos A - \sin(n-1)A,$$

$$\cos(n+1)A = 2 \cos nA \,.\, \cos A - \cos(n-1)A,$$

$$2\sin^2 A = 1 - \cos 2A,$$

$$4\sin^3 A = 3 \sin A - \sin 3A,$$

$$8\sin^4 A = 3 - 4\cos 2A + \cos 4A,$$

$$16\sin^5 A = 10 \sin A - 5 \sin 3A + \sin 5A,$$

$$\&\text{c.} = \&\text{c.}$$

$$2\cos^2 A = 1 + \cos 2A,$$

$$4\cos^3 A = 3\cos A + \cos 3A,$$

$$8\cos^4 A = 3 + 4\cos 2A + \cos 4A,$$

$$16\cos^5 A = 10\cos A + 5\cos 3A + \cos 5A,$$

$$\&\text{c.} = \&\text{c.}$$

$$\tan^2 A = \frac{1 - \cos 2A}{1 + \cos 2A}\,;\quad \tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\,.\,\tan B}\,,$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}\,;\quad \sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B$$

$$= \cos^2 B - \cos^2 A.$$

In any plane triangle,

$$a : b : c :: \sin A : \sin B : \sin C\,;$$

$$\frac{a + b}{a - b} = \frac{\tan \frac{1}{2}(A + B)}{\tan \frac{1}{2}(A - B)}\,;\quad \cos A = \frac{b^2 + c^2 - a^2}{2bc}\,;$$

$$\sin^2 \frac{1}{2} A = \frac{(s - b)(s - c)}{bc}\,;\quad \cos^2 \frac{1}{2} A = \frac{s\,.(s - a)}{bc}\,;\quad \tan^2 \frac{1}{2} A = \frac{(s - b)(s - c)}{s\,.\,(s - a)}.$$

Spherical Trigonometry. Napier's Rules.

(1) Sine of middle part = product of tangents of adjacent parts.

(2) Sine of middle part = product of cosines of opposite parts.

In any spherical triangle,

$$\cos a = \cos b\,.\,\cos c - \sin b\,.\,\sin c\,.\,\cos A\,;$$

$$\sin^2 \tfrac{1}{2} A = \frac{\sin(s - b)\sin(s - c)}{\sin b\,.\,\sin c}\,;\quad \cos^2 \tfrac{1}{2} A = \frac{\sin s\,.\,\sin(s - a)}{\sin b \sin a}\,;$$

$$\tan^2 \tfrac{1}{2} A = \frac{\sin(s - b)\,\sin(s - c)}{\sin s\,.\,\sin(s - a)}\,;$$

$$\sin a : \sin b : \sin c :: \sin A : \sin B : \sin C\,;$$

$$\tan \tfrac{1}{2}(A + B) = \frac{\cos \frac{1}{2}(a - b)}{\cos \frac{1}{2}(a + b)}\,.\,\cot \tfrac{1}{2} C,$$

$$\tan \tfrac{1}{2}(A - B) = \frac{\sin \frac{1}{2}(a - b)}{\sin \frac{1}{2}(a + b)}\,.\,\cot \tfrac{1}{2} C,$$

$$\tan \tfrac{1}{2}(a + b) = \frac{\cos \frac{1}{2}(B - A)}{\cos \frac{1}{2}(B + A)}\,.\,\tan \tfrac{1}{2} c,$$

$$\tan \tfrac{1}{2}(a - b) = \frac{\cos \frac{1}{2}(A - B)}{\cos \frac{1}{2}(A + B)}\,.\,\tan \tfrac{1}{2} c.$$

TABLE OF ASTRONOMICAL MEAN REFRACTIONS.

Zenith Distance.	Mean Refraction.	Zenith Distance.	Mean Refraction.	Zenith Distance.	Mean Refraction.
° ′	′ ″	° ″	′ ″	° ′	° ″
1 0	0 1	73 30	3 15	85 10	10 11
10 0	0 10	74 0	3 21	85 20	10 28
20 0	0 21	74 30	3 28	85 30	10 46
30 0	0 34	75 0	3 35	85 40	11 6
35 0	0 41	75 30	3 42	85 50	11 26
40 0	0 49	76 0	3 50	86 0	11 47
45 0	0 58	76 30	3 59	86 10	12 10
50 0	1 10	77 0	4 8	86 20	12 34
52 0	1 15	77 30	4 18	86 30	13 0
54 0	1 20	78 0	4 28	86 40	13 27
56 0	1 26	78 30	4 40	86 50	13 56
58 0	1 33	79 0	4 52	87 0	14 27
60 0	1 41	79 20	5 1	87 10	15 0
61 0	1 45	79 40	5 10	87 20	15 35
62 0	1 49	80 0	5 20	87 30	16 12
63 0	1 54	80 20	5 31	87 40	16 52
64 0	1 59	80 40	5 42	87 50	17 35
65 0	2 5	81 0	5 54	88 0	18 21
66 0	2 10	81 20	6 7	88 10	19 11
67 0	2 17	81 40	6 20	88 20	20 5
68 0	2 24	82 0	6 35	88 30	21 3
69 0	2 31	82 20	6 50	88 40	22 5
69 30	2 35	82 40	7 7	88 50	23 13
70 0	2 39	83 0	7 25	89 0	24 27
70 30	2 44	83 20	7 45	89 10	25 47
71 0	2 48	83 40	8 7	89 20	27 14
71 30	2 53	84 0	8 30	89 30	28 50
72 0	2 58	84 20	8 55	89 40	30 33
72 30	3 3	84 40	9 23	89 50	32 15
73 0	3 9	85 0	9 54	90 0	34 32

MENSURATION.

Triangle $= \frac{1}{2} bp = \frac{1}{2} ab \sin C = \sqrt{\{s . (s-a)(s-b)(s-c)\}}$.

Trapezoid $= \frac{1}{2}(a+b)p$.

Area of regular polygon of n sides

$$\frac{n}{4} a^2 \cot \frac{180^\circ}{n} = \frac{1}{2} nR^2 \sin \frac{360}{n} = nr^2 \tan \frac{180^\circ}{n},$$

R and r being the radii of the circumscribed and inscribed circles.

Perimeter of regular polygon $= 2nr \tan \frac{180^\circ}{n}$.

Perimeter of circle $= 2\pi r$; area of circle $= \pi r^2 = \frac{\pi d^2}{4} = \frac{c^2}{4\pi}$.

Area of sector of circle $= \frac{ar}{2} = \frac{A^\circ}{360^\circ} \pi r^2$ (a being the length of arc).

Area of segment of circle $= \frac{r}{2}(a - r . \sin A^\circ)$; ring $= \pi(R^2 - r^2)$.

Vol. of prism $= B^2 . h$ (B^2 being area of base).

Vol. of cylinder $= \pi r^2 h$; vol. of cone $= \pi r^2 \frac{h}{3}$.

Vol. of pyramid $= B^2 . \frac{h}{3}$; vol. of frustum $= \frac{h}{3}(A^2 + B^2 + A . B)$.

Surface of prism $= ph + 2B^2$ (p being perimeter and B^2 the area of base).

Surface of cone $= \pi rh + \pi r^2$.

Surface of sphere $= 4\pi R^2$; surface of segment $= 2\pi Rh$.

Vol. of sphere $= \frac{4}{3}\pi r^3$; vol. of segment $= \frac{\pi h^2}{3}(3R - h)$.

Vol. of circular ring (having circular section rad. a)

$$= 2\pi^2 a^2 (a + r).$$

CO-ORDINATE GEOMETRY. EQUATIONS TO LINES, &c.
(RECTANGULAR CO-ORDINATES.)

Straight line; $y = ax + b$.

Straight line through point (h, k); $y - k = a(x - h)$.

Straight line through points (h, k), (h', k'); $y - k = \dfrac{k' - k}{h' - h}(x - h)$.

Straight line perpendicular to $y = ax + b$ is $y = -\dfrac{1}{a}x + b'$,

.... parallel to $y = ax + b$ is $y = ax + b'$.

Perpendicular on $y = ax + b$ from $(h, k) = \dfrac{k - ah - b}{\sqrt{(1 + a^2)}}$.

Circle; origin at centre; $y^2 + x^2 = r^2$.

.... circumference, centre on axis of x;

$$y^2 + x^2 - 2rx = 0.$$

.... circumference, centre not on axis of x;

$$y^2 + x^2 - 2\alpha x - 2\beta y = 0.$$

.... anywhere, $(y - \beta)^2 + (x - \alpha)^2 = r^2$.

.... tangent, origin at centre; $ky + hx = r^2$, (h, k) being a point on the circumference.

Circle; chord of contact to tangents drawn from exterior point (h', k'), $k'y + h'x = r^2$.

Conic sections, $y^2 = px + qx^2$ origin at vertex; axis on axis of x, $p =$ latus-rectum; in ellipse q is negative, in hyperbola q is positive; in parabola q is 0; in circle q is -1.

Ellipse, origin at centre; $y^2 = \dfrac{b^2}{a^2}(a^2 - x^2)$,

.... vertex; $y^2 = \dfrac{b^2}{a^2}(2ax - x^2)$.

Hyperbola, origin at centre; $y^2 = \frac{b^2}{a^2}(x^2 - a^2)$,

…………………… vertex; $y^2 = \frac{b^2}{a^2}(x^2 + 2ax)$.

Parabola, origin at vertex; $y^2 = 4mx$.

Cissoid; $y^2 = \frac{x^3}{2a - x}$; conchoid; $x^2y^2 = (b^2 - x^2)(a + x)^2$.

Witch; $xy^2 = 4a^2(2a - x)$.

$$\text{Cycloid, } \begin{cases} \text{origin at base; } & x = a \operatorname{vers}^{-1}\frac{x}{a} - \sqrt{(2ay - y^2)}, \\ \text{origin at vertex; } & y = a \operatorname{vers}^{-1}\frac{x}{a} + \sqrt{(2ax - x^2)}. \end{cases}$$

Tangent to any curve; $y - k = \frac{dy}{dx}(x - h)$.

Subtangent $= y\frac{dx}{dy}$.

Normal to any curve; $y - k = -\frac{dx}{dy}(x - h)$.

$$\text{Intercepts of asymptotes; } \begin{cases} y - x\frac{dy}{dx}, & \text{when } x = \infty, \\ x - y\frac{dx}{dy}, & \text{when } x = \infty. \end{cases}$$

If equation to curve be reducible to the form

$$y = ax + b + \frac{c}{x} + \frac{d}{x^2} + \text{\&c.},$$

then equation to rectilinear asymptote is

$$y = ax + b.$$

At a singular point, being

a point of contrary flexure, $\frac{d^2y}{dx^2} = 0$, or $\frac{1}{0}$, and changes its sign;

a multiple point, $\frac{dy}{dx} = \frac{0}{0}$, and $\frac{dy}{dx}$ or $\frac{d^2y}{dx^2}$ will have multiple values.

A conjugate point; $\frac{dy}{dx} = \frac{0}{0}$, and is impossible for $x = a \pm h$.

A cusp; $\frac{dy}{dx}$ will have but one value; $\frac{d^2y}{dx^2}$ two, or more.

The radius of curvature is $\rho = \frac{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{-\frac{d^2y}{dx^2}}$.

To find the evolute; $\beta - y = \rho^{\frac{2}{3}} . \left(\frac{d^2y}{dx^2}\right)^{-\frac{1}{3}}$, and $a - x = \frac{dy}{dx}(y - \beta)$.

STATICS.

FORCES IN ONE PLANE.

CONDITIONS OF EQUILIBRIUM.

Forces through one point; $\begin{cases} f_1 \cos a_1 + f_2 \cos a_2 + f_3 \cos a_3 + \&c. = 0, \\ f_1 \sin a_1 + f_2 \sin a_2 + f_3 \sin a_3 + \&c. = 0. \end{cases}$

Forces not all meeting in one point;

$$\begin{cases} f_1 \cos a_1 + f_2 \cos a_2 + f_3 \cos a_3 + \&c. = 0, \\ f_1 \sin a_1 + f_2 \sin a_2 + f_3 \sin a_3 + \&c. = 0, \\ \left.\begin{array}{l} f_1 y_1 \cos a_1 + f_2 y_2 \cos a_2 + f_3 y_3 \cos a_3 + \&c. \\ - f_1 x_1 \sin a_1 - f_2 x_2 \sin a_2 - f_3 x_3 \sin a_3 - \&c. \end{array}\right\} = 0. \end{cases}$$

CENTRE OF GRAVITY.

A system of bodies; $\begin{cases} \bar{x} = \frac{w_1 x_1 + w_2 x_2 + w_3 x_3 + \&c.}{w_1 + w_2 + w_3 + \&c.}; \\ \bar{y} = \frac{w_1 y_1 + w_2 y_2 + w_3 y_3 + \&c.}{w_1 + w_2 + w_3 + \&c.}. \end{cases}$

For any body; $\bar{x} = \frac{\int x \frac{dM}{dx} dx}{\int \frac{dM}{dx}}$; and $\bar{y} = \frac{\int y \frac{dM}{dy} dy}{\int \frac{dM}{dy} dy}$.

In triangle; $\bar{x} = \frac{2}{3}h$; parallelogram; $\bar{x} = \frac{h}{2}$.

Trapezoid; $\bar{x} = \frac{h}{3} \cdot \frac{2a+b}{a+b}$; semicircle; $\bar{x} = \frac{2}{3} \cdot \frac{2r}{\pi}$ (from centre).

Sector of circle; $\bar{x} = \frac{2}{3} \frac{\sin a}{a}$.

Segment of circle; $\bar{x} = \frac{4}{3}r \cdot \frac{\sin^3 a}{2a - \sin 2a}$ (measured from centre).

Arc of circle; $\bar{x} = \frac{r \cdot \text{chord } 2a}{2a}$; arc of semicircle $\bar{x} = \frac{2r}{\pi}$.

Arc of semicycloid; $\bar{x} = \frac{2}{3}r$; $\bar{y} = \left(\pi - \frac{4}{3}\right)r$.

Area of cycloid; $\bar{y} = \frac{5}{6}r$; area of parabola; $\bar{x} = \frac{3}{5}h$.

Area of semi-ellipse; $\bar{x} = \frac{4a}{3\pi}$ from centre.

Cone; $\bar{x} = \frac{3h}{4}$; pyramid; $\bar{x} = \frac{3h}{4}$.

Conical surface; $\bar{x} = \frac{2h}{3}$; volume of hemisphere; $\bar{x} = \frac{3r}{8}$.

Volume of spherical segment; $\bar{x} = \frac{3}{4} \cdot \frac{(2r-h)^2}{3r-h}$ (from centre).

Surface of spherical segment; $\bar{x} = \frac{h}{2}$ (from base).

Spherical sector; $\bar{x} = \frac{3}{8} \cdot (2r-h)$ from centre.

Conic frustum; $\bar{x} = \frac{h \cdot (a^2 + 2ab + 3b^2)}{4 \cdot (a^2 + ab + b^2)}$ (from πa^2).

Paraboloid; $\bar{x} = \frac{2}{3}h$.

Parabolic frustum; $\bar{x} = \frac{h}{3} \cdot \frac{a^2 + 2b^2}{a^2 + b^2}$ (from πa^2).

$$\text{Cycloid; } \begin{cases} \text{arc; } \bar{x} = \frac{2}{3} r, \\ \text{surface of solid of revolution; } \bar{x} = \frac{2r}{15} \cdot \frac{15\pi - 8}{3\pi - 4}, \\ \text{vol. of solid of revol. } \bar{x} = \frac{r}{3} \cdot \frac{9\pi^2 - 32}{9\pi^2 - 16} \text{ (from base).} \end{cases}$$

MACHINES.

Pulley; $W_1 = nP$; $W_2 = 2^n P$; $W_3 = (2^n - 1) P$;
and, if weight of each pulley be w,

$$W_1 = n(P - W); \quad W_2 = 2^n . P - (2^n - 1) w;$$
$$W_3 = (2^n - 1) . P + (2^n - 1 - n) w.$$

Inclined plane, without friction; $P = W . \frac{\sin i}{\cos a}$,

.... with friction; $P = W . \frac{\sin (i \pm e)}{\cos (a \mp e)}$.

Screw, without friction; $P = W . \frac{h}{2\pi R}$,

.... with friction; $P = W . \frac{h \pm \mu r}{r \mp \mu h} \cdot \frac{r}{R}$.

DYNAMICS.

$$\text{Impact, direct; } \begin{cases} V_a' = V_a - \frac{b(1+e).(V_a - V_b)}{a+b}, \\ V_b' = V_b + \frac{a(1-e).(V_a - V_b)}{a+b}. \end{cases}$$

Impact, oblique;

$$\begin{cases} V_a'.\cos a' = V_a \cos a - \dfrac{b(1-e).(V_a.\cos a - V_b \cos \beta)}{a+b}, \\ V_a' \sin a' = V_a.\sin a. \end{cases}$$

Uniform acceleration: $s = \frac{1}{2} ft^2 = \frac{v^2}{2f} = \frac{1}{2} tv,$

$$t = \frac{v}{f} = \frac{2s}{v} = \sqrt{\frac{2s}{f}},$$

$$v = ft = \frac{2s}{t} = \sqrt{2fs},$$

$$f = \frac{v}{t} = \frac{v^2}{2s} = \frac{2s}{t^2}.$$

If the body be projected, with velocity V, in line of action of f,

$$s = Vt \pm \frac{1}{2} ft^2; \quad V_1^2 = V^2 \pm 2fs.$$

Variable acceleration: $f = \frac{dv}{dt}$; $v = \frac{ds}{dt}$; $f = \frac{d^2 s}{dt^2}$.

Projectiles: $y = x \tan a - \frac{gx^2}{2V^2.\cos^2 a}$;

$$t = \frac{2.V.\sin(a-i)}{g.\cos i}; \quad r = \frac{2V^2.\sin(a-i)\cos a}{g.\cos^2 i};$$

$$\sin(2a-i) = \frac{gr}{V^2} \cos^2 i + \sin i.$$

Simple pendulum: $t = \pi.\sqrt{\frac{l}{g}}$;

Conical pendulum: $t = 2\pi \sqrt{\frac{h}{g}}$.

Pressure on a curve $= \left(\frac{v^2}{\rho} \pm g \cos \theta\right).m.$

MOMENT OF INERTIA.

$$MK^2 = \int x^2 . \frac{dM}{dx} . dx.$$

For an arc of a plane curve; $\frac{dM}{dx} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{1}{2}}$.

For an area of a plane curve; $\frac{dM}{dx} = y$.

For a surface of revolution; $\frac{dM}{dx} = 2\pi y . \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{1}{2}}$.

For a solid of revolution; $\frac{dM}{dx} = \pi y^2$.

For any solid; $\frac{dM}{dx} = \iint dy . dz$.

CENTRE OF OSCILLATION.

When moment of inertia about axis of suspension $= MK^2$, and distance of centre of gravity from the same axis $= h$,

$$\text{length of simple pendulum} = \frac{K^2}{h}.$$

SPECIFIC GRAVITIES.

Cork	·240	Zinc	7·190
Larch	·522	Tin	7·291
Elm	·588	Iron (cast)	7·207
Teak	·657	Iron (wrought)	7·778
Fir	·753	Steel	7·816
Beech	·852	Brass	7·824
Oak	·934	Copper (cast)	8·788
Lignum Vitæ	1·334	Copper (wire)	8·878
Coal	1·270	Silver	10·474
Sand	1·886	Lead	11·352
Clay	1·919	Mercury	13·568
Brick	2·000	Gold (standard)	17·486
Portland Stone	2·145	Gold (pure)	19·258
Chalk	2·315	Platinum (pure)	19·500
Granite	2·651	Platinum (laminated)	21·041

One cubic foot of brickwork weighs 1 cwt.

One linear foot of rope weighs in lbs. ·045 × (circumf.)^2 in inches.

Proof strength of rope in cwt. = 2 × (circumf.)^2 in inches.

	Elasticity.	Tenacity.	Resistance.		Expansion.
	Elongation or compression in Millionths of length by 1 lb. per sq. inch.	Breaking weight in lbs. per sq. inch of section.	Crushing pressure in lbs. per sq. inch.		Expansion in Millionths of length for 1° of Fahrenheit.
Elm	1·430	13500	10300	Mercury in vol.	104·15
Larch	·950	10000	5500		
Fir	·752	12000	6000		
Oak	·690	17300	9500		
Brass	·112	18000	10300		10·52
Copper	·059	60000			9·44
Iron (cast)	·059	16500	112000		6·17
Iron (wrought)	·035	67200	3600		6·42
Steel	·035	120000			6·36
Lead	1·390	3300			15·92
Glass	·125	9400			4·05

THERMOMETER SCALES.

	Freezing point of water.	Boiling point of water.
Fahrenheit	32	212
Centigrade	0	100
Reaumur	0	80

$$\therefore \begin{cases} C = \frac{5}{9}(F - 32), \\ R = \frac{4}{9}(F - 32). \end{cases}$$

HYDROSTATICS.

Normal pressure on a surface = area × depth of centre of gravity.

Depth of centre of pressure $= X$; then:

$$\text{for a line; } X = \frac{\int x^2 \frac{ds}{dx} \,.\, dx}{\int x \frac{ds}{dx} \,.\, dx};$$

$$\text{for a plane area; } X = \frac{\int x^2 y dx}{\int y dx}.$$

If H be the horizontal pressure upon the vertical projection of a given surface immersed in a fluid, and W be the weight of the superincumbent fluid, the

$$\text{Resultant pressure upon the surface} = \sqrt{(H^2 + W^2)};$$

$$\text{and its inclination to the horizon} = \tan^{-1} \frac{W}{H}.$$

Principle of Archimedes: "If a body be wholly or partially immersed in a fluid it will be pressed upwards with a force which is equal to the weight of the fluid which it has displaced."

Also, in the case of floatation: The whole volume of the body will be to that of the portion immersed as the specific gravity of the liquid to that of the body.

Boyle's Law: "The elastic force of a given mass of gas at a constant temperature varies inversely as the space it occupies."

Dalton's Law: "The change in volume of a given mass of gas under a constant pressure varies directly as the number of degrees of its temperature above 32° of Fahrenheit." The change in volume from 32° to 212° is $V' - V = \cdot 366\, V$.

If t and t' be the temperatures, p and p' the pressures, and V and V' the volumes of the same mass of gas;

$$V' = \frac{460 + t'}{460 + t} \times \frac{p}{p'} \,.\, V.$$

HEIGHTS OF MOUNTAINS.

Height, in feet, by Barometer $= H = 60000 \log_{10} \frac{h}{h'}$ in inches of mercury reduced to 32°; the temperature of the air being also supposed to be 32°. More accurately

$$H = 67 \,.\, (836 + t + t') \,.\, \log_{10} \frac{h}{\{1 + \cdot 0001\,(\tau - \tau')\}\, h'},$$

t and t' being the temperatures of the air at the two stations; τ and τ' the temperatures of the columns of mercury h and h'.

AIR PUMP.

$$\text{Elastic force after } n \text{ strokes} = \left(\frac{R}{R + C}\right)^n;$$

R being the volume of the receiver and connecting tubes, and C the volume of one cylinder minus the volume of the piston.

In Diving Bell:

$$\text{Internal pressure} = \frac{V}{V'} \text{ atmospheres,}$$

V and V' being the volumes of air in bell before and after immersion, or:

$$\text{Internal pressure} = \left(1 + \frac{x}{33}\right) \text{ atmospheres,}$$

x being the depth of water from surface to surface.

APPENDICES.

APPENDIX I.

ON SYNTHETIC DIVISION.

THE process of Synthetic Division is derived from the ordinary operation of algebraic division with detached coefficients, by a careful rejection of all redundant details. The operation of subtraction of the successive partial products is converted into addition by changing the signs of all the terms of the divisor, except the first (which is made unity), and the several coefficients of the quotient are evolved as the sums of the successive columns of coefficients of the partial products which, in the ordinary process, produce, by subtraction, the first terms of the successive remainders.

Thus, supposing it were required to divide

$$3x^5 - 4x^4 + 7x^3 + 5x^2 - 2x + 10 \text{ by } 2x^3 - 4x^2 - 3x + 2,$$

making the first coefficient of the divisor unity and changing the signs of all the terms except the first, we have, by the ordinary process,

$$\begin{array}{r|cccccc|l}
1+2+\frac{3}{2}-1 & 3 & -4 & +7 & +5 & -2 & +10 & 3+2+\frac{31}{2}\ldots \\
 & 3 & +6 & +\frac{9}{2} & -3 & & & \\
\cline{2-5}
 & & 2 & +\frac{23}{2} & +2 & -2 & & \\
 & & 2 & +4 & +3 & -2 & & \\
\cline{3-7}
 & & & \frac{31}{2} & +5 & -4 & +10 & \\
 & & & \frac{31}{2} & +31 & +\frac{93}{4} & -\frac{31}{2} & \\
\cline{4-7}
 & & & & 36 & -\frac{77}{4} & -\frac{11}{2} &
\end{array}$$

and dividing the terms of the quotient by 2, to compensate for the corresponding division of the original divisor, we have, for the coefficients of the quotient, $\frac{3}{2}+1+\frac{31}{4}$; and for those of the remainder (which are unaffected by the change in the divisor)

$$36-\frac{77}{4}-\frac{11}{2}.$$

By this example it is seen that the coefficients of the terms of the modified quotient are identical with those of the first terms of the successive remainders (converted into sums by the change of the signs of the divisor), and that they can readily be obtained without the intervention of the subordinate summations: thus the coefficient $\frac{31}{2}$ can be obtained from its components $7+\frac{9}{2}+4$ without the intervention of the $\frac{23}{2}$, and so of all others, to whatever extent we choose to carry the division.

To put the matter in a general form let the coefficients of the divisor be represented by $1+b+c$, and those of the dividend by $A+B+C+D+E+F+G$; we may then symbolize the operations thus,

Form 1.

$$\begin{array}{llllllll}
1+b+c)\,A & +B & +C & +D & +E & +F & +G & (A+B_{\prime}+C_{\prime\prime}+D_{\prime\prime}+E_{\prime\prime} \\
\quad\;\; A & +Ab & +Ac & & & & & \\
\cline{1-3}
 & B_{\prime} & +C_{\prime} & +D & & & & \\
 & B_{\prime} & +B_{\prime}b & +B_{\prime}c & & & & \\
\cline{2-4}
 & & C_{\prime\prime} & +D_{\prime} & +E & & & \\
 & & C_{\prime\prime} & +C_{\prime\prime}b & +C_{\prime\prime}c & & & \\
\cline{3-5}
 & & & D_{\prime\prime} & +E_{\prime} & +F & & \\
 & & & D_{\prime\prime} & +D_{\prime\prime}b & +D_{\prime\prime}c & & \\
\cline{4-6}
 & & & & E_{\prime\prime} & +F_{\prime} & +G & \\
 & & & & E_{\prime\prime} & +E_{\prime\prime}b & +E_{\prime\prime}c & \\
\cline{5-7}
 & & & & & F & +G &
\end{array}$$

using $B_{\prime}$, $C_{\prime\prime}$, &c. to express the sums of the terms immediately above them; so that

$$\begin{aligned} B_{\prime} &= B + Ab \\ C_{\prime\prime} &= C + Ac + B_{\prime}b \\ D_{\prime\prime} &= D + B_{\prime}c + C_{\prime\prime}b \\ E_{\prime\prime} &= E + C_{\prime\prime}c + D_{\prime\prime}b \end{aligned}$$

And the operation might be arranged, more concisely, thus:

Form 2.

$$\begin{array}{lccccccc} 1+b+c) & A+ & B+ & C+ & D+ & E+ & F+ & G \\ & A & +Ab & +Ac & & & & \\ & & B_{\prime} & +B_{\prime}b & +B_{\prime}c & & & \\ & & & C_{\prime\prime} & +C_{\prime\prime}b & +C_{\prime\prime}c & & \\ & & & & D_{\prime\prime} & +D_{\prime\prime}b & +D_{\prime\prime}c & \\ & & & & & E_{\prime\prime} & +E_{\prime\prime}b & +E_{\prime\prime}c \\ & & & & & & +F_{\prime\prime} & +G_{\prime} \end{array}$$

where each line, except the last, is obtained by multiplying its first term into the several terms of the divisor, those successive first terms being identical with the coefficients of the quotient. It thus becomes unnecessary to take the trouble to again write these coefficients of the quotient in their ordinary position on the right of the dividend.

If now the successive lines be written diagonally instead of horizontally, and the divisor be written vertically downwards, the process will be precisely the same as before; but the arrangement will be very much more convenient, and the quotient will be exhibited in a more intelligible form, thus:

Form 3.

$$\begin{array}{r|ccccccc} 1 & A & +B & +C & +D & +E & +F & +G \\ +b & & +Ab & +B_{\prime}b & +C_{\prime\prime}b & +D_{\prime\prime}b & +E_{\prime\prime}b & \\ +c & & & +Ac & +B_{\prime}c & +C_{\prime\prime}c & +D_{\prime\prime}c & +E_{\prime\prime}c \\ \hline & A & +B_{\prime} & +C_{\prime\prime} & +D_{\prime\prime} & +E_{\prime\prime} & +F_{\prime\prime} & +G_{\prime} \end{array}$$

Comparing this with Form 2, it is seen that it differs from that process merely in the arrangement of the terms; and by comparison with Form 1, we see that, the divisor being of the n^{th} degree, the terms of the lowest line of Form 3, *except the last* $(n-1)$, will be the coefficients of the quotient; the last $n-1$ being the coefficients of the remainder left when the process is stopped at the usual point: viz. when a continuation of the operation would involve negative indices in the quotient, supposing the indices of the dividend and divisor to descend in order from m to 0 and from n to 0, m being greater than n. This remainder is called, for distinction, the "*final remainder.*"

It will be observed that the remainder at any point of the division beyond this, is obtained in precisely the same manner, and that, in all cases, its terms will involve the same indices as the terms of the dividend immediately over them; which is not the case with the terms of the quotient, the indices of which are derived in succession from that of the first term, in the usual way, or by subtracting n from the exponent of the corresponding term of the dividend immediately above.

This is the operation of "Synthetic Division," so called from the synthetic formation of the successive coefficients. If we apply it to a few numerical examples it will help to familiarize the process.

Taking, first, the example above worked out by the ordinary method, we have:

$$\begin{array}{r|rrrrrr}
1 & 3 & -4 & +7 & +5 & -2 & +10 \\
+2 & & +6 & +4 & +31 & & \\
+\frac{3}{2} & & & +\frac{9}{2} & +3 & +\frac{93}{4} & \\
-1 & & & & -3 & -2 & -\frac{31}{2} \\
\hline
 & 3 & +2 & +\frac{31}{2} & +36 & -\frac{77}{4} & -\frac{11}{2}
\end{array}$$

And dividing the terms of the quotient by 2, the coefficient of the first term of the original divisor, we have for quotient

$$\frac{3}{2}x^2 + x + \frac{31}{4};$$

and for final remainder $36 - \frac{77}{4}x - \frac{11}{2}$.

Let it be required to divide

$$x^9 - 3x^8 - 31x^7 + 25x^6 + 3x^5 - 15x^4 - 8x^3 + 19x^2 + 3x + 10$$

by

$$3x^4 - 21x^3 + 9x - 6.$$

$$\begin{array}{r|l}
1 & 1 - 3 - 31 + 25 + 3 - 15 - 8 + 19 + 3 + 10 \\
+7 & \quad + 7 + 28 - 21 + 7 \pm 0 + 14 \\
\pm 0 & \qquad \pm 0 \pm 0 \pm 0 \pm 0 \pm 0 \pm 0 \\
-3 & \qquad\quad - 3 - 12 + 9 - 3 \pm 0 - 6 \\
+2 & \qquad\qquad + 2 + 8 - 6 + 2 \pm 0 + 4 \\
\hline
 & 1 + 4 - 3 + 1 \pm 0 + 2 - 3 + 21 - 3 + 14
\end{array}$$

So that the quotient is

$$\frac{1}{3}x^5 + \frac{4}{3}x^4 - x^3 + \frac{1}{3}x^2 + 2,$$

and the final remainder

$$-3x^3 + 21x^2 - 3x + 14.$$

Supposing it were required to continue the division, it is only necessary to carry on the same process, thus:

$$\begin{array}{r|l}
1 & \ldots - 8 + 19 + 3 + 10 \\
+7 & \ldots + 14 - 21 \pm 0 - 21 + 14 + 56 \\
\pm 0 & \ldots \pm 0 \pm 0 \pm 0 \pm 0 \pm 0 \pm 0 \pm 0 \\
-3 & \ldots - 3 \pm 0 - 6 + 9 \pm 0 + 9 - 6 - 24 \\
+2 & \ldots - 6 + 2 \pm 0 + 4 - 6 \pm 0 - 6 + 4 + 16 \\
\hline
 & \ldots - 3 \pm 0 - 3 + 2 + 8 + 65 - 12 - 20 + 16
\end{array}$$

And so on, to any extent, the remaining terms of the quotient, thus far, being

$$-x^{-1}-x^{-2}+\frac{2}{3}x^{-4}+\frac{8}{3}x^{-5}.$$

And the remainder at this point,

$$65x^{-2}-12x^{-3}-20x^{-4}+16x^{-5}.$$

There is one modification of this process which is of very great importance from its adaptation to the operation of the extraction of the roots of numerical equations, of which we shall have to speak. It consists in placing (*in the exceptional case of division by a binomial*) the second term of the binomial on the right-hand of the dividend in the place that is usually occupied by the quotient. Thus, supposing it were required to divide

$$x^5-7x^3-4x^2+5x-2 \text{ by } x-3,$$

the work would be arranged thus:

$$\begin{array}{l}
1+0-7-4+\ \ 5-\ \ 2\ (3 \\
\ \ \ +3+9+6+\ \ 6+33 \\
\hline
1+3+2+2+11+31
\end{array}$$

The quotient being $x^4+3x^3+2x^2+2x+11$, and the final remainder $+31$.

A little practice upon the examples given in the body of the work will accustom the student to this elegant and most useful operation.

APPENDIX II.

THEORY AND NUMERICAL SOLUTION OF EQUATIONS OF ALL DEGREES BY HORNER'S METHOD.

PROP. I.

IF a polynomial in x be divided by $x-r$, the final remainder will be the same as the original polynomial, only having r in the place of x.

Let $$ax^6+bx^5+cx^4+dx^3+ex^2+fx+g,$$

be the given polynomial: then, dividing $x-r$, we have

$$\begin{array}{lllllll}
a+b & +c & +d & +e & +f & +g & (+r \\
+ar & +(ar^2+br) & +(ar^3+br^2+cr) & +(ar^4\ldots+dr) & +(ar^5\ldots+er) & +(ar^6+\ldots+fr) & \\
\hline
a+(ar+b) & +(ar^2+br+c) & +(ar^3+br^2+cr+d) & +(ar^4\ldots\ldots+e) & +(ar^5\ldots+f) & +(ar^6+\ldots+g) &
\end{array}$$

The final remainder is therefore,

$$ar^6+br^5+cr^4+dr^3+er^2+fr+g,$$

which is the same as the original dividend, only having r in the place of x.

Cor. It follows from this that, r being any number, the final remainder arising from the division of a polynomial in x, having numerical coefficients, by $x-r$, will be the numerical value of the polynomial when the number r is substituted for x.

EXAMPLES.

1. What are the respective values of

$$x^8-6x^7+9x^5+2x^4-4x^3+8x^2-2x-5,$$

when $x=3$, $x=5$, and $x=7$?

Ans. -4259; -49065; $+978609$.

2. Find the numerical values of

$$y^5 - 3y^4 + 5y^2 + 2y - 18,$$

when $y = 2$; $y = 4$; $y = 5$; $y = \cdot 5$; and $y = -\cdot 3$.

Ans. -10; $+326$; $+1367$; $-15\cdot 90625$; $-18\cdot 17673$.

Every general equation having real coefficients is supposed to be reduced to the form

$$ax^n + bx^{n-1} + cx^{n-2} + dx^{n-3} \ldots\ldots + kx^2 + lx + m = 0.$$

Def. A "Root" of an equation is any quantity which, being substituted for the unknown quantity in it, verifies the equation.

Prop. II.

If r be a root of the equation $X = 0$; then X will be divisible by $x - r$.

For if X be divided by $(x - r)$ the final remainder is, by Prop. I, the numerical value of X when, in it, r is substituted for x; but when r is a root of the equation $X = 0$, the result of this substitution must be 0; and therefore the final remainder must be 0 when the equation is divided by $x - r$.

Cor. 1. The converse of the proposition will also be true: viz. if an equation be divisible by $x - r$, r is a root of that equation; for the final remainder being 0 shows that the substitution of r for x verifies the equation; and therefore r is a root.

Cor. 2. If the quotient from the above division be divisible by $x - r_1$, then r_1 will also be a root of the original equation; for that must also be divisible by $x - r_1$, and therefore by Cor. 1 r_1 must be a root.

Cor. 3. The quotient arising from the division of the equation

$$ax^6 + bx^5 + cx^4 + dx^3 + ex^2 + fx + g = 0 \ldots\ldots\ldots\ldots (1)$$

by $x - r_1$ is

$$ax^5 + (ar + b)\,x^4 + (ar^2 + br + c)\,x^3 + (ar^3 + br^2 + cr + d)\,x^2$$
$$+ (ar^4 + br^3 + cr^2 + dr + e)\,x + (ar^5 + br^4 + cr^3 + dr^2 + er + f),$$

and this when put equal to 0, will, according to the last corollary, contain the remaining roots of the equation. If therefore we suppose that the original equation (1) have n roots each equal to r, $n-1$ of them will also be roots of this new or "reduced" equation, and consequently if, in it, we substitute x for r, we shall still retain r as the value of $n-1$ of the roots of the resulting equation: making this substitution we have

$$\left.\begin{array}{l} ax^5 \\ +ax^5+bx^4 \\ +ax^5+bx^4+cx^3 \\ +ax^5+bx^4+cx^3+dx^2 \\ +ax^5+bx^4+cx^3+dx^2+ex \\ +ax^5+bx^4+cx^3+dx^2+ex+f \end{array}\right\} = 0,$$

or $$6ax^5+5bx^4+4cx^3+3dx^2+2ex+f=0 \ldots\ldots\ldots\ldots (2)$$

If therefore the original equation (1) have n roots each equal to r, this last (which is called the "derived equation") will have $n-1$ roots each equal to r; and therefore it and the original equation (1) will have a common measure $(x-r)^{n-1}$, since they must both be divisible by that quantity.

Now a little consideration will show that this derived equation is immediately formed from the original by multiplying each term by the exponent of x in that term and diminishing the exponent by unity; we have therefore a simple method of determining the equal roots of an equation when any number of them are each equal to the same value r.

The same investigation will hold true if, besides the n roots each equal to r, there be p roots each equal to s and q roots each equal to t; for we shall arrive at the derived equation whether we suppose the original to have been divided by $x-r$, by $x-s$, or by $x-t$, and hence the derived equation must be divisible by $(x-r)^{n-1}$, by $(x-s)^{p-1}$, and by $(x-t)^{q-1}$; consequently it and the original must have a common measure $(x-r)^{n-1}.(x-s)^{p-1}.(x-t)^{q-1}$.

Cor. 4. It is obvious from Cor. 2, that any equation whose roots are r, $r_{,}$, $r_{,,}$, $r_{,,,}$ &c. may be represented under the form

$$(x-r)(x-r_{,})(x-r_{,,})(x-r_{,,,})\ldots\ldots(x-r_{n-1})=0\,;$$

and, by successively performing the multiplication indicated, and arranging the coefficients vertically, we have :

$$\begin{array}{r|l}
x^2 - r & x + rr_{,} \\
- r_{,} &
\end{array}$$

$$\begin{array}{r|r|l}
x^3 - r & x^2 + rr_{,} & x - rr_{,}r_{,,} \\
- r_{,} & + rr_{,,} & \\
- r_{,,} & + r_{,}r_{,,} &
\end{array}$$

$$\begin{array}{r|r|r|l}
x^4 - r & x^3 + rr_{,} & x^2 - rr_{,}r_{,,} & x + rr_{,}r_{,,}r_{,,,} \\
- r_{,} & + rr_{,,} & - rr_{,}r_{,,,} & \\
- r_{,,} & + r_{,}r_{,,} & - rr_{,,}r_{,,,} & \\
- r_{,,,} & + rr_{,,,} & - r_{,}r_{,,}r_{,,,} & \\
 & + r_{,}r_{,,,} & & \\
 & + r_{,,}r_{,,,} & &
\end{array}$$

Without carrying the process further, it will be obvious that the coefficients in the result of the continuous product will be as follows :

The coefficient of the first term being unity,

The coefficient of the *second* term is the sum of all the roots, with their signs changed.

The coefficient of the *third* term is the sum of the products of the roots, taken *two* and *two*, with their signs changed.

The coefficient of the *fourth* term is the sum of the products of the roots, taken *three* and *three*, with signs changed, and so on.

The coefficient of x^0, or the *last* term, being the product of all the roots with their signs changed.

Cor. 5. Every equation has the same number of roots as there are units in the highest exponent of the unknown quantity in it.

PROP. III.

If the signs of the alternate terms in any equation be changed, the signs of all the roots will be changed.

For if, in any equation, we substitute $-x$ for x, all the terms involving odd powers of x will be changed; while all the terms involving even powers of x will retain their original signs. Consequently the effect of such substitution will be to change the alternate signs of the equation; but the substitution of $-x$ for x is equivalent to changing the signs of all the roots. Hence, if an equation have its alternate signs changed, the signs of all the roots will be changed.

PROP. IV.

Every equation, all whose roots are real, that is, which are not of the form $a \pm \beta \sqrt{-1}$, has the same number of positive roots as there are variations of sign, and the same number of negative roots as there are permanences of sign in passing successively from term to term of the equation.

It is evident that the number of variations together with the number of permanences make up one less than the number of terms, and are therefore equal to the number of roots.

Now, supposing the series of signs in any equation to be

$$+ - - + - + + + - +$$

if we introduce a new positive root, by multiplying by $x - r$, we shall have

$$\begin{array}{ccccccccccc}
+ & - & - & + & - & + & + & + & - & + & \\
 & - & + & + & - & + & - & - & - & + & - \\
\hline
+ & - & \pm & + & - & + & \pm & \pm & - & + & - \\
\hline
\end{array}$$

in which result the doubtful signs will depend upon the relative magnitude of the contiguous coefficients of the original equation. But we can see that the number of these doubtful signs will be the same as the number of permanences in the original, and that they will moreover correspond to them; consequently these signs can never tend to diminish the number of variations.

It is also evident that the additional sign introduced by the multiplication must be different from the last of the original series and must therefore give an additional variation at the end of the result. Each positive root therefore introduces at least one variation which did not previously exist in the equation.

Similarly, taking the same series of signs and performing the multiplication by $x+r$, which is equivalent to introducing a new negative root, we have

$$\begin{array}{ccccccccccc}
+ & - & - & + & - & + & + & + & - & + & \\
 & + & - & - & + & - & + & + & + & - & + \\
\hline
+ & \pm & - & \pm & \pm & \pm & + & + & \pm & \pm & + \\
\hline
\end{array}$$

Here the doubtful signs are the same in number and correspond to the *variations* of the original, and they cannot therefore tend to diminish the number of permanences; and the additional sign at the end of the series must have the effect of introducing a new permanence. Consequently this result must contain at least one more permanence than the original.

We therefore find that each positive root introduces at least one variation, and each negative root at least one permanence; but the number of variations and the number of permanences together, being equal to the number of roots, it follows that each positive root will introduce one, and only one, variation, and each negative root one, and only one, permanence; that is, there must be the same number of positive roots as there are variations of sign, and the same number of negative roots as there are permanences of sign in the successive terms of the equation.

DEGUA'S CRITERION FOR IMAGINARY ROOTS.

COR. It follows from the above proposition, that *if any coefficient of an equation be* $=0$ *and the sign of the preceding* *term be the same as the sign of that which follows it, the equation must have two roots which (under the condition of all the roots being real) would be both positive and negative, and which must therefore be imaginary.*

PROP. V.

Imaginary roots enter an equation in pairs; that is, if a root of an equation be $\alpha+\beta\sqrt{-1}$, then $\alpha-\beta\sqrt{-1}$ must also be a root of that equation.

Let $$ax^3+bx^2+cx+d=0,$$

be an equation one root of which is $\alpha+\beta\sqrt{-1}$, then, dividing by $x-(\alpha+\beta\sqrt{-1})$ we have

$$\begin{array}{l}
a+b \qquad\qquad +c \qquad\qquad\qquad\qquad +d \qquad\qquad\qquad (\alpha+\beta\sqrt{-1}\\
\underline{\;\; a(\alpha+\beta\sqrt{-1})+(\alpha'+\beta'\sqrt{-1})(\alpha+\beta\sqrt{-1})+(\alpha''+\beta''\sqrt{-1})(\alpha+\beta\sqrt{-1})}\;;\\
a+(\alpha'+\beta'\sqrt{-1})+(\alpha''+\beta''\sqrt{-1}) \qquad\qquad +(\alpha'''+\beta'''\sqrt{-1})
\end{array}$$

but, since $\alpha+\beta\sqrt{-1}$ is a root of the equation, we must have

$$\alpha'''+\beta'''\sqrt{-1}=0,$$

which can only be the case when $\alpha'''=0$ and $\beta'''=0$.

If now we proceed to divide by $x-(\alpha-\beta\sqrt{-1})$ we shall have

$$\begin{array}{l}
a+b \qquad\qquad +c \qquad\qquad\qquad\qquad +d \qquad\qquad\qquad (\alpha-\beta\sqrt{-1}\\
\underline{\;\; a(\alpha-\beta\sqrt{-1})+(\alpha'-\beta'\sqrt{-1})(\alpha-\beta\sqrt{-1})+(\alpha''-\beta''\sqrt{-1})(\alpha-\beta\sqrt{-1})}\\
a+(\alpha'-\beta'\sqrt{-1})+(\alpha''-\beta''\sqrt{-1}) \qquad\qquad +(\alpha'''-\beta'''\sqrt{-1})
\end{array}$$

in which it will be observed that α', α'', α''', β', β'', β''' are precisely the same as in the preceding divisor; but it has there been shown that $\alpha'''=0$ and $\beta'''=0$; hence $\alpha'''-\beta'''\sqrt{-1}=0$, and therefore the equation is divisible by $x-(\alpha-\beta\sqrt{-1})$; and consequently, if $\alpha+\beta\sqrt{-1}$ be a root, $\alpha-\beta\sqrt{-1}$ will also be a root.

EXAMPLES.

1. Determine whether 8, 6, 4 or 2 are roots of the equation

$$x^4-19x^3+128x^2-356x+336=0.$$

Ans. 2, 4 and 6 are roots.

2. Two roots of the equation

$$x^5 - 2x^4 - 67x^3 + 200x^2 + 588x - 1440 = 0,$$

are 2 and 6; determine the equation containing the remaining roots. Ans. $x^3 + 6x^2 - 31x - 120 = 0$.

3. One root of the equation

$$x^4 - 19x^3 + 132x^2 - 302x + 56 = 0,$$

is 4; what is the equation containing the remaining roots?

Ans. $x^3 - 15x^2 + 72x - 14 = 0$.

4. Two roots of the equation

$$x^4 - 16x^3 + 86x^2 - 176x + 105 = 0,$$

are 1 and 5; what are the other roots? Ans. 3 and 7.

5. Has the equation $x^3 - 2x^2 - 15x + 36 = 0$ any equal roots?

Ans. 2, each equal to 3.

6. Determine the equal roots of the equation

$$x^6 + 3x^5 - 6x^4 - 6x^3 + 9x^2 + 3x - 4 = 0,$$

and thence the other roots. Ans. 1, 1, 1, −1, −1, −4.

7. Determine the equal roots of the equation

$$x^4 - 6x^3 + 12x^2 - 10x + 3 = 0.$$

Ans. 1, 1, 1.

8. By the application of the law of the coefficients, form the four equations whose roots are respectively (1, 2 and 3); (1, 2 −3 and 4); (2, 3, −4 and 5) and (3 and $2 \pm 3\sqrt{1}$).

(1) $x^3 - 6x^2 + 11x - 6 = 0$; (3) $x^4 - 6x^3 + 9x^2 - 94x - 120 = 0$;

(2) $x^4 - 4x^3 - 7x^2 + 34x - 24 = 0$; (4) $x^3 - 7x^2 + 25x - 39 = 0$.

9. One root of the equation

$$x^5 - 12x^4 + 67x^3 - 212x^2 + 366x - 260 = 0,$$

is $2 + 3\sqrt{-1}$; determine the equation containing the remaining roots; and show, by division, that one of them is $2 - 3\sqrt{-1}$.

Prop. VI.

If the coefficients of an equation be taken in inverted order, the roots of the new equation will be the reciprocals of those of the original.

Let the given equation be

$$ax^n + bx^{n-1} + cx^{n-2} + \ldots\ldots + kx^2 + lx + m = 0 \ldots\ldots\ldots(1),$$

then the new equation will be

$$my^n + ly^{n-1} + ky^{n-2} \ldots\ldots + cy^2 + by + a = 0 \ldots\ldots\ldots(2),$$

or, dividing throughout by y^n,

$$\frac{a}{y^n} + \frac{b}{y^{n-1}} + \frac{c}{y^{n-2}} \ldots\ldots + \frac{k}{y^2} + \frac{l}{y} + m = 0 \ldots\ldots\ldots (3),$$

and, comparing (3) with (1) we see that $x = \frac{1}{y}$, hence the values of y in (2) are the reciprocals of the values of x in (1).

Cor. 1. If the coefficients of the proposed equation be the same when taken in inverted order as when taken in direct order, it is clear that the new equation will be identical with the original and the roots of the equation and their reciprocals must furnish the same series of numbers; that is, to each root there must be a corresponding root which is its reciprocal and the roots must therefore be of the form:

$$a,\ \frac{1}{a},\ a_1,\ \frac{1}{a_1},\ a_2,\ \frac{1}{a_2},\ a_3,\ \frac{1}{a_3}\ \&c.$$

Equations of the above form are called "reciprocal equations," or "recurring equations."

Cor 2. In a recurring equation of an odd degree one root will be + 1 or − 1 according as the sign of the last term is − or +; for the equation must have one root which is the same as its reciprocal and it must therefore be ± 1; since also the other roots consist of pairs which have the same sign, the last term of the equation, which is the product of all the roots with their signs changed, must have a contrary sign to the root unity.

Cor. 3. Let $ax^n + bx^{n-1} + cx^{n-2} \dots\dots + cx^2 + bx + a = 0 \dots\dots (1)$

be a recurring equation; then dividing by $x^{\frac{n}{2}}$ we have

$$a\left(x^{\frac{n}{2}} + \frac{1}{x^{\frac{n}{2}}}\right) + b\left(x^{\frac{n}{2}-1} + \frac{1}{x^{\frac{n}{2}-1}}\right) + c\left(x^{\frac{n}{2}-2} + \frac{1}{x^{\frac{n}{2}-2}}\right) +$$

$$k + \left(x^2 + \frac{1}{x^2}\right) + l\left(x + \frac{1}{x}\right) + m = 0 \dots\dots\dots (2)$$

if n be even, which can always be made the case, since if n be odd one of the roots is known.

But if $x + \frac{1}{x} = y$, we have

$$y^2 = x^2 + \frac{1}{x^2} + 2;$$

$$y^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right);$$

$$y^4 = x^4 + \frac{1}{x^4} + 4\left(x^2 + \frac{1}{x^2}\right) + 6,$$

and so on.

Hence the values of $x^2 + \frac{1}{x^2}$; $x^3 + \frac{1}{x^3}$ &c. can be determined in terms of

$$y^2;\ y^3; \dots\dots y^{\frac{n}{2}-1};\ y^{\frac{n}{2}},$$

and those being substituted in the equation (2) the resulting equation will be of the form

$$\alpha y^{\frac{n}{2}} + \beta y^{\frac{n}{2}-1} + \gamma y^{\frac{n}{2}-2} \dots\dots + \lambda y^2 + \mu y + \nu = 0,$$

and from the values of y in this we shall obtain the values of x in the original (1) by means of the equation $x + \frac{1}{x} = y$.

Hence, *if n be even*, the solution of the original equation can be made to depend upon that of an equation of the degree $\frac{n}{2}$.

If *n be odd*, one root of the equation is $+1$ or -1, and the solution of the original may therefore be made to depend upon that of an equation of the degreee $\frac{n-1}{2}$, since $n-1$ is even.

PROP. VII.

To transform an equation into another whose roots shall be greater or less by a given quantity, than those of the original.

Let $ax^6 + bx^5 + cx^4 + dx^3 + ex^2 + fx + g = 0$,

be the given equation, it is required to determine the coefficients of the equation

$$a_1(x\pm r)^6 + b_1(x\pm r)^5 + c_1(x\pm r)^4 + d_1(x\pm r)^3 + e_1(x\pm r)^2 + f_1(x\pm r) + g_1 = 0.$$

Since both polynomials are equal to 0, they must be equal to one another; if therefore we divide the original by any quantity we must have a quotient and a remainder the same in value as if we divide the other by that quantity. Consequently, if we divide the original by $x\pm r$ we shall have, for quotient, the value of

$$a_1(x\pm r)^5 + b_1(x\pm r)^4 + c_1(x\pm r)^3 + d_1(x\pm r)^2 + e_1(x\pm r) + f_1,$$

and for remainder the value of g_1.

Again, if we divide this quotient by $x\pm r$ we shall have for a new quotient the value of

$$a_1(x\pm r)^4 + b_1(x\pm r)^3 + c_1(x\pm r)^2 + d_1(x\pm r) + e_1,$$

and for remainder the value of f_1.

Thus, by continually dividing by $x\pm r$ we shall obtain the values of the remaining coefficients e_1, d_1, c_1, b_1 and a_1.

COR. By a process of this kind, the second term of an equation may readily be eliminated, since in the successive divisions the value r will be added or subtracted n times to the coefficient of the second term; and the resulting coefficient of the reduced equation will be 0 if the value of r be taken equal to $-\frac{b}{na}$, b and a being the coefficients of the second and first terms of the original equation. Thus, to eliminate the second term from the equation

$$2x^3-12x^2-7x+2=0 \text{ .... } (1),$$

we have $r=\frac{12}{3\times 2}=2$, and, dividing continuously by $x-2$, we get

$$\begin{array}{rrrrl}
2 & -12 & -\ 7 & +\ 2 & (2 \\
 & +\ 4 & -16 & -46 & \\
\hline
 & -\ 8 & -23 & -44 & \\
 & +\ 4 & -\ 8 & & \\
\cline{2-3}
 & -\ 4 & -31 & & \\
 & +\ 4 & & & \\
\hline
2 & \pm\ 0 & -31 & -44 & \\
\hline
\end{array}$$

So that the new equation $2x_1^2-31x_1-44=0$, will have its roots less by 2 than those of the original (1).

METHOD OF SOLUTION.

If we reduce the roots of an equation by a number which is greater than one, only, of its positive roots, that root will appear as a negative root in the reduced equation, and consequently, assuming all the roots to be real, the number of variations in the reduced equation will be one less than the number in the original.

Again, if we reduce the roots of this equation by a number which, added to the former number, is greater than two, only, of the roots of the original, another root will become negative, and we shall therefore, in this reduced equation, find one variation less than in the last, that is, two less than in the original.

And so on.

We can therefore determine the positions of the positive roots in the scale of numbers by continually diminishing them by unity; for when the roots have been diminished by a number which is greater than any one or more of the positive roots, we shall immediately be made aware of the fact by the loss of a corresponding number of variations in the reduced equation.

If we change the alternate signs of the equation, the negative roots will become positive and the positive negative, we can therefore determine the positions of the negative roots in precisely the same manner as we can those of the positive roots.

If, in any such reduction of the equation, we should lose two or more variations, the roots thus indicated may be either equal, nearly equal, or, an even number of them may be imaginary. If they be nearly equal, we shall obtain their positions by reducing the roots of the equation by a number less than that last employed. If they be equal, they will be determined by the aid of the derived equation. And if they be imaginary, we can have recourse to the equation whose roots are the reciprocals of those of the original: and the following considerations will show how it may be thus employed.

BUDAN'S CRITERION FOR IMAGINARY ROOTS.

If the roots of an equation be reduced by a number ρ which is greater than m of its roots, the reciprocals of those roots will be greater than the reciprocal of that number, or $\frac{1}{\rho}$, and therefore, if those roots be real, the number of variations left when the reciprocal equation is reduced by $\frac{1}{\rho}$ will correspond to the number of variations lost when the original is reduced by ρ: hence if m be the number of variations lost in the latter case and q the number left in the former, there must be $q-m$ roots which are imaginary, since, if real, they must be less than ρ and their reciprocals also less than $\frac{1}{\rho}$, which is absurd.

EXTENSION OF BUDAN'S CRITERION; AND METHOD OF SEPARATION OF NEARLY EQUAL ROOTS.

It does not follow that, when $q-m=0$, (that is, when the number of variations left in the reduced reciprocal equation is the same as the number lost from the original) there are no imaginary roots indicated by the lost variations: but these will seldom fail to be made evident by continuing the reduction of the reduced reciprocal equation until all the variations left disappear, and then applying the same test as before; thus

Let the given equation be $x^5-2x^4-13x^3+39x^2-20x+4=0$.

$1-\ 2-13+39-20+\ 4\ (1$
$\ \ -\ 1-14+25+\ 5+\ 9$
$\ \ \pm\ 0-14+11+16$
$\ \ +\ 1-13-\ 2$
$\ \ +\ 2-11$ Two var. lost $m=2$.
$1+\ 3-11-\ 2+16+\ 9\ (1$
$\ \ +\ 4-\ 7-\ 9+\ 7+16$
$\ \ +\ 5-\ 2-11-\ 4$
$\ \ +\ 6+\ 4-\ 7$
$\ \ +\ 7+11$
$1+\ 8+11-\ 7-\ 4+16\ (1$
$\ \ +\ 9+20+13+\ 9+25$
Two lost
$16-\ 4-\ 7+11+\ 3+\ 1\ (1$
$\ \ +12+\ 5+\quad +\quad +$ None left.

Reciprocal equation.

$4-20+39-13-\ 2+\ 1\ (1$
$\ \ -16+23+10+\ 8+\ 9$
$\ \ -12+11+21+29$
$\ \ -\ 8+\ 3+24$
$\ \ -\ 4-\ 1$ Two var. left $q=2$
$4\pm\ 0-\ 1+24+29+\ 9\ (1$
$\ \ +\ 4+\ 3+\quad +\quad +$ Two lost.

Reciprocal of reduced reciprocal equation.

$9+29+24-\ 1\pm\ 0+\ 4\ (1$
$\ \ +38+62+61+61+65$
None left, $\therefore$ roots imaginary.

Should it prove necessary to apply this extended criterion, it is quite certain that the roots, if real, must be very nearly equal; and this process, if carried far enough, cannot fail to separate them; for suppose two roots r and r_1, each less than 1, to be very nearly equal, then the difference of their reciprocals $\frac{1}{r}-\frac{1}{r_1}$, or

$\frac{r_1 - r}{rr_1}$ is evidently greater than their difference $r_1 - r$; and so it will be also in every successive reciprocal transformation. It will appear moreover that the positions of these corresponding roots in the successive reduced equations will enable us to determine their actual values; for, calling the original equation A, we will suppose that two variations are lost between 3 and 4; then, calling the reciprocal equation B or $\frac{1}{A_3}$, (the subscript 3 indicating that the roots of A have been reduced by 3) we will suppose that the corresponding roots of this reciprocal equation lie between 2 and 3, and taking the equation which has the roots of equation B reduced by 2 and indicating it as before by B_2, its reciprocal equation may be indicated by C or $\frac{1}{B_2}$. Supposing the roots of C to be between 5 and 6, we should have the roots of the original, A, between the values expressed by the two continued fractions

$$3 + \cfrac{1}{2 + \cfrac{1}{5}} \quad \text{and} \quad 3 + \cfrac{1}{2 + \cfrac{1}{6}}$$

that is, between the values $\frac{38}{11}$ and $\frac{45}{13}$, or between 3·45 and 3·46.

By continuing this process we must at length arrive at a stage where one variation is lost between two particular numbers, and the other between two other numbers; and the position of each root can therefore be obtained. Thus, supposing the above operation had been carried a step further, and we found that one of the variations in C disappeared between 2 and 3, and the other between 4 and 5, we should then have the first root limited by the fractions

$$3 + \cfrac{1}{2 + \cfrac{1}{5 + \cfrac{1}{2}}}, \quad \text{and} \quad 3 + \cfrac{1}{2 + \cfrac{1}{5 + \cfrac{1}{3}}},$$

and the other by the fractions

$$3+\cfrac{1}{2+\cfrac{1}{5+\cfrac{1}{4}}},\qquad\text{and}\qquad 3+\cfrac{1}{2+\cfrac{1}{5+\cfrac{1}{5}}},$$

that is, one of the roots would lie between 3·458 and 3·457; and the other between 3·4565 and 3·4560. The roots would thus be separated, and we could obtain their actual values, either by continuing the process, or by using the initial figures above obtained, in Horner's process upon the original equation.

Horner's process for obtaining the roots of a numerical equation consists in finding figure by figure a value which, when used in diminishing the roots of the given equation, will tend, more nearly than any other value, to make the last term of the reduced equation equal to zero, without necessitating the loss of the particular variation which is dependent upon the root of which we are in search. It is thus, in fact, a mere extension of the process already pointed out for the determination of the position of the roots. Thus, supposing we had to find one of the roots of the equation $x^3-7x+7=0$. For the determination of the positions of the roots we have:

Positive roots.

$$\begin{array}{ll}
(A) & 1\pm0-7+7\ (1\\
 & \quad +1-6+1\\
 & \quad +2-4\\
(A_1) & 1+3-4+1\ (1\\
 & \quad +4\pm0+1\\
 & \quad +5+5\\
 & 1+6+5+1
\end{array}$$

Two variations lost.

$$\begin{array}{ll}
\left(B\text{ or }\frac{1}{A_1}\right) & 1-4+3+1\ (1\\
 & \quad -3\pm0+1\\
 & \quad -2-2\\
 & 1-1-2+1
\end{array}$$

Two variations left.

Negative root.

$$\begin{array}{l}
1\mp0-\ 7-7\ (3\\
\quad\ 3+\ 9+6\\
\hline
\quad +3+\ 2-1\\
\quad +3+18\\
\hline
\quad +6+20\\
\quad +3\\
\hline
1+9+20-1
\end{array}$$

The root is evidently between -3 and -4.

∴ these roots are not imaginary.

In order to find further figures of the negative root, to which we will first turn our attention, we must employ a figure in the first place of decimals which will diminish the terminal coefficient − 1 as much as possible without changing its sign. A little consideration will show that ·1 is too large a value for this purpose, for, proceeding as before; we have

$$\begin{array}{rrrrl}
1 & +9 & +20 & -1 & (\cdot 1 \\
 & +\cdot 1 & +\cdot 91 & +2\cdot 091 & \\
\hline
 & 9\cdot 1 & +20\cdot 91 & + &
\end{array}$$

and the variation upon which everything depends is gone.

We must therefore in this case use a figure in the second place of decimals, and a few trials made mentally will show that ·04 is the value to be used; thus,

$$\begin{array}{rrrrl}
1 & +9 & +20 & -1 & (\cdot 04 \\
 & +\cdot 04 & +\cdot 3616 & +\cdot 814464 & \\
\hline
 & +9\cdot 04 & +20\cdot 3616 & -\cdot 185536 & \\
 & \cdot 04 & +\cdot 3632 & & \\
\cline{2-3}
 & 9\cdot 08 & +20\cdot 7248 & & \\
 & \cdot 04 & & & \\
\cline{2-2}
 & 9\cdot 12 & & &
\end{array}$$

So that the reduced equation having its roots less by 3·04 than those of the original, is

$$x_1^3 + 9\cdot 12x_1^2 + 20\cdot 7248x_1 - \cdot 185536,$$

and, if this last term had been reduced actually to zero, the value of the root would have been exactly 3·04.

We may continue the same process to any extent, and the next step would be the employment of the digit 8 in the third place of decimals, since the ·185 of the last coefficient divided by the 20·7 of the preceding sum gives ·89 as an approximate multiplier. Using this figure we now get:

$$\begin{array}{rrrl}
1+9{\cdot}12. & +20{\cdot}7248\ldots & -{\cdot}185536\ldots & (\,{\cdot}008\\
{\cdot}008 & {\cdot}073024 & +{\cdot}166382592 & \\
\hline
+9{\cdot}128 & +20{\cdot}797824 & -{\cdot}019153408 & \\
+\ {\cdot}008 & +\ {\cdot}073088 & & \\
\cline{1-2}
+9{\cdot}136 & +20{\cdot}870912 & & \\
{\cdot}008 & & & \\
\hline
9{\cdot}144 & +20{\cdot}870912 & -{\cdot}019153408 &
\end{array}$$

It is obvious that a continuation of this process will encumber us with a large number of decimals which have no influence on the result, unless it be required in a very extended form; we may therefore contract the work by cutting off the same number of figures from each coefficient, allowance being made for the increase in each which would otherwise take place, that is, 3 in the last, 2 in the last but one, and 1 in the last but two, or second coefficient. So that in order to prevent any further increase in the figures of the last term we have only to cut off one figure from the last term but one; 2 from the last but two; 3 from the last but three, and so on, according to the degree of the equation; thus,

$$\begin{array}{|r|r|rl}
1+9{\cdot}1 & 44+20{\cdot}87091 & 2-{\cdot}019153408 & (\,{\cdot}0009\\
 & 823 & +\quad 18791226 & \\
\cline{3-3}
 & 20{\cdot}87914 & -\quad 362182 & \\
 & 823 & & \\
\cline{2-2}
 & 20{\cdot}88737 & &
\end{array}$$

Since the two remaining figures of the second term will be cut off at the next step, the process now becomes one of simple contracted division, and we have

$$\begin{array}{rrl}
20{\cdot}8|8|7|3|7 & -{\cdot}000362182 & (\,{\cdot}00001734\\
 & 208874 & \\
\cline{2-2}
 & 153308 & \\
 & 146211 & \\
\cline{2-2}
 & 7097 & \\
 & 6266 & \\
\cline{2-2}
 & 831 & \\
 & 835 & \\
\cline{2-2}
\end{array}$$

and the complete root is − 3·04891734.

This is the process for the determination of any real root of an equation, and it may be carried out to any required extent: but it is obviously unnecessary to interrupt the continuity of the operation at each successive figure of the root by bringing down into a horizontal line the coefficients of the reduced equation; omitting this the work would stand thus:

```
1 ± 0          −  7               − 7 (3·04891734
  + 3          +  9               + 6
  ───          ────               ═══
  + 3          +  2               − 1······
  + 3          + 18                 814464
  ───          ════                ══════
  + 6          + 20····             185536···
  + 3          +     3616           166382592
  ═══          ─────────           ═════════
  + 9··        + 20·3616             19153408
  +  04        +    ·3632             18791226
   ────        ═════════            ════════
  + 9·04       + 20·7248··             362182
  +  04        +      73024            208874
   ─────       ───────────           ──────
  + 9·08       + 20·797824             153308
  +  04        +      73088            146211
  ══════       ═══════════           ══════
  + 9·12       + 20·87091|2              7097
         8            823|               6266
   ──────      ──────────             ════
  + 9·128      + 20·87914                 831
         8            823                 835
   ──────     ═══════════                ═══
  + 9·136     +20·8|8|7|3|7
         8
  ═══════
 +|9·1|44
```

The successive double lines in the several columns indicate the terminations in those columns, of the successive stages of the operation as the root is evolved figure by figure.

Reverting now to the two positive roots between 1 and 2, we may attempt to separate them either by the use of decimal reductions or by employing the extension of Budan's Criterion, which has already shown them to be probably real and nearly, if not quite, equal. By the first method we have only to look for a change of sign in the last term, since, if the roots be separable, one of the variations must disappear without the other, and this too will be shown by the change of sign above mentioned. Using therefore decimal reductions, we have

$$\begin{array}{ll}
1+3 \quad -4 \quad +1 & (\cdot 1 \\
\quad \cdot 1 + \cdot 31 - \cdot 369 & \\
\hline
+3\cdot 1 - 3\cdot 69 + & \text{too small,} \\
\hline\hline
1+3 \quad -4 \quad +1 & (\cdot 2 \\
+ \cdot 2 + \cdot 64 - \cdot 672 & \\
\hline
+3\cdot 2 - 3\cdot 36 + & \text{too small,} \\
\hline\hline
1+3 \quad -4 \quad +1 & (\cdot 3 \\
+ \cdot 3 + \cdot 99 - \cdot 903 & \\
\hline
+3\cdot 3 - 3\cdot 01 + & \text{too small,} \\
\hline\hline
1+3 \quad -4 \quad +1 & (\cdot 4 \\
+ \cdot 4 + 1\cdot 36 - 1\cdot 056 & \\
\hline
+3\cdot 4 - 2\cdot 64 - & \text{too great, } \therefore \text{ root between } 1\cdot 3 \text{ and } 1\cdot 4, \\
\hline\hline
1+3 \quad -4 \quad +1 & (\cdot 5 \\
+ \cdot 5 + 1\cdot 75 - 1\cdot 125 & \\
\hline
+3\cdot 5 - 2\cdot 25 - & \text{too small,} \\
\hline\hline
1+3 \quad -4 \quad +1 & (\cdot 6 \\
+ \cdot 6 + 2\cdot 16 - 1\cdot 104 & \\
\hline
+3\cdot 6 - 1\cdot 84 - & \text{too small,} \\
\hline\hline
1+3 \quad -4 \quad +1 & (\cdot 7 \\
+ \cdot 7 + 2\cdot 59 - \cdot 987 & \\
\hline
+3\cdot 7 - 1\cdot 41 + & \text{too great, } \therefore \text{ root between } 1\cdot 6 \text{ and } 1\cdot 7. \\
\hline\hline
\end{array}$$

By the method of reciprocals, we have, in continuation of that portion of the process already worked at p. 396:

$(B_1)\ 1-1-2+1\ (1$ One root of this equation less than 1;
$\pm 0-2-1$ ∴ one root of reciprocal equation (B) at p. 396.
$+1-1$ less than 2.
$(B_2)\ 1+2-1-1\ (1$ One root of this equation less than 1;
$+3+2+1$ ∴ one root of reciprocal equation (B) at p. 396.
less than 3.

The roots of the original are therefore separated, and are consequently neither equal nor imaginary; and, in order to find their approximate distinguishing values, we have only to revert to the original equation and, by noting the successive steps of our transformations, find the continued fractions which express the approximate values of the two roots: we thus find one of the roots to be nearly

$$x = 1 + \frac{1}{2}, \text{ and the other nearly, } x = 1 + \frac{1}{3};$$

$$\text{or} \qquad x = 1\cdot5 \text{ and } x = 1\cdot3.$$

These values cannot of course be viewed as more than a somewhat rough approximation to the true values; but if we choose to carry the process a step further we can readily obtain more complete results; thus,

$\left(C \text{ or } \frac{1}{B_1}\right)\ 1-2-1+1\ (1$
$-1-2-1$
$\pm 0-2$
$(C_1)\ 1+1-2-1\ (1$
$+2\pm 0-1$

$\left(D \text{ or } \frac{1}{B_2}\right)\ 1+1-2-1\ (1$
$+2\pm 0-1$

One root of (C) less than 1, and one root between 2 and 3; the first of these refers to the positive root of (B_2) and the latter to that of (B_1).

And the approximate values will now be

$$x = 1 + \cfrac{1}{1 + \cfrac{1}{2}} \quad \text{and} \quad x = 1 + \cfrac{1}{2 + \cfrac{1}{1}},^{*}$$

$$\text{or } x = 1{\cdot}61 \text{ and } x = 1{\cdot}33.$$

The equation (D) is the reciprocal of (B_2), but it happens to be identical with (C_1); hence it appears that from this point the process for both roots gives identical results. It will be found in fact that one of the roots is represented by the continued fraction

$$x = 1 + \frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{20} + \frac{1}{2} + \frac{1}{3} + \frac{1}{1} + \frac{1}{6} + \frac{1}{1} + \frac{1}{2},$$

which gives the value $x = 1{\cdot}692021473$;

And the other by the continued fraction

$$x = 1 + \frac{1}{2} + \frac{1}{1} + \frac{1}{4} + \frac{1}{20} + \frac{1}{2} + \frac{1}{3} + \frac{1}{1} + \frac{1}{6} + \frac{1}{1} + \frac{1}{2},$$

which gives the value $x = 1{\cdot}35689587$.

The sum of these values is $3{\cdot}04891734$, the value of the negative root, as it ought to be, since the coefficient of the second term of the original equation is ± 0.

To obtain these roots by Horner's operation is now quite simple; since we have previously determined the first decimal of each root, we have only to make use of it, and act, afterwards, according to circumstance, as in the determination of the negative root. The operation will stand thus:

1 ± 0
+ 1
+ 1
+ 1
+ 2
+ 1
+ 3·3
+ ·3
+ 3·6
+ ·3
+ 3·95
+ 5
+ 4·00
+ 5
+ 4·056
+ 6
+ 4·062
+ 6
+ 4·0|68

− 7
+ 1
− 6
+ 2
− 4 . .
+ ·99
− 3·01
+ 1·08
− 1·93
+ 1975
− 1·7325
+ 2000
− 1·5325
+ 24336
− 1·508164
+ 24372
− 1·48379 2
+ 325|
− 1·48054|
+ 325|
− 1·4772|9
+ 3|
− 1·4769|
+ 3|
− 1·476|6

+ 7
− 6
+ 1 . . .
903
97 . .
86625
10375 . . .
9048984
1326016
1184429
141587
132922
8665
7382
1283
1181
102
89
13
10
3

(1·356895867

And, for the root between 1·6 and 1·7, thus:

1 ± 0	− 7	+ 7 (1·69202147
+ 1	+ 1	− 6
+ 1	− 6	+ 1 . . .
+ 1	+ 2	1·104
+ 2	− 4 . .	+ ·104 . . .
+ 1	+ 2·16	100809
+ 3·6	− 1·84	3191 . . .
+ 6	+ 252	3156888
+ 4·2	+ 68 . .	34112
+ 6	+ 4401	31772
+ 4·89	+ 11201	2340
+ 9	+ 4482	1589
+ 4·98	+ 15683 . .	751
+ 9	+ 10144	635
+ 5·072	+ 1578444	116
+ 2	+ 10148	111
5·074	+ 15885|92	5
2	+ 1|	
|5·0|76	15886|	
	1|	
	15887|	

For further examples, the student is referred to the body of the work.

CAMBRIDGE: PRINTED AT THE UNIVERSITY PRESS.